The Use of ADI-FDTD Subgrids in FDTD Ground Penetrating Radar Modeling

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Abstract—Realistic numerical modeling of ground penetrating radar (GPR) using the traditional finite-difference time-domain (FDTD) method could benefit from the implementation of subgrids, especially when fine details of targets need to be modeled. In this paper, we investigate the use of the alternating-direction-implicit (ADI) FDTD technique as a subgrid into the conventional FDTD method. As ADI-FDTD is unconditionally stable, it is envisaged that a common time-step could be used in both the main FDTD grid and in the subgird. We examine the performance of ADI-FDTD subgrids when implemented into the standard FDTD method, using different communication schemes for the information exchange between the two grids. In addition, results from the comparison between the proposed algorithms and a commonly employed purely FDTD subgridding technique are presented.

1. Introduction

One of the major limitations of the FDTD approach to the numerical solution of Maxwell's equations [1], [2], is its conditionally stable nature. The FDTD stability condition, known as the CFL condition, after the initials of Courant, Freidrichs, and Lewy, states that a maximum allowable time-step is limited by a minimum cell size in the computational domain. When structures of fine geometry need to be modeled, the spatial-step should be small enough so as all the objects in the computational domain are adequately resolved. For this class of models, an FDTD solution using a uniform small spatial-step would result in substantial computer memory requirements and an increase in execution time. To alleviate this problem a possible solution is the introduction of subgrids to model fine details into the FDTD method and therefore, economize on computational resources. Most studies on the introduction of subgrids into the FDTD method employ the standard explicit method in order to calculate the fields in both the coarse and the fine grids [3]. As a result, the different spatial-steps used in the two grids lead to the requirement for different time-steps, as the CFL condition must be satisfied in both grids. Therefore, apart from spatial interpolation of the fields at the boundary between the coarse and the fine grid, time interpolation has to be also carried out. In addition, implementation could become cumbersome when trying to keep the calculations synchronized in time. To overcome the time interpolation requirement and make our calculations simpler, the ADI technique, which was initially introduced for solving parabolic differential equations [4], is implemented in the subgrid [5], instead of the explicit FDTD method. As this technique is used for the calculations in the subgrid and it is unconditionally stable, it could operate using the same time-step as the one that the traditional FDTD method uses in the main grid. A dispersion analysis [6], has proven that for the same temporal and spatial step the FDTD method is more accurate than the ADI-FDTD method. But for spatially over-resolved problems, known as ADI class problems, when a part or the whole computational domain needs to be resolved in great detail, the ADI-FDTD method provides satisfactory results while reducing the simulation run times.

The focus of our research is the FDTD simulation of GPR responses from small targets for the non-destructive testing of structures and of near surface features. Therefore, we present different implementation procedures of ADI-FDTD subgrids into the standard FDTD grid and discuss the communication between the two methods, for a two-dimensional TM_z GPR modeling case. In all our numerical tests, all media are modeled as low-loss dielectrics with constitutive parameters independent of frequency. Furthermore, all the media are considered to be non-magnetic.

2. Numerical Formulation of the ADI-FDTD Scheme

Applying the ADI-FDTD method to Maxwell's equations [7], [8], a full time-step, from n to n+1, is split into two. In the first sub-step the fields in one direction are calculated implicitly while in the other direction explicitly. For the second half sub-step the implicit and explicit evaluations are alternated. For the first half sub-step calculations, the procedure is based on (1) to (3),

$$H_x^{n+1/2}(i+1/2,j) = H_x^n(i+1/2,j) - \frac{\Delta t}{2\mu\Delta y} \cdot \left[E_z^n(i+1/2,j+1/2) - E_z^n(i+1/2,j-1/2)\right],$$
(1)

$$H_{y}^{n+1/2}(i,j+1/2) = H_{y}^{n}(i,j+1/2) + \frac{\Delta t}{2\mu\Delta x} \cdot \left[E_{z}^{n+1/2}(i+1/2,j+1/2) - E_{z}^{n+1/2}(i-1/2,j+1/2)\right],$$
(2)

$$E_{z}^{n+1/2}(i+1/2,j+1/2) = E_{z}^{n}(i+1/2,j+1/2) + \frac{\Delta t}{2\epsilon\Delta x} \Big[H_{y}^{n+1/2}(i+1,j+1/2) - H_{y}^{n+1/2}(i,j+1/2) \Big] - \frac{\Delta t}{2\epsilon\Delta y} \cdot \Big[H_{x}^{n}(i+1/2,j+1) - H_{x}^{n}(i+1/2,j) \Big].$$
(3)

The indices *i*, *j* denote the position of the cell in the computational grid. Equations (2) and (3) cannot be used directly for calculations since they have field variables at the same time-step in both left and right hand sides of the equation. Therefore, the $E_z^{n+1/2}$ component is eliminated. In this way, we get (4) for calculating the $H_y^{n+1/2}(i, j+1/2)$ component implicitly,

$$-\left(\frac{\Delta t}{2\Delta x}\right)^{2} \frac{1}{\mu\varepsilon} H_{y}^{n+1/2}(i-1,j+1/2) + \left[1 + 2\left(\frac{\Delta t}{2\Delta x}\right)^{2} \frac{1}{\mu\varepsilon}\right] H_{y}^{n+1/2}(i,j+1/2) - \left(\frac{\Delta t}{2\Delta x}\right)^{2} \frac{1}{\mu\varepsilon} H_{y}^{n+1/2}(i+1,j+1/2) \\ = H_{y}^{n}(i,j+1/2) + \frac{\Delta t}{2\mu\Delta x} \left[E_{z}^{n}(i+1/2,j+1/2) - E_{z}^{n}(i-1/2,j+1/2)\right] \\ - \left(\frac{\Delta t}{2}\right)^{2} \frac{1}{\mu\varepsilon\Delta x\Delta y} \left[H_{x}^{n}(i+1/2,j+1) - H_{x}^{n}(i+1/2,j)\right] + \left(\frac{\Delta t}{2}\right)^{2} \frac{1}{\mu\varepsilon\Delta x\Delta y} \left[H_{x}^{n}(i-1/2,j+1) - H_{x}^{n}(i+1/2,j)\right].$$
(4)

For updating the H_y field from time-step *n* to n+1/2, equation (4) requires the solution of a tridiagonal system and it can be solved in a computational efficient way with back-substitution.

For brevity, the formulation for the second half time-step, from n+1/2 to n+1, is not shown here but it is similar to the one described above. The only difference from the first procedure is that for the second half time-step, the $H_x^{n+1}(i+1/2, j)$ component is calculated implicitly.

3. Numerical Results

Fig. 1(a) shows the geometry of the computational domain. For our tests, a small rectangular void, with dimensions 20cm x 1cm, is buried in a homogeneous dielectric medium of ε_r =6 and zero conductivity. The GPR transmitting antenna is modeled as a line source (Ricker wavelet, 900 MHz). The arrangement of the electromagnetic (EM) fields is the same in both the traditional FDTD and the ADI-FDTD methods (Fig. 1(b)). In order to have a reference model, we computed the fields for our model using the standard FDTD method for the whole computational domain, for two cases: first the domain was meshed employing a very fine spatial step (domain: 500x500 cells, $\Delta x=\Delta y=1$ mm), and then setting a coarser spatial step (domain: 100x100 cells, $\Delta x=\Delta y=5$ mm). Then, we introduced an ADI-FDTD subgrid into the standard coarser FDTD grid. The main grid to subgrid ratio is 1/5 (Fig 1(b)), but this approach can be easily extended to other odd ratios. The odd integer ratio provides collocated fields in space at the subgrid and the main grid [3] (Fig. 1(b)). We use upper and lower case letters to indicate the coarse and the fine grid EM fields, respectively.

To truncate the computational domain we used a non-split Perfectly Matched Layer (PML) absorbing boundary condition (ABC). This ABC is called Recursive Integration PML and has been recently developed for FDTD elastic wave modeling [9]. This ABC does not require splitting of the fields and is based on the coordinate stretching formulation [10]. Using the RIPML, we do not need to modify the updating field equations to implement the ABC into the FDTD method; we just need to add the RIPML equations as a correction to the standard formulas.

The main issue discussed in this study relates to the communication, concerning the information exchange, between the main FDTD grid and the ADI-FDTD subgrid. This matter is not as straightforward as it has been portrayed in the literature. Here, we present two communication

schemes at the interface of the two grids, out of a number that we have tested. In the first scheme, we pass the collocated H_x , H_y main grid fields to the ADI-FDTD grid, at the boundary of the two grids (Fig. 2(a), left). The non-collocated h_x , h_y fields required for the fine grid calculations at the boundary of the two grids, are obtained using a simple linear interpolation scheme. Then, we transfer the e_z fields, which have collocated E_z fields, to the coarse grid, that are located half of a coarse grid cell inside the fine grid. For the second communication scheme, we provide the fine grid with the collocated H_x , H_y fields at the boundary of the two grids, together with the H_x , H_y fields one of a coarse grid cell into the subgrid. This leads to an overlapping region of the two grids (Fig. 2(a), right). The non-collocated magnetic fields required for the ADI-FDTD calculations are obtained using linear interpolation. Then, the E_z fields that are located one and a half of a coarse grid cell into the fine grid, are replaced by their collocated e_z fields, which are calculated by the ADI-FDTD method. To evaluate further our numerical results using the ADI-FDTD method in the subgrid, we also conducted our model simulation using the subgridding procedure proposed by Chevalier et al. [3]. This algorithm uses the standard FDTD method in both the coarse and the fine grids. In this case, as the two grids are using different time-steps, apart from the spatial interpolation, time interpolation has to be employed also, in order for the calculations in both grids to be synchronized.

Fig. 2(b) illustrates the results for the cases described above. The traditional FDTD method with a uniformly fine grid is considered to be in general the most accurate. However, using a very small spatial step (1mm) for the whole computational domain, when we want to resolve in detail only a part of it, is computationally expensive. On the other hand, a coarser spatial step (5mm) does not provide the desired resolution at some parts of the model. Using the first communication scheme between the main grid and the subgrid, the amplitude of the electric field attenuates slightly, while the second communication scheme and the algorithm proposed in [3] perform better, with the former giving the best performance (Fig. 2(b), right). Though not presented here for brevity, the proposed ADI-FDTD formulation with an overlapping cell has the smallest computed local error.

4. Conclusions

We have studied the performance of an ADI-FDTD subgrid when it is implemented into the standard explicit FDTD method. This method is particularly effective when a fine grid needs to be modeled in an area of the computational domain. Since the ADI-FDTD technique does not have to comply with the CFL stability condition, long simulation times due to small time-steps are no longer required when parts into the computational space need to be modeled in detail. We have tested the performance of our subgridding method setting the coarse to fine grid cell ratio equal to 1/5. We have described two schemes for the communication between the standard FDTD method and the ADI-FDTD technique at the boundary of the two grids. To evaluate our results, we have also developed an algorithm using a commonly applied subgridding procedure [3]. When we pass to the fine grid information from the coarse grid only at the boundary of the two grids, the calculated electric field amplitude slightly attenuates. While in the case of an overlapping cell, the ADI-FDTD method has a very good overall performance and a slightly better performance than the subgridding scheme presented in [3]. The big advantage of the proposed algorithm is that the time-step is uniform across the whole computational domain. In addition, we have not observed any instability due to the introduction of the ADI-FDTD subgrid. In all, for over-resolved problems, the merits of the computational effectiveness of the ADI-FDTD method, makes it a very powerful tool for EM modeling, with acceptable levels of accuracy. The 3-D case is subject to current ongoing work.

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Figure 1: (a) Geometry of the computational domain. (b) EM components arrangement and enlargement of the area at the coarse-fine grid boundary. The coarse to fine grid ratio is 1/5.



