ANALYSIS OF 2-D TE SCATTERING WITH IMPEDANCE BOUNDARY CONDITION USING DUALITY OF THE IE-MEI METHOD

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Abstract

We show that an integral equation formulation of the measured equation of invariance (IE-MEI) fulfills a duality and the duality is useful to solve the TE scattering problem for a 2-D object by reusing the same matrices as those used to solve the corresponding TM scattering problem. This means that TM and TE scattering problems are solved with the same matrices at the same time. We show that a formulation of the IE-MEI derives the duality of the IE-MEI quite naturally. The application range of the IE-MEI method and its duality is very vast: it can treat scattering problems for perfect electric conductor (PEC), lossy materials, and lossless materials illuminated by TE and TM sources in near or far field region. In this paper, we show the validity of the duality in the case of scattering with impedance boundary condition (IBC) by numerical examples of a 2-D circular and square cylinders illuminated by TE plane waves.

1. Introduction

The measured equation of invariance (MEI) has been proposed as an efficient alternative to absorbing boundary condition (ABC) to put the truncating boundary very close to a scattering object in the finite difference (FD) method [1]. It has been successfully applied to many problems such as static and dynamic problems for many kinds of materials.

On the other hand, Rius et al. [2] derived an integral equation formulation of the MEI, which is similar to the method of moments (MoM). In the IE-MEI method, we postulate a certain local linear relationship between the electric and the magnetic fields on the surface of a scattering object. The local linear relationship is represented by two cyclic band matrices, whose bandwidths are typically three independently of the number of unknowns in the problem. In spite of the success of the IE-MEI, the derivation by Rius et al relied on some vague conjectures.

Recently we have reformulated the IE-MEI using another reciprocity theorem and showed a postulate required for the IE-MEI to be plausible [3]; the postulate is convincing from intuitive considerations. The postulate is that there exit locally confined electric and magnetic sources on the surface of a PEC scattering object at good approximation. From the derivation, we have found that the local linear relationship in the IE-MEI is applicable to scattering objects whose electromagnetic characteristics are arbitrary: PEC, lossy materials, or lossless materials [3, 4]. In a latest issue, Lan et al. [5] have obtained the other formulation of the IE-MEI (they call it on-surface MEI) by using the reaction integral equation and extended the application range of the IE-MEI to include wire antenna.

In this paper, we derive the duality of the local linear relationship. Numerical examples are given to show the validity of the duality of the IE-MEI for scattering problems to a circular and square cylinders with an impedance boundary condition illuminated by TE waves.

2. Duality of the IE-MEI method

Let us consider a problem in Fig. 1. From the reciprocity theorem and the postulate of the existence of the locally confined sources and equivalent surface currents, the final form of the IE-MEI can be written as [3]

$$\int_{C_0} \left(\mathbf{E}^s \bullet \mathbf{J}_h^t - \mathbf{H}^s \bullet \mathbf{M}_h^t \right) dl' = 0 \tag{1}$$

where C_0 is the local portion of C: on the portion, the equivalent sources $\mathbf{J}_h^t$ and $\mathbf{M}_h^t$ are essentially non-zero. $\mathbf{E}^s$ and $\mathbf{H}^s$ are the scattered electric and magnetic fields of the problem. This equation is the same form as the original form of the IE-MEI in [2]. Discretizing $\mathbf{E}^s$, $\mathbf{H}^s$, $\mathbf{J}_h^t$, and $\mathbf{M}_h^t$, we obtain the matrix form of the local linear relationship of the IE-MEI [2, 3] as

$$\mathrm{A}\left(\mathbf{e}^s/\eta\right) - \mathrm{B}\,\mathbf{h}^s = 0 \tag{2}$$

where $\mathbf{e}^s$ and $\mathbf{h}^s$ are column vectors representing $\boldsymbol{E}^s$, $\boldsymbol{H}^s$ respectively, η is the intrinsic impedance of S, and A and B are sparse matrices representing $\mathbf{J}^t_h$ and $\mathbf{M}^t_h$ in Eq.(1) respectively.

To explain the duality, let us consider a TM problem of PEC as in Fig. 1. First, we calculate $\mathrm{E}^s_{z,m}$ and $\mathrm{H}^s_{l,m}$ produced by electric currents on C, called as metrons:

$$J_z^m(l) = \exp(j\tfrac{2\pi l}{L} m) \;\; for \;\; m = -N_p, \ldots, N_p \tag{3.}$$

Inserting $\mathrm{E}^s_{z,m}$ and $\mathrm{H}^s_{l,m}$ into Eq.(2) and solving the equations regarding to rows of A and B, we can finally obtain two sparse matrices A and B. Then, the matrices A and B are used to solve TM scattering problems for arbitrary materials. Next, let us consider the corresponding TE problem. Let us take metrons of the form in Eq.(3) as magnetic currents on the Perfect Magnetic Conductor (PMC) that has the same shape as the PEC. Noting that the relationship of the IE-MEI (or A and B) is independent on the characteristics of the material in S_2, and using the duality of Maxwell's equations, we finally attain

$$\tfrac{E^s_{l,m}}{\eta} = -H^s_{l,m} \;\; , \qquad H^s_{z,m} = \tfrac{E^s_{z,m}}{\eta} . \tag{4}$$

Inserting Eq. (4) into Eq.(2) for the TE problem, we obtain the duality relation between two sparse matrices A and B for the TE problem and those for the TM problem (Liu et al [6] have derived the duality of FD-MEI method):

$$\mathrm{A}^{\mathrm{TE}} = -\mathrm{B}, \qquad \mathrm{B}^{\mathrm{TE}} = \mathrm{A} . \tag{5}$$

In the same procedure as that for TM problem with IBC [3], the electric current J_l on the object with IBC for TE problem is written as

$$J_l = -\left(\mathrm{A} - \tfrac{Z_S}{\eta}\mathrm{B}\right)^{-1}\left\{H_z^i + B\left(E_l^i/\eta\right)\right\}. \tag{6}$$

where E_l^i and H_z^i are the incident electric and magnetic fields respectively, and Z_s is the surface impedance of the object. Because A and B are cyclic band sparse matrices whose bandwidth are typically 3 independently of the number of unknowns N in the problems, we can solve the TE problem with IBC with the same efficient procedure as that of a TM problem for PEC: $O(N)$ memory storage and $O(N^2 \log N)$ operation counts.

3 Numerical examples

To show that the duality of the IE-MEI method works well, we calculate the electric currents on a circular and square cylinders with the IBC (ε_r=1, σ=0.3 S/m, Z_s/η= 0.171+j0.162 at 300 MHz) as examples with a smooth perimeter and with sharp corners repectively. We choose the sizes of both cylinders which may suffer from internal resonance. Comparison is made for the examples between the solutions by the IE-MEI method and those by the method of moments using a combined field integral equation (CMoM).

Figure 2 illustrates the circular cylinder which are internally resonant at $TM_{1,13}$ and $TE_{0,13}$ modes. Figure 3 shows the electric current distribution on the circular cylinder illuminated by the TE plane wave from the $-x$ direction. The normalized length is defined as the length along the perimeter of the cylinder originated from the point $(a,0)$ and is normalized by the total length of the perimeter. The amplitude and the phase are normalized with respect to $2H_z^i$. Since the IBC holds within relative error of 3%, the magnetic current is almost the same as the electric current. In the IE-MEI method, the current converges when N = 512 (N_p=54) while the current in the CMoM almost converges when N=2048. Both results agree with each other except around the shadow region. However, the maximum relative error is below 7 % and the average relative error is 2 %. Figure 4 depicts the scattering cross section that is normalized with respect to the scattering width πa. Both results are in good agreement as expected.

Figure 5 illustrates the square cylinder which is internally resonant at $TM_{18,18}$ and $TE_{18,18}$ modes. Figure 6 shows the electric current for the TE plane wave incident at φ=135 degrees. The result of the IE-MEI method agrees with that of the CMoM. N=512 (N_p=73) and N=2048 are used in the IE-MEI method and CMoM respectively. Although the maximum relative error (24 %) occurs around the corners, the average relative error is 10 %; on average, they agree with each other. Figure 7 depicts the scattering cross section that is normalized with respect to the scattering width $\pi w/2$. Both results are in good agreement except around the angle range between 247 to 276 degrees.

Summary

We have shown that the duality of the local relationship between the electric and magnetic field in the IE-MEI method is applicable to scattering problems with IBC and the duality can be derived from the interpretation of the new formulation of the IE-MEI.

The circular and square cylinders were considered as numerical examples. The electric current distribution and the scattering cross section of the cylinders with the IBC were calculated by using the duality of the IE-MEI method and the combined field method of moments. The results by both methods agree with each other. Although not shown here, we have found that the duality holds for PEC, lossy objects, or lossless objects at a good approximation; therefore the duality is independent of the constituents of the objects. They all indicate that the MEI method is right approximately.

The next important issue on the IE-MEI method will be to solve 3-D scattering problems efficiently as 2-D problems so as to become a powerful method in computational electromagnetics.

References

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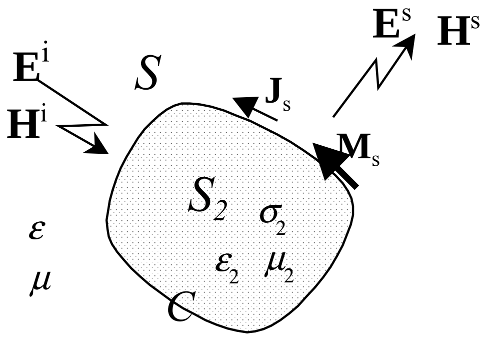

Fig. 1 Configuration of a problem.

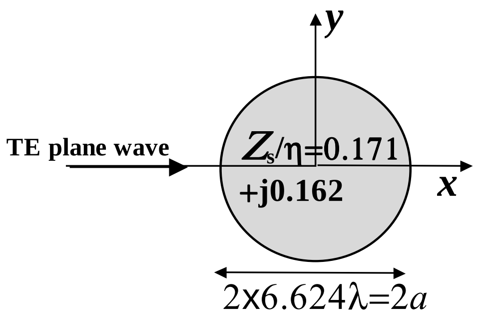

Fig. 2 Lossy circular cylinder.

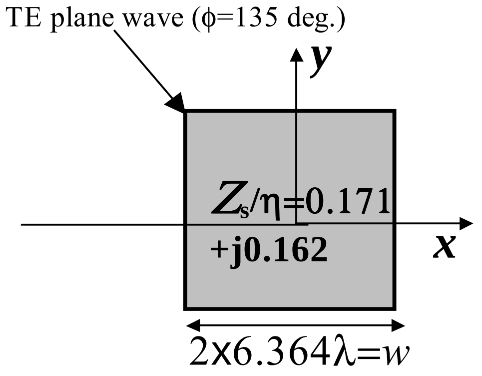

Fig. 5 Lossy square cylinder.

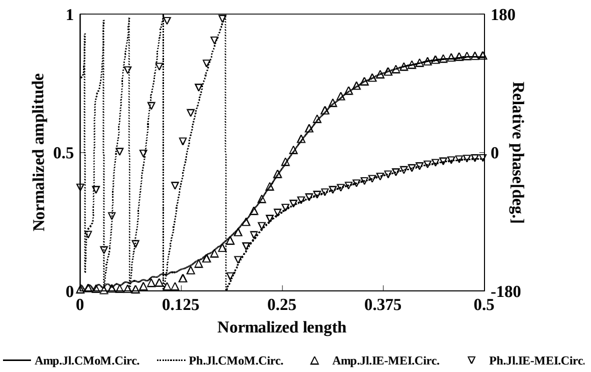

Fig. 3 Normalized electric current on the circular cylinder illuminated by TE plane wave.

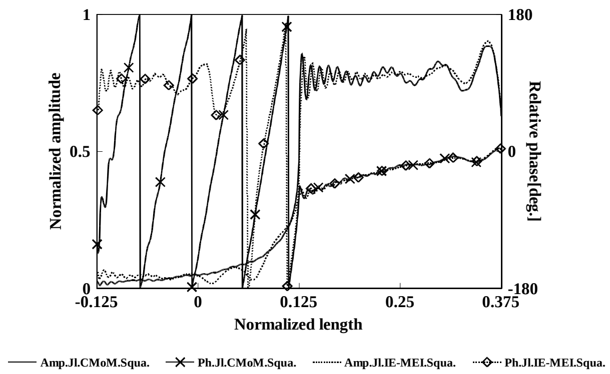

Fig. 6 Normalized electric current on the square cylinder illuminated by TE plane wave.

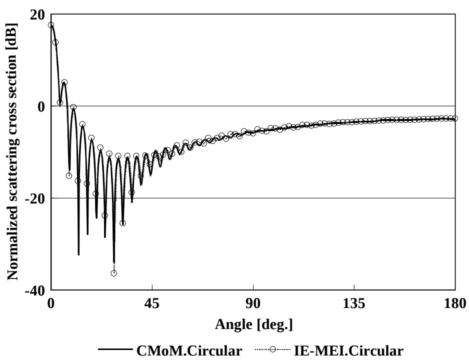

Fig. 4 Normalized scattering cross section of the circular cylinder illuminated by TE plane wave.

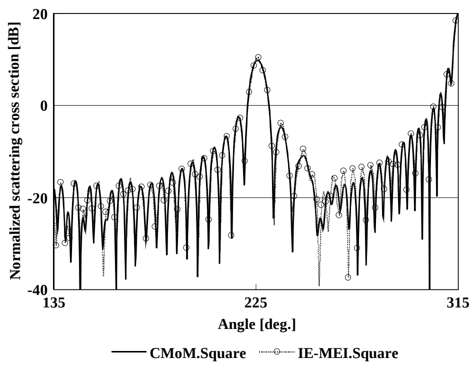

Fig. 7 Normalized scattering cross section of the square cylinder illuminated by TE plane wave.