

TIME DOMAIN MODELLING OF 3D PERIODIC STRUCTURES USING FDTD IN A NON ORTHOGONAL 4D BASIS

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INTRODUCTION

Periodic structures such as active phased array antennas, gate arrays, frequency selective surfaces (FSS) and Photonic Band Gap (PBG) crystals are of considerable interest for a wide range of communications and radar applications. Modelling of planar arrays of 3-D periodic elements represents a significant theoretical challenge, particularly when non-linear devices form part of the system. Methods employing a Floquet mode representation and a Method of Moments (MoM) solution are widely available for planar structures, but these cannot directly cater for planar arrays of arbitrary 3-D elements and material properties, without custom modification to suit individual scenarios. The Finite Difference Time Domain (FDTD) method is well suited to modelling generalised 3-D structures and also non-linear devices. This paper presents a modification to the standard FDTD implementation, which significantly increases the utility of the method as a design and optimisation tool. It provides the ability to model infinite periodic antenna arrays or scatterers and in particular the behaviour of elements on triangular lattices. The theoretical foundation of these extensions is explained in the early sections of this paper. A code incorporating the new FDTD model has been developed and has already proven to be of considerable practical benefit in the design of microwave and millimetre-wave structures. A series of examples are presented which illustrate its accuracy and flexibility for solving a wide range of complex problems. The capability to model arbitrary planar periodic structures has also been applied to assess and compensate for effects imposed by manufacturing techniques, including the radii of cutters used to machine waveguide aperture arrays. In this sense the code has many real-life applications in the design of low cost antennas, where it is desirable to circumvent the use of expensive manufacturing techniques, such as electronic discharge machining (EDM), or to assess the impact of draft angles used in moulded or cast arrays.

FDTD offers a very flexible and therefore powerful tool for modelling infinite arrays and makes it possible to directly obtain the broadband response of a given structure (using a pulse excitation), including incorporation of non-linear media and devices. To enable the FDTD technique to be applied to more realistic scenarios, and at the same time obtain a rapid solution, periodic boundary conditions are introduced. As with frequency domain methods, it is only necessary to model a single array element by enforcing the periodicity of the fields. However, special problems arise when time domain methods model phase shifted arrays or non-normal incidence of plane waves using a single periodic cell, Tsay and Pozar [1]. In these cases, the field excitation is generated at each array element at different time instants, Fig.1a. In addition, in order to make the technique applicable to a wider range of practical problems, it is desirable to introduce non-orthogonal periodic boundary conditions, which enable triangular lattices to be simulated, Fig.1b. For oblique plane wave incidence and phase shifted arrays, this non-orthogonality must also extend to the time domain. The meaning of the non-orthogonality of the time axis can be explained using Special Relativity, as detailed in the following section.

In order to model these structures, the standard FDTD technique has been extended to a general 4-D space-time non-orthogonal grid. Doubly periodic structures, such as FSS, are very demanding in terms of the Absorbing Boundary Conditions (ABCs), due to the propagation of grating lobes when illuminated at high incidence angles. As a consequence, the ABCs reduce the overall speed of the computations. Promising new ABCs, based on an analytical Discrete Green's Functions (DGF), have therefore been implemented, [2].

The technique described here is a general approach to the problem, producing an extended FDTD method that can treat phased shifted infinite planar arrays of 3-D elements with arbitrary pulsed excitation by modelling a single array element. The method is based on a true 4-D periodic representation of the fields in the time domain. For the phased array (or a dichroic surface) under non-normal incidence, the scattered field appears to be periodic in a 4-D basis. The FDTD technique is implemented solving Maxwell's equations in an arbitrary non-orthogonal 4-D basis. A summary of the technique is given below, but was set out in more detail in a recent paper, [3].

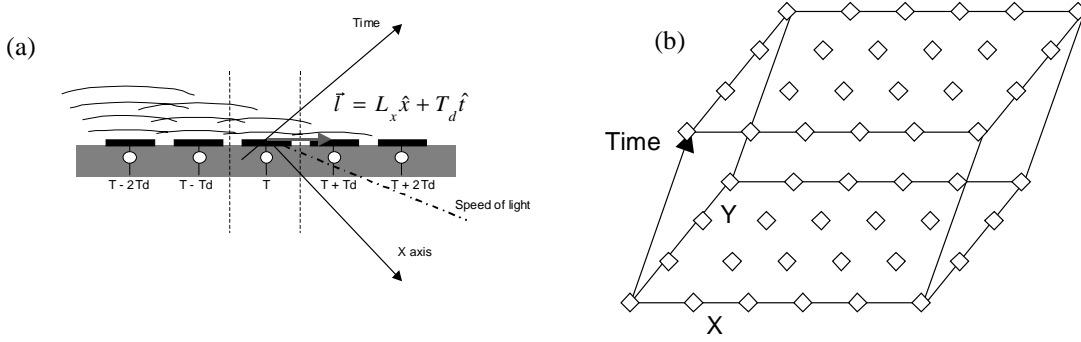


Fig.1: (a) Field at a given periodic cell (depends on the scattering at adjacent cells on previous time instants), (b) skewed lattice cell

ARBITRARY 4-D FDTD BASIS FOR INFINITE ARRAYS

The boundary condition assumes that the field solution and its derivatives can be obtained by a translation of the field cell. Thus, the fields outside the periodic cell, that are necessary to calculate the fields at the boundary, can be estimated by the fields inside the cell using a translation (Eq.1). In order to apply this condition it is necessary that the boundary of the region being modelled is the same as that forming the periodic cell.

$$\vec{E}(\vec{r}, t) = \vec{E}(\vec{r} + \sum_{i=1}^3 l_i \vec{A}_i, t + \sum_{i=1}^3 l_i T_i) \quad \vec{H}(\vec{r}, t) = \vec{H}(\vec{r} + \sum_{i=1}^3 l_i \vec{A}_i, t + \sum_{i=1}^3 l_i T_i) \quad (1)$$

If the FDTD grid is maintained, an extrapolation procedure is required to predict the fields at future time instants. This is possible for sinusoidal signals in steady state, since the estimation of the field at future time instants can be obtained as a simple phase delay on the sinusoidal response. However, in that case the periodic boundary conditions for the FDTD method is restricted to monochromatic sources and linear media. For the more general case of pulse excitation, the problem of using periodic boundary conditions can be tackled using a change of basis for the time co-ordinate to match the lattice in the real 4-D space. In this way, the periodic boundary condition can be applied directly in the new time co-ordinates, but Maxwell's equations in the new basis have extra terms that must be included in the FDTD algorithm, [3]. An alternative approach to avoid additional terms in Maxwell's equations can be implemented using the Lorentz transform (Eq.2), as detailed below.

$$x' = \gamma \cdot (x - vt) \quad y' = y \quad z' = z \quad t' = \gamma \cdot (t - \frac{v}{c^2} x) \quad \gamma = 1 / \sqrt{1 - v^2/c^2} \quad (2)$$

Maxwell equations are invariant to the Lorentz transform. This means that the equations remain formally the same after the transformation, without any additional term. The theory of Special Relativity can be used to understand the physical meaning of the method. In his famous work published in the year 1905, Einstein [4] set the background for modern physical theories, stating two postulates which are the basis of the theory of Special Relativity: the first regarding the covariant form of laws of physics and the second which refers to the fact that the speed of light is constant for all observers situated in an inertial reference frame. To give form to these two ideas, the concept of space-time was born, together with the use of the Lorentz transformations of four-dimensional vectors ensuring the validity of the second postulate.

Using the Lorentz transform, the frame of reference corresponds to a relativistic observer. For this observer, physical laws (including Maxwell equations) remain unchanged, but the perception of an outside static observer can be rather different. Non simultaneous events (radiation from phase shifted array elements) in the frame of reference can appear to be simultaneous for the moving observer. However, the Lorentz transformation approach requires that the structure is moving with respect to the FDTD grid, resulting in practical problems in the implementation of the numerical algorithm. This can be overcome for a phase shifted array or (FSS at oblique incidence) by transforming the problem into a 'constant phase' array or an FSS at normal incidence. This introduces a transformation of Maxwell's equations into the new set given in (3) below, [3].

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}h}{\partial t'} + \vec{j}' \quad \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}e}{\partial t'} \quad \vec{E}h = \vec{E} + Z_0 / \epsilon_r \cdot (\vec{\alpha} \times \vec{H}) \quad \vec{H}e = \vec{H} - (\vec{\alpha} \times \vec{E}) / Z_0 \quad (3)$$

Where \vec{E}_h \vec{H}_e and \vec{E} \vec{H} should be taken as the contravariant co-ordinates, ∇ the covariant co-ordinates of the lattice basis and the vector $\alpha = v / c^2$.

These modified Maxwell's equations conform to their usual form for $\vec{\alpha} = 0$ (normal incidence). The structure of the equations is similar to Maxwell's equations for the new field functions (E_h, H_e). The algorithm stability is critical when α/c is close to one. This happens for angles of incidence close to 90° to the normal ($\vec{\alpha} = c$), where there is a singularity in the Lorentz transform. This is equivalent in Einstein's theory of Special Relativity to an object approaching the speed of light. It is interesting to postulate the equivalence between evanescent and surface wave phenomena observed in the antenna problem and the paradox encountered by Einstein, consistent with this method, of an object moving faster than the speed of light, as seen by a static observer.

PRACTICAL APPLICATIONS

A computer code has been implemented in order to demonstrate the 4-D FDTD method. Perfect Matched Layer (PML) boundary conditions have been implemented in this 4-D basis, the field splitting being performed in E_h and H_e according to the relations of Eq.3, relating these to the physical fields E and H. This software calculates the transmission coefficient of a plane wave on a variety of structures, including FSS, PBGs and corrugated scatterers, as well as impedance and radiation pattern characteristics of phased array antennas, including triangular lattices. For analysis of FSS, the code simulates the incidence of a plane wave Gaussian pulse and outputs the broadband transmission and reflection of the 0th order Floquet mode at the given angle of incidence. The grating lobe response can also be extracted by considering the higher order propagating modes.

Figure 2 compares the transmission characteristics of an FSS structure modelled using the 4-D FDTD method with results presented by Zarrillo and Aguiar [5] produced using a mode-matching technique (MM). This analysis used a $60 \times 21 \times 21$ grid, with 16 layers PML ABCs and took 6 minutes CPU time on a 450MHz Pentium.

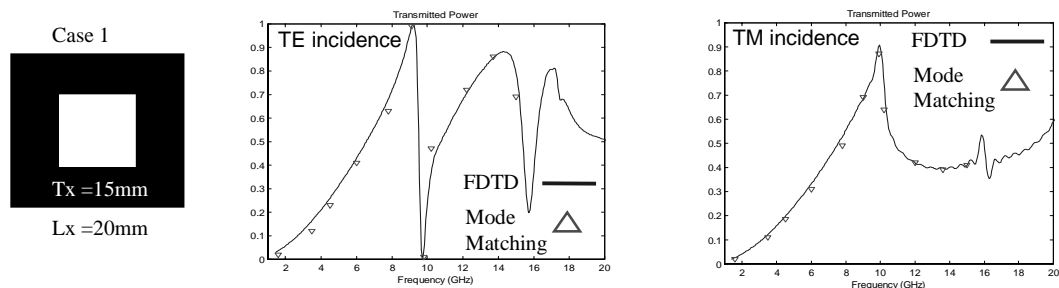


Fig 2: TE and TM transmission through a screen of square holes (30° incidence)

A Photonic Band Gap crystal was also modelled by way of a demonstration. Such structures have received considerable attention in the literature and offer considerable potential to suppress surface waves in printed antenna substrates. One well-known structure is the so-called wood-pile. Ho et al [6] have modelled this configuration using a MoM approach; Fig.3a,b compares this solution with the ERA 4-D FDTD result.

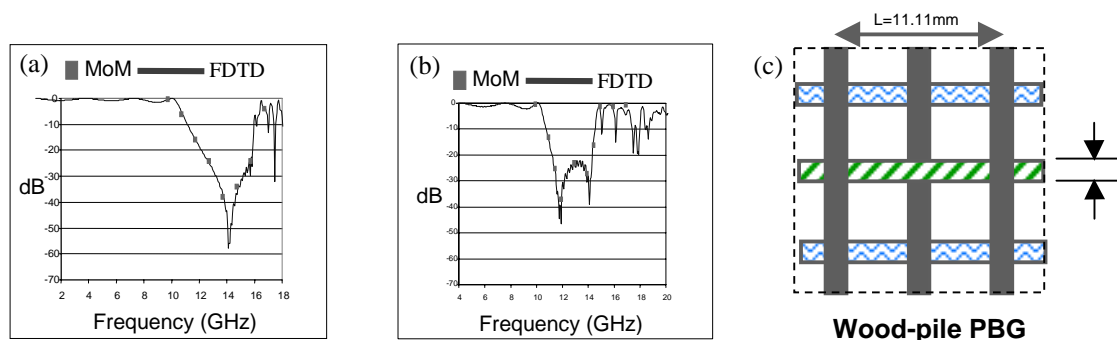


Fig 3: Predicted performance of Photonic Band Gap wood-pile structure at (a) 0° incidence, (b) 30° incidence, (c) geometry

A complex FSS comprising electrically thick and thin FSS on different periodic (but harmonically related) lattices was also modelled. This structure was designed to produce pass-bands at L and S band, with a reflection

band at 8 GHz, as shown in Fig.4. The code was applied to model this structure without need for modification.

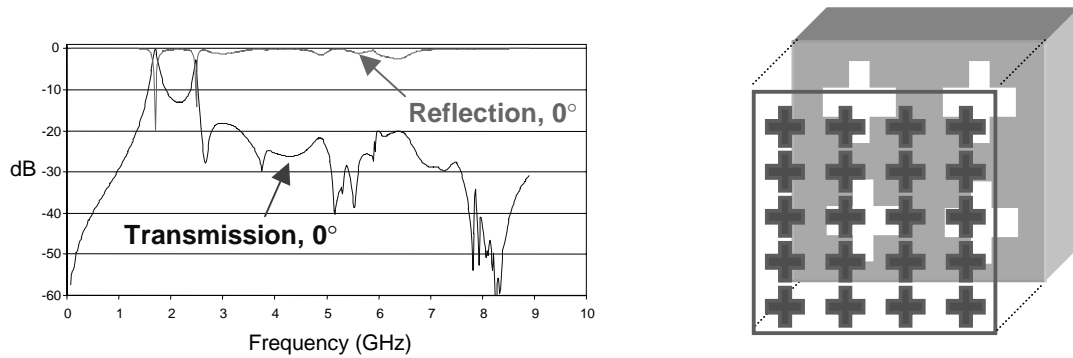


Fig.4: Multilayer FSS comprising electrically thick screen with apertures, plus separate printed array

Work is also ongoing into modelling active structures. An example of such a structure is shown schematically in Figure 5. Here crossed dipoles are connected by diodes to form a conducting, and hence reflecting, mesh when the diodes are forward biased and a narrow-band reflection resonance when under reverse bias conditions. Other non-linear devices can be modelled in this configuration to provide quasi-optical down-conversion, or to act as a frequency doubler, for example in millimetre-wave applications. The 4-D FDTD code is also being used in the design of fixed beam and phased scanned antennas on triangular array lattices.

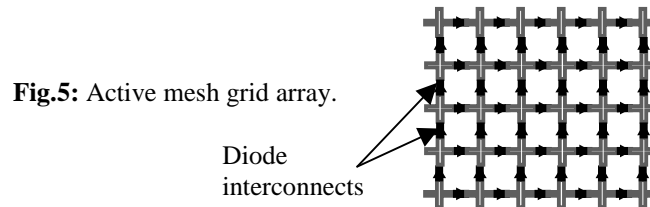


Fig.5: Active mesh grid array.

CONCLUSIONS

This paper has described the fundamental framework of the 4-D FDTD method and has illustrated the application of the method to periodic arrays with skewed lattices. The generalisations detailed in this paper should enable realistic structures to be modelled, including active elements, provided that representative data is available on the time varying behaviour of these components. It also opens up the possibility of accurately modelling a broader range of structures, including Photonic Band Gap (PBG) materials. Crucial to this is the flexibility of the technique to analyse the wideband behaviour of infinite planar arrays of arbitrary 3-D elements, without recourse to modification of the code to accommodate individual geometries. In addition, the 4-D FDTD reduces the memory and computational requirements by modelling only a single periodic cell, making it feasible to simulate and optimise realistic configurations, including features introduced by manufacturing processes.

As Einstein showed it is sometimes necessary to stand outside your own paradigm to solve a problem. By using the Lorentz transform it could be said that the techniques described in this paper are not new, but nevertheless their novel application may have a considerable impact on the utility of the FDTD method. Parallels with the speed of light paradox also provide an interesting new perspective on evanescent and surface wave phenomena.

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