Proceedings of ISAP2007, Niigata, Japan

New Electric Field Integral Equations Using the Orthogonal Properties of the Modes for 3-Dimensional Waveguide

Masahiro Tanaka and Kazuo Tanaka
Department of Information Science, Gifu University
1-1 Yanagido, Gifu-shi, Gifu 501-1193, Japan, masahiro@info.gifu-u.ac.jp

1. Introduction

We proposed electric field integral equations (EFIE) which are suitable for a basis theory of a computer aided design (CAD) of a 3-dimensional (3D) single-mode waveguide [1]. In the EFIE, we determined the transmission and reflection coefficients by using the asymptotic expression of Green's function. The EFIE can be numerically solved by the standard method of moment (MoM) directly. However, it is difficult to extend the EFIE method to multimode waveguide.

In this paper, we propose new EFIE which can extend to 3D multimode waveguide. In the new EFIE, we determine the transmission and reflection coefficients by using the orthogonal properties of the modes. In this paper, we consider a 3D single-mode waveguide, in order to simplify the formulation. But it is straightforward to extend the EFIE for a 3D single-mode waveguide to that for a 3D multimode waveguide. Numerical results are finally shown.

2. Waveguide Model

In this paper, we consider a 3D single-mode waveguide as shown in Fig. 1(a). The two multimode waveguides 1 and 2 (regions Ω_1 and Ω_2), whose cross-sections are rectangle $a \times b$, are connected by the junction (region Ω_0), where the junction is an arbitrary shape, even though the junction is an iris in Fig. 1(a). The incident wave comes from the waveguide 2.

Let S_0, S_1 and S_2 denote the side surface of the junction, the waveguide 1 and the waveguide 2, respectively. Note that S_0 has finite in extent, and S_1 and S_2 have infinite in extent. Surfaces S_{10} and S_{20} denote the virtual surface between the junction and the waveguide 1, and between the junction and the waveguide 2, respectively. Surfaces Γ_{10} and Γ_{20} also denote virtual surfaces, which are cross-section surfaces within the junction $S_0 + S_{10} + S_{20}$.

3. New EFIE

For the waveguide as shown in Fig. 1(a), we obtain an EFIE

$$\boldsymbol{E}(\boldsymbol{r}) = \int_S \left\{ j\omega\mu \boldsymbol{J}(\boldsymbol{r}')G(\boldsymbol{r}|\boldsymbol{r}') - \frac{j}{\omega\varepsilon} [\nabla' \cdot \boldsymbol{J}(\boldsymbol{r}')] \nabla' G(\boldsymbol{r}|\boldsymbol{r}') \right\} dS' \tag{1}$$

where

$$G(\boldsymbol{r}|\boldsymbol{r}') = \frac{1}{4\pi} \frac{\exp(-jkn|\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|} \tag{2}$$

$$\boldsymbol{J}(\boldsymbol{r}') = \hat{\boldsymbol{n}} \times \boldsymbol{H}(\boldsymbol{r}') \tag{3}$$

and $\hat{\boldsymbol{n}}$ denotes the unit normal vector to surface.

In order to derive a new EFIE, we first decompose the total field to guided fields and unguided fields in the waveguides 1 and 2. Namely, the total field $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r})$ in the waveguide 1 are decomposed as

$$\boldsymbol{E}(\boldsymbol{r}) = T_{10} \boldsymbol{E}^t_{10}(\boldsymbol{r}) + \boldsymbol{E}^C(\boldsymbol{r}) \tag{4}$$

$$\boldsymbol{H}(\boldsymbol{r}) = T_{10} \boldsymbol{H}^t_{10}(\boldsymbol{r}) + \boldsymbol{H}^C(\boldsymbol{r}) \tag{5}$$

ISBN: 978-4-88552-223-9 C3055©IEICE

and those in the waveguide 2 as

$$\boldsymbol{E}(\boldsymbol{r}) = R_{10}\boldsymbol{E}^r_{10}(\boldsymbol{r}) + \boldsymbol{E}^i(\boldsymbol{r}) + \boldsymbol{E}^C(\boldsymbol{r}) \tag{6}$$

$$\boldsymbol{H}(\boldsymbol{r}) = R_{10}\boldsymbol{H}^r_{10}(\boldsymbol{r}) + \boldsymbol{H}^i(\boldsymbol{r}) + \boldsymbol{H}^C(\boldsymbol{r}) \tag{7}$$

where T_{10} and R_{10} are transmission and reflection coefficients of TE_{10} mode, respectively. $\boldsymbol{E}^t_{10}$ and $\boldsymbol{H}^t_{10}$ are transmitted guided modes in the waveguide 1, $\boldsymbol{E}^r_{10}$ and $\boldsymbol{H}^r_{10}$ are reflected guided modes in the waveguide 2, and $\boldsymbol{E}^i$ and $\boldsymbol{H}^i$ are incident guided fields. $\boldsymbol{E}^C$ and $\boldsymbol{H}^C$ denotes unguided fields in the waveguides 1 and 2, which are the sum of cutoff modes.

Substituting Eqs. (4)-(7) into Eq. (1), we obtain

$$\left.\begin{array}{cc} \boldsymbol{E}(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_0) \\ T_{10}\boldsymbol{E}^t_{10}(\boldsymbol{r}) + \boldsymbol{E}^C(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_1) \\ R_{10}\boldsymbol{E}^r_{10}(\boldsymbol{r}) + \boldsymbol{E}^i(\boldsymbol{r}) + \boldsymbol{E}^C(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_2) \end{array}\right\}$$

$$\begin{aligned} &= \int_{S_0}\left\{j\omega\mu\boldsymbol{J}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}\right]\nabla'G\right\}dS' + \int_{S_1+S_2}\left\{j\omega\mu\boldsymbol{J}^C G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^C\right]\nabla'G\right\}dS' \\ &\quad + T_{10}\int_{S_1}\left\{j\omega\mu\boldsymbol{J}^t_{10}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^t_{10}\right]\nabla'G\right\}dS' + R_{10}\int_{S_2}\left\{j\omega\mu\boldsymbol{J}^r_{10}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^r_{10}\right]\nabla'G\right\}dS' \\ &\quad + \int_{S_2}\left\{j\omega\mu\boldsymbol{J}^i G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^i\right]\nabla'G\right\}dS' \end{aligned} \tag{8}$$

where

$$\boldsymbol{J}^p_{10}(\boldsymbol{r}) = \hat{\boldsymbol{n}}\times\boldsymbol{H}^p_{10}(\boldsymbol{r}), \quad p = t, r \tag{9}$$

$$\boldsymbol{J}^q(\boldsymbol{r}) = \hat{\boldsymbol{n}}\times\boldsymbol{H}^q(\boldsymbol{r}), \quad q = i, C. \tag{10}$$

The transmitted electric field $\boldsymbol{E}^t_{10}$, the reflected electric field $\boldsymbol{E}^r_{10}$ and the incident electric field $\boldsymbol{E}^i$ satisfy

$$\left.\begin{array}{cc} \boldsymbol{E}^t_{10}(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_1) \\ 0 & (\boldsymbol{r} \text{ in } \Omega_0, \Omega_2) \end{array}\right\} = \int_{S_1}\left\{j\omega\mu\boldsymbol{J}^t_{10}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^t_{10}\right]\nabla'G\right\}dS' + \boldsymbol{U}^t_{10}(\boldsymbol{r}) \tag{11}$$

$$\left.\begin{array}{cc} \boldsymbol{E}^r_{10}(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_2) \\ 0 & (\boldsymbol{r} \text{ in } \Omega_0, \Omega_1) \end{array}\right\} = \int_{S_2}\left\{j\omega\mu\boldsymbol{J}^r_{10}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^r_{10}\right]\nabla'G\right\}dS' + \boldsymbol{U}^r_{10}(\boldsymbol{r}) \tag{12}$$

$$\left.\begin{array}{cc} \boldsymbol{E}^i(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_2) \\ 0 & (\boldsymbol{r} \text{ in } \Omega_0, \Omega_1) \end{array}\right\} = \int_{S_2}\left\{j\omega\mu\boldsymbol{J}^i G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^i\right]\nabla'G\right\}dS' + \boldsymbol{U}^i(\boldsymbol{r}) \tag{13}$$

where

$$\boldsymbol{U}^t_{10}(\boldsymbol{r}) = \int_{S_{10}}\left\{j\omega\mu\boldsymbol{J}^t_{10}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^t_{10}\right]\nabla'G + \boldsymbol{M}^t_{10}\times\nabla'G\right\}dS' \tag{14}$$

$$\boldsymbol{U}^r_{10}(\boldsymbol{r}) = \int_{S_{20}}\left\{j\omega\mu\boldsymbol{J}^r_{10}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^r_{10}\right]\nabla'G + \boldsymbol{M}^r_{10}\times\nabla'G\right\}dS' \tag{15}$$

$$\boldsymbol{U}^i(\boldsymbol{r}) = \int_{S_{20}}\left\{j\omega\mu\boldsymbol{J}^i G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^i\right]\nabla'G + \boldsymbol{M}^i\times\nabla'G\right\}dS' \tag{16}$$

$$\boldsymbol{M}^p_{10}(\boldsymbol{r}) = \boldsymbol{E}^p_{10}(\boldsymbol{r})\times\hat{\boldsymbol{n}}, \quad p = t, r \tag{17}$$

$$\boldsymbol{M}^q(\boldsymbol{r}) = \boldsymbol{E}^q(\boldsymbol{r})\times\hat{\boldsymbol{n}}, \quad q = i, C. \tag{18}$$

Substituting Eqs. (11)-(13) into Eq. (8), we finally obtain

$$\begin{aligned} \left.\begin{array}{cc} \boldsymbol{E}(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_0) \\ \boldsymbol{E}^C(\boldsymbol{r}) & (\boldsymbol{r} \text{ in } \Omega_1, \Omega_2) \end{array}\right\} &= \int_{S_0}\left\{j\omega\mu\boldsymbol{J}G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}\right]\nabla'G\right\}dS' \\ &\quad + \int_{S_1+S_2}\left\{j\omega\mu\boldsymbol{J}^C G - \frac{j}{\omega\varepsilon}\left[\nabla'\cdot\boldsymbol{J}^C\right]\nabla'G\right\}dS' \\ &\quad - T_{10}\boldsymbol{U}^t_{10}(\boldsymbol{r}) - R_{10}\boldsymbol{U}^r_{10}(\boldsymbol{r}) - \boldsymbol{U}^i(\boldsymbol{r}). \end{aligned} \tag{19}$$

The integral equation (19) is the one which we have proposed so far. The properties of Eq. (19) are following: (i) Eq. (19) is the similar formula to Eq. (1); (ii) S_1 and S_2 can be regarded as finite in extent, because $\boldsymbol{J}^C$ which is the sum of cutoff modes vanishes at the far point from the iris. According to the properties, it is possible to apply the standard MoM to Eq. (19). However, Eq. (19) can not be solved, because the unknowns are not only $\boldsymbol{J}$ and $\boldsymbol{J}^C$ but also T_{10} and R_{10}. Therefore, we need to obtain another equations.

In order to derive another equations, we applied the asymptotic expression of Green's function in Ref. [1]. However, it is difficult to extend this procedure to multimode waveguides.

As a procedure which can be extent to multimode waveguides, we derive another equations applying the orthogonal properties of the modes [2]. Multiplying Eq. (19) by the transmitted guided mode $\boldsymbol{E}^t_{10}(\boldsymbol{r})$ in the waveguide 1, and integrating the resultant equation on the cross-section surface Γ_{10} gives

$$\begin{aligned}\int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r}) dS &= \int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \int_{S_0} \left\{ j\omega\mu \boldsymbol{J} G - \frac{j}{\omega\varepsilon} \left[\nabla' \cdot \boldsymbol{J}\right] \nabla' G \right\} dS' dS \\ &+ \int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \int_{S_1+S_2} \left\{ j\omega\mu \boldsymbol{J}^C G - \frac{j}{\omega\varepsilon} \left[\nabla' \cdot \boldsymbol{J}^C\right] \nabla' G \right\} dS' \\ &- T_{10} \int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \boldsymbol{U}^t_{10}(\boldsymbol{r}) dS - R_{10} \int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \boldsymbol{U}^r_{10}(\boldsymbol{r}) dS - \int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \boldsymbol{U}^i(\boldsymbol{r}) dS.\end{aligned} \tag{20}$$

Using the orthogonal properties of mode, the left-side hand of Eq. (20) becomes

$$\int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r}) dS = T_{10} \int_{\Gamma_{10}} \boldsymbol{E}^t_{10}(\boldsymbol{r}) \cdot \boldsymbol{E}^t_{10}(\boldsymbol{r}) dS. \tag{21}$$

Similarly, multiplying Eq. (19) by the reflected guided mode $\boldsymbol{E}^r_{10}(\boldsymbol{r})$ in the waveguide 2, and integrating the resultant equation on Γ_{20} gives

$$\begin{aligned}\int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r}) dS &= \int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \int_{S_0} \left\{ j\omega\mu \boldsymbol{J} G - \frac{j}{\omega\varepsilon} \left[\nabla' \cdot \boldsymbol{J}\right] \nabla' G \right\} dS' dS \\ &+ \int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \int_{S_1+S_2} \left\{ j\omega\mu \boldsymbol{J}^C G - \frac{j}{\omega\varepsilon} \left[\nabla' \cdot \boldsymbol{J}^C\right] \nabla' G \right\} dS' \\ &- T_{10} \int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \boldsymbol{U}^t_{10}(\boldsymbol{r}) dS - R_{10} \int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \boldsymbol{U}^r_{10}(\boldsymbol{r}) dS - \int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \boldsymbol{U}^i(\boldsymbol{r}) dS\end{aligned} \tag{22}$$

where

$$\int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r}) dS = R_{10} \int_{\Gamma_{20}} \boldsymbol{E}^r_{10}(\boldsymbol{r}) \cdot \boldsymbol{E}^r_{10}(\boldsymbol{r}) dS. \tag{23}$$

The equations (19), (20) and (22) are the new EFIE which we propose in this paper. Since S_1 and S_2 can be regarded as finite in extent, we can expand $\boldsymbol{J}^C$ in finite number of basis functions. When we apply the standard MoM to Eq. (19), (20) and (22) using N basis functions, we obtain a matrix equation, where the size of a coefficient matrix is $(N + 2) \times (N + 2)$ and the size of an unknown vector is $N + 2$. Namely, Eqs. (19), (20) and (22) can be numerically solved by the standard MoM with no use of mode expansion technique.

4. Numerical Simulations

Figure 2 shows the transmission and reflection coefficients and transmitted and reflected energies. The size of waveguide is $a = 15.8$mm and $b = 7.9$mm, and the size of iris is $w = a/\sqrt{2}$, $h = b/\sqrt{2}$ and $t = 2$mm. The method #1 is the proposed method in this paper. The method #2 is the method proposed in Ref. [1]. The results of #1 and #2 are compared with those in Ref. [3]. The results of #1 and #2 are good agreement with those of Ref. [3] from Fig. 2(a). The total energies satisfy the energy conservation law within an accuracy of 1% except $a/\lambda = 0.6$.

5. Conclusions

We have proposed the new EFIE (19), (20) and (22) which are suitable for a basis theory of computer aided design of a 3-dimensional waveguide. The new EFIE can be numerically solved by the standard MoM with no use of mode expansion technique. It is straightforward to extend this procedure to a multimode waveguide. We have also shown numerical calculations of the iris waveguide.

References

[1] M. Tanaka and K. Tanaka: "New Electric Field Integral Equation for Computer Aided Design of 3-Dimensional Waveguide," in Proc. IEEE Antennas and Propag. Symp., pp. 1599-1602, 2006.

[2] R. E. Collin: "Field Theory of Guided Waves," New York: McGraw-Hill, 1960.

[3] H. Patzelt and F. Arndt: "Double-Plane Steps in Rectangular Waveguides and their Application for Transformers, Irises, and Filters," IEEE Trans. on Microwave Theory Tech., vol. MTT-30, pp. 771-776, 1982.

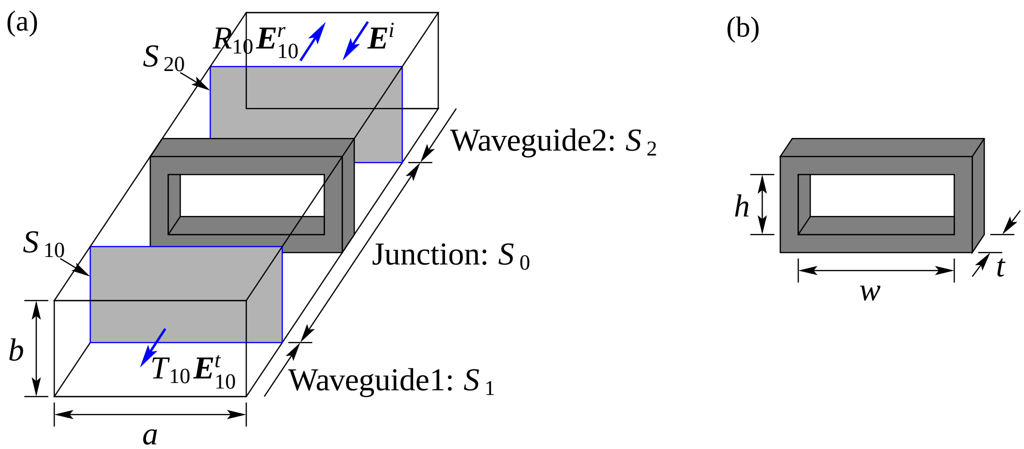

Figure 1: (a) Iris waveguide model, and (b) parameters of iris.

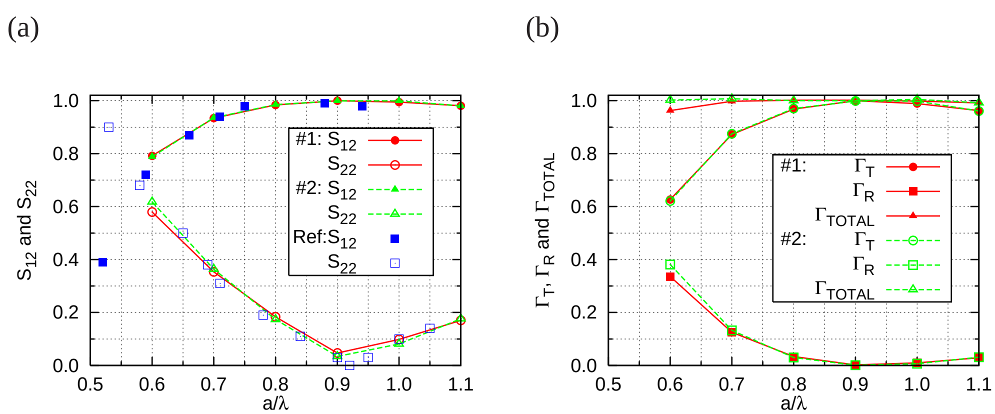

Figure 2: (a) Transmission coefficient (S_{12}) and reflection coefficient (S_{22}) of iris waveguide, and (b) transmitted energy (Γ_T), reflected energy (Γ_R) and their total energy Γ_{TOTAL}.