

Backscattering of laser from targets in random media for E-Wave polarization

Hosam El-Ocla

Department of Computer Science, Lakehead University
955 Oliver Road, Thunder Bay, Ontario, Canada P7B 5E1

E-mail: hosam@lakeheadu.ca

Abstract: The characteristics of radar cross section (RCS) of partially convex targets with large sizes up to five wavelengths in free space and random media are studied in this work. The nature of incident wave is an important factor in the remote sensing and radar detection applications. Here I investigate the effects of a beam wave incidence on the performance of RCS as I have handled the plane wave incidence case in a previous study. Targets are taking large sizes to be bigger enough than the beam width with putting into consideration a horizontal incident wave polarization (E-wave incidence). The effects of the target configuration, random medium parameters, and the beam width on the laser RCS and the enhancement in the radar cross-section (ERCS) are numerically analyzed. Therefore, we will be able to have some sort of control on radar detection using beam wave incidence.

1 Introduction

A number of methods proposed to formulate the scattering wave were presented: examples are in [1]–[3]. In this regard, some years ago, a method has been presented for solving the scattering problem as a boundary value problem [4]–[8]. This method is characterized by the estimation of the current on the whole surface and not only on the illumination region as in the physical optics method. Therefore this method gives a precious calculation of the wave intensity.

Studying the backscattering enhancement implemented in ERCS of electromagnetic waves propagating in random media has attracted researchers in the fields of radar engineering and remote sensing as in [9]–[12]. As a result of the double passage effect on waves backscattered from point targets, RCS in random medium is enhanced to be twice that one in free space. As the more practical models, numerical results have been shown for RCS of conducting convex bodies such as circular and elliptic cylinders [4]. Later, the effects of target configuration, random media, and

polarization on the RCS and ERCS for plane wave incidence were analyzed in many of my publications (e.g. [5]–[8], where other references are available). It was found that these parameters have an obvious influence on the RCS in addition to the double passage effect.

In electromagnetic wave propagation and scattering, the effects of incident wave become significant, depending upon its nature and polarization. It should be noted that for generating waves of infinitely large plane wave fronts, an infinitely large source should be used. This can not be available easily especially for plane waves wide sufficiently at the fronts of large size targets in the far field. In an attempt to generate plane wave, an expansion of plane wave into Gaussian beam waves was derived [13]. Gaussian beams play a key role in different fields of physics; let us mention applications in lasers, electromagnetic waves, etc. Many problems of propagation and scattering of Gaussian beams have been solved (see [7, 14, 15], where other references can be found). On the other hand, the research on laser radar [16] for target ranging, detection, and recognition [17] has become the one key technology to evaluate and model the characteristics of scattering from a complex target in the military and civil applications.

In this paper, the scattering characteristics are analyzed through studying the behavior of laser RCS (LRCS) of a complex target. In doing that, one can calculate the LRCS by assuming a beam wave incident on a nonconvex cross section. In fact, we can consider the beam wave as a plane wave when the mean size of the scatterer becomes smaller than the beam width, however, this is not usually the general case practically. To detect targets of larger sizes, we should, therefore, handle the case where the beam width is smaller than the target size.

In this study I consider the scattering problems where beam wave incidence is backscattered from targets in free space and continuous random media of different strengths. Effects evaluation of the target configuration including size and curvature on the LRCS

and the enhancement phenomenon in LRCS (ELRCS) is investigated. To achieve this aim, we draw on our method described earlier to conduct numerical results for the LRCS of concave-convex targets of large sizes up to about five wavelengths to be bigger enough than the beam width. We deal with the scattering problem two-dimensionally assuming horizontal polarization (E-wave incidence). In the previous work [5], it has been clarified that the RCS changes obviously with the illumination region curvature. In this study, it is concentrated on the wave backscattering from convex illumination portion only. The time factor $\exp(-i\omega t)$ is assumed and suppressed in the following section.

2 Formulation

Geometry of the problem is shown in Figure 1. A random medium is assumed as a sphere of radius L around a target of the mean size $a \ll L$, and also to be described by the dielectric constant $\varepsilon(\mathbf{r})$, the magnetic permeability μ , and the electric conductivity ν . For simplicity $\varepsilon(\mathbf{r})$ is expressed as

$$\varepsilon(\mathbf{r}) = \varepsilon_0[1 + \delta\varepsilon(\mathbf{r})] \quad (1)$$

where ε_0 is assumed to be constant and equal to free space permittivity and $\delta\varepsilon(\mathbf{r})$ is a random function with

$$\langle \delta\varepsilon(\mathbf{r}) \rangle = 0, \quad \langle \delta\varepsilon(\mathbf{r}) \delta\varepsilon(\mathbf{r}') \rangle = B(\mathbf{r}, \mathbf{r}') \quad (2)$$

and

$$B(\mathbf{r}, \mathbf{r}) \ll 1, \quad kl(\mathbf{r}) \gg 1 \quad (3)$$

Here, the angular brackets denote the ensemble average and $B(\mathbf{r}, \mathbf{r})$, $l(\mathbf{r})$ are the local intensity and local scale-size of the random medium fluctuation, respectively, and $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space. Also μ and ν are assumed to be constants; $\mu = \mu_0$, $\nu = 0$. For practical turbulent media the condition (3) may be satisfied. Therefore, we can assume the forward scattering approximation and the scalar approximation [18]. Consider the case where a directly incident beam wave is produced by a line source $f(\mathbf{r}')$ along the y axis. Here, let us designate the incident wave by $u_{in}(\mathbf{r})$, the scattered wave by $u_s(\mathbf{r})$, and the total wave by $u(\mathbf{r}) = u_{in}(\mathbf{r}) + u_s(\mathbf{r})$. The target is assumed to be a conducting cylinder of which cross-section is expressed by

$$r = a[1 - \delta \cos 3(\theta - \phi)] \quad (4)$$

where ϕ is the rotation index and δ is the concavity index. We can deal with this scattering problem two dimensionally under the condition (3); therefore, we represent \mathbf{r} as $\mathbf{r} = (x, z)$. Assuming a horizontal polarization of incident waves (E-wave incidence), we can impose the Dirichlet boundary condition for wave field $u(\mathbf{r})$ on the cylinder surface S . That is, $u(\mathbf{r}) = \mathbf{0}$, where $u(\mathbf{r})$ represents E_y .

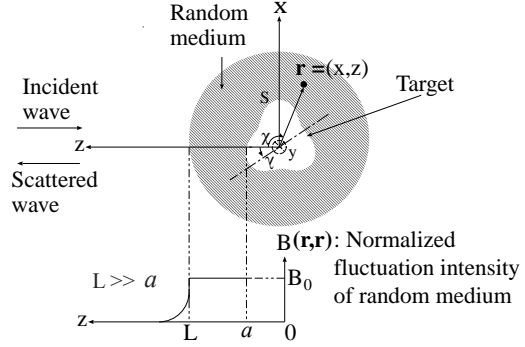


Figure 1: Geometry of the problem of wave scattering from a conducting cylinder in a random medium.

Using the current generator Y_E and Green's function in random medium $G(\mathbf{r} | \mathbf{r}')$, we can express the scattered wave as

$$u_s(\mathbf{r}) = \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 [G(\mathbf{r} | \mathbf{r}_2) Y_E(\mathbf{r}_2 | \mathbf{r}_1) u_{in}(\mathbf{r}_1 | \mathbf{r}_t)] \quad (5)$$

where \mathbf{r}_t represents the source point location and it is assumed as $\mathbf{r}_t = (0, z)$ in section 3. We consider $u_{in}(\mathbf{r}_1 | \mathbf{r}_t)$, whose dimension coefficient is understood, to be represented as:

$$u_{in}(\mathbf{r}_1 | \mathbf{r}_t) = G(\mathbf{r}_1 | \mathbf{r}_t) \exp[-(\frac{kx_1}{kW})^2] \quad (6)$$

where W is the beam width. The beam expression is approximately useful only around the cylinder. Here, Y_E is the operator that transforms incident waves into surface currents on S and depends only on the scattering body. The current generator can be expressed in terms of wave functions that satisfy Helmholtz equation and the radiation condition. More details about Y_E are available in [4]–[8].

Therefore, the average intensity of backscattering wave for E-wave incidence is given by

$$\begin{aligned} \langle |u_{se}(\mathbf{r})|^2 \rangle &= \int_S d\mathbf{r}_{01} \int_S d\mathbf{r}_{02} \int_S d\mathbf{r}'_1 \int_S d\mathbf{r}'_2 \\ & Y_E(\mathbf{r}_{01} | \mathbf{r}'_1) Y_E^*(\mathbf{r}_{02} | \mathbf{r}'_2) \\ & \langle G(\mathbf{r} | \mathbf{r}_{01}) G(\mathbf{r} | \mathbf{r}_{02}) G^*(\mathbf{r} | \mathbf{r}'_1) G^*(\mathbf{r} | \mathbf{r}'_2) \rangle \end{aligned} \quad (7)$$

We can obtain the LRCS σ using equation (7)

$$\sigma = \langle |u_s(\mathbf{r})|^2 \rangle \cdot k(4\pi z)^2 \quad (8)$$

3 Numerical Results

In the following, we conduct numerical results for LRCS and normalized LRCS (NLRCS), defined as the ratio of LRCS in random media σ to LRCS in free space σ_0 .

3.1 Radar cross-section RCS

First, we discuss, the numerical results for LRCS shown in figures 2 and 3. Obviously and as well known that for bigger kW , LRCS becomes closer to RCS in case of plane wave incidence that was shown in [6] owing to the extension of effective illumination region (EIR). Also, we observe that when δ decreases, the LRCS oscillates more largely in descending manner. The vast oscillations in LRCS can be explained as follows: with increasing δ , there is increase in the number of nonspecular points in the vicinity of convex-to-concave inflection points and these points are located in the shadow region on the scatterer surface. The contributions from these points, in addition to the specular reflections from the illuminated portion, progress in opposite directions and be out of phase so they cancel out and therefore the fluctuations decrease with δ . However, with small δ , the number of inflection points reduces and the scattering waves become sometimes in phase so they add up and sometimes out of phase so they cancel out depending on the scattering rays directions and that leads to such large oscillated behavior. This analysis agrees with a previous investigation dealing with scattering returns from illuminated and shadow portions of smooth targets with inflection points [21, 22]. The descending behavior is attributed to the EIR_b gradual shrink with ka , which in turn reduces the contribution to the scattered waves.

As kW increases as the shadow region gets smaller which accordingly reduces the number of nonspecular points and their effects as well and therefore the peak-to-peak fluctuations band be narrower as shown in figure 2. For random medium case, we notice that as the SCL increases, the behavior of LRCS in random media becomes closer to its behavior in free space except for the magnitude of LRCS. The demonstration of SCL impact on the LRCS will be analyzed in more details when discuss the ELRCS shortly in the next section.

As getting ka larger than kW , the LRCS decreases as a result of the shortage in the surface current and that leads to the gradual decrease in the scattered wave contribution with ka . In contrast, RCS with plane wave incidence is invariant with ka because the generated surface current does not alter since the target's front region facing the incident wave is always illuminated and covered by the plane wave.

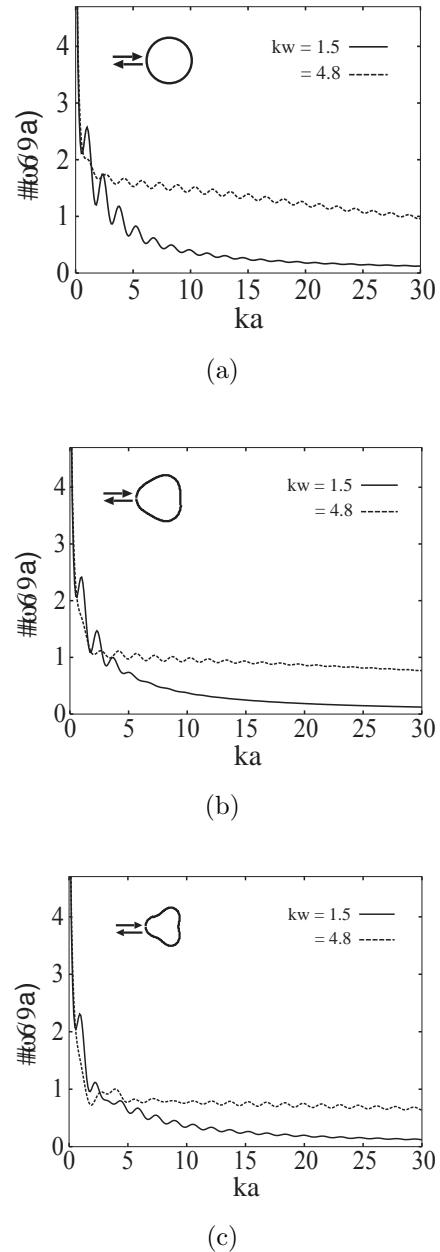


Figure 2: LRCS vs. target size in free space where (a) $\delta = 0$, (b) $\delta = 0.1$, (c) $\delta = 0.2$.

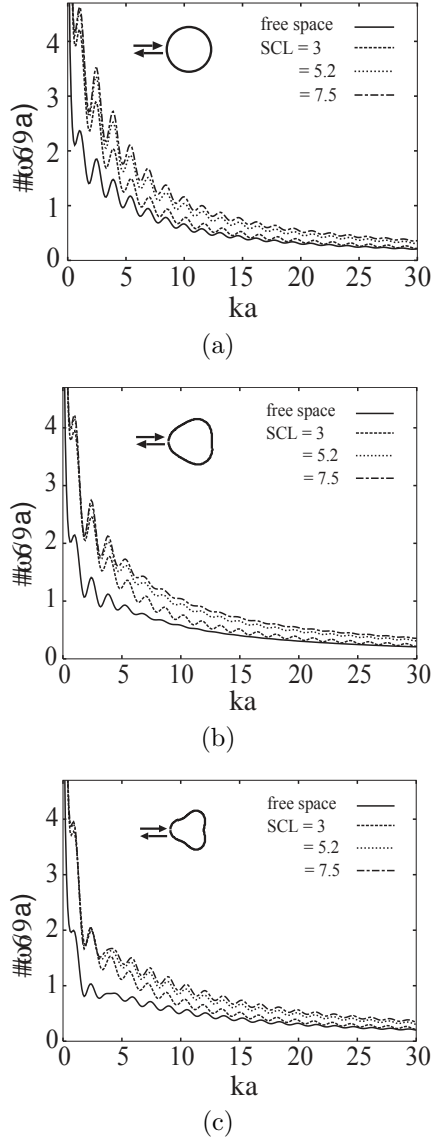


Figure 3: LRCS vs. target size in free space and at three different SCLs for $kW=2$ where (a) $\delta = 0$, (b) $\delta = 0.1$, (c) $\delta = 0.2$.

3.2 Backscattering enhancement

We consider next the NLRCS to manifest the ELRCS in random media compared to free space propagation and hence we present numerical results for NLRCS in figure 4 where $kW = 2$.

For $ka \ll SCL$, the NLRCS equals two, due to the double passage effect, and this value of NLRCS is realized, independent of illumination portion curvature, i.e., independent of the concavity index δ . In this range, beam wave seems as if it is a plane wave for the small ka .

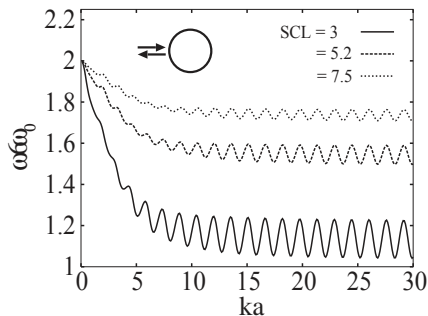
For $ka \simeq SCL$, the NLRCS changes remarkably and irregularly deviating away from two as a result of the inflection points effect discussed above that influences LRCS differently in both cases of free space and random medium trajectories. Inflection points may locate inside kW , however, they locate outside SCL depending on ka . In this case, contributions from inflection points in free space are coherent, but on the contrary they are incoherent in random medium and that makes such difference in the impact of these points and that is clear when $SCL = 3$ while $2kW = 4$ in figure 4. As δ increases, as number of inflection points increases as previously mentioned and that in turn magnify the incoherent contributions. Therefore the difference in RCS between free space and random medium becomes bigger leading to larger fluctuations in NLRCS. On the other hand when $SCL > 2kW$, the impact of inflection points in random medium becomes similar to that one in free space and, therefore, the deviation of NRCS from two is not that massy and does not change much with δ .

For $ka > SCL$, NLRCS oscillates regularly in sinusoidal behavior owing to the random medium effect, with frequency approximately equals $\pi/2$ in almost same manner irrespective of δ as the case with plane wave incidence [6]. The oscillated behavior with plane wave has waning amplitude with ka approaching to certain values. However and as shown in figure 5, the NLRCS dwindles with ka monotonically while oscillating. When $ka \gg kW$, NLRCS would diminish with large enough target and the beam wave becomes incapable of target detection.

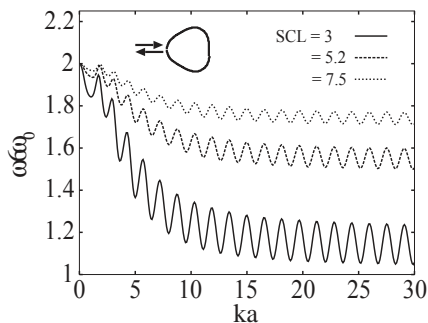
One can behold that when $SCL < 2kW$, not all EIR is used which makes NLRCS is quite far from that one when $SCL > 2kW$ (in figure 4, compare the case of NLRCS when $SCL = 3$ and that ones at $SCL = 5.2, 7.5$). Similarly, we can understand that if we set $SCL = 2kW = 4$ in figure 4, NLRCS behavior will be improved in the sense of lessening the strength of oscillations and be closer to two. On the other hand, when we let $2kW = 3$ for $SCL = 3$ as shown in figure 6, NLRCS rises up closing to 2, however, the peak-to-peak oscillations get extended more than the case with $2kW = 4$. Therefore, to reduce the strength of the fluctuations in NLRCS, the condition $SCL \geq 2kW$ should be realized. This relation proves that EIR of targets in random media depends on SCL apart from the nature of incident wave. In addition to the previous condition, SCL and accordingly kW should have wider sizes to have NLRCS closer to 2.

4 Conclusion

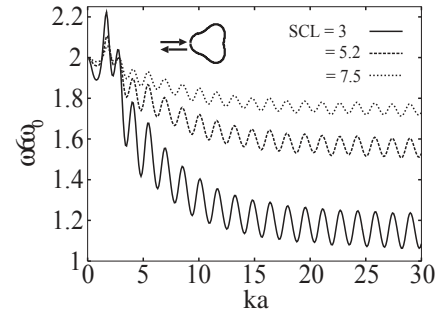
The characteristics of RCS of smooth targets with inflection points are influenced by a way that can be correlated with the contributions from the vicinity of convex-to-concave transitions on the scatterer surface in addition to the effective illumination region. These features are more obvious with beam wave incidence case compared to the plane wave incidence problem that was shown in a previous study [6]. Numerical results show that laser RCS (LRCS) suffers from oscillated behavior in random medium in a similar way to the case with plane wave incidence. However, LRCS diminishes with large enough target and the beam wave becomes incapable of target detection. To have a control over the backscattering enhancement, double passage should be the only effect that can be existed. To reach this objective, the fluctuations strength in the backscattering waves in random medium can be reduced by having SCL larger than or at least equals to the beam size. Moreover, SCL and accordingly the beam size should have wider sizes to have the enhancement in LRCS closer to two. This conclusion is valid irrespective of target configuration.



(a)



(b)



(c)

Figure 4: Normalized LRCS vs. target size at different δ for $kW=2$ where (a) $\delta = 0$, (b) $\delta = 0.1$, (c) $\delta = 0.2$ and σ, σ_0 are LRCS in random media and in free space, respectively.

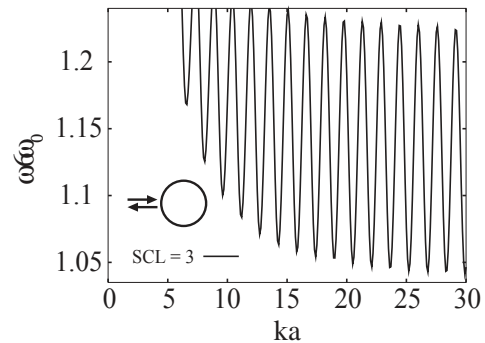


Figure 5: Enlargement of figure 4-a

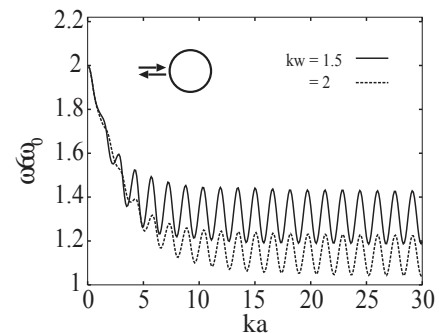


Figure 6: Normalized LRCS vs. target size for $SCL = 3$ and $\delta = 0$ at different kW .

References

- [1] Joseph B. Keller and William Streifer, "Complex rays with an application to Gaussian beams", J. Opt. Soc. Am., Vol. 61, No. 1, pp. 40–43, 1971.

- [2] Hiroyoshi Ikuno, "Calculation of far-scattered fields by the method of stationary phase", *IEEE Transactions on Antennas and Propagation*, Vol. AP-27, No. 2, pp. 199–202, 1979.
- [3] Bennett, C.L., Mieras, "Time domain scattering from open thin conducting surfaces", *Radio Science*, Vol. 16, No. 6, pp. 1231–1239, 1981.
- [4] Z. Q. Meng and M. Tateiba, "Radar cross sections of conducting elliptic cylinders embedded in strong continuous random media", *Waves in Random Media*, Vol. 6, pp. 335–345, 1996.
- [5] H. El-Ocla and M. Tateiba, "Strong backscattering enhancement for partially convex targets in random media", *Waves in Random Media*, Vol. 11, No. 1, pp. 21–32, 2001.
- [6] H. El-Ocla and M. Tateiba, "Backscattering enhancement for partially convex targets of large sizes in continuous random media for E-wave incidence", *Waves in Random Media*, Vol. 12, No. 3, pp. 387–397, 2002.
- [7] H. El-Ocla and M. Tateiba, "An indirect estimate of RCS of conducting cylinder in random medium", *IEEE Antennas and Wireless Propagation Letters*, Vol. 2, pp. 173–176, 2003.
- [8] H. El-Ocla, "Backscattering from conducting targets in continuous random media for circular polarization", *Waves in Random and Complex Media*, Vol. 15, No. 1, pp. 91–99, 2005.
- [9] Yu. A. Kravtsov and A. I. Saishev, "Effects of double passage of waves in randomly inhomogeneous media", *Sov. Phys. Usp.*, Vol. 25, pp. 494–508, 1982.
- [10] E. Jakeman, "Enhanced backscattering through a deep random phase screen", *J. Opt. Soc. Am.*, Vol. 5, No. 10, pp. 1638–1648, 1988.
- [11] Akira Ishimaru, "Backscattering enhancement: from radar cross sections to electron and light localizations to rough surface scattering", *IEEE Antennas and Propagation Magazine*, Vol. 33, No. 5, pp. 7–11, 1991.
- [12] M. I. Mishchenko, "Enhanced backscattering of polarized light from discrete random media: calculation in exactly the backscattering direction", *J. Opt. Soc. Am.*, Vol. 9, No. 6, pp. 978–82, 1992.
- [13] Vlastislav Cerveny, "Expansion of a plane wave into Gaussian beams", *Studia Geophysica et Geodaetica*, Volume: 46, Suppl. Special Issue, pp. 43–54, 2002.
- [14] Judd S. Gardner and R.E. Collin, "Scattering of a Gaussian Laser beam by a large perfectly conducting cylinder: physical optics and exact solutions", *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 3, pp. 642–652, 2004.
- [15] Haruo Sakurai, Makoto Ohki, Kuniyuki Motojima, and Shogo Kozaki, "Scattering of Gaussian beam from a hemispherical boss on a conducting plane", *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 3, pp. 892–894, 2004.
- [16] A. V. Jelalian, *Laser Radar Systems* (Artech House, Boston, Mass., 1992).
- [17] O. Steinvall, H. Olsson, G. Bolander, C. Carlsson, and D. Letalick, "Gated viewing for target detection and recognition", in *Laser Radar Technology and Applications IV*, G. W. Kamerman and C. Werner, eds., *Proc. SPIE 3707*, pp. 432–448, 1999.
- [18] Akira Ishimaru, *Wave propagation and scattering in random media*, IEEE press, 1997.
- [19] V. H. Rumsey, "Reaction concept in electromagnetic theory", *Physical Review*, Vol. 94, pp. 1483–1491, 1954.
- [20] M. Tateiba, "Numerical analysis of nonreciprocity for spatial coherence and spot dancing in random media", *Radio Science*, Vol. 17, No. 6, pp. 1531–1535, 1982.
- [21] Hiroyoshi Ikuno and Leopold B. Felsen, "Complex ray interpretation of reflection from concave-convex surface", *IEEE Transactions on Antennas and Propagation*, Vol. 36, No. 9, pp. 1260–1271, 1988.
- [22] Hiroyoshi Ikuno and Leopold B. Felsen, "Complex rays in transient scattering from smooth targets with inflection points", *IEEE Transactions on Antennas and Propagation*, Vol. 36, No. 9, pp. 1272–1280, 1988.

Acknowledgment

This work was supported in part by National Science and Engineering Research Council of Canada (NSERC) under Grant 250299-02.