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ROBUST ADAPTIVE BEAMFORMING BASED ON A GRADIENT PROJECTION METHOD

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1 Introduction

Recently, adaptive beamforming has been widely used in wireless communications, microphone array speech processing and so on. One of the adaptive beamforming methods is directionally constrained minimization of power. However, this method is known to degrade if some of underlying assumptions on the environment, sources, or sensor array become violated [5]. To resolve this disadvantage, some methods have been proposed [7, 8, 10], whereas these methods does not sufficiently exploit the degree of freedom of the step parameter in the recursive rule, or take relatively large computational complexity.

In this paper, we propose an algorithm of robust adaptive beamforming. First, we apply a gradient projection method to the minimization problem with linear constraints and derive a generalized recursive rule. We next present the convergence condition of the recursive rule and propose a design method of the step parameter in the recursive rule such that the convergence condition is approximately satisfied. Finally, we show that the proposed algorithm gives more robust performance than the conventional algorithm.

Notation: For a complex matrix A, A^T, $\bar{A}$ and A^* stand for the transpose, complex conjugate and complex conjugate transpose of A, respectively.

2 Conventional adaptive beamforming

Consider an adaptive array antenna system with N array elements as shown in Fig. 1.

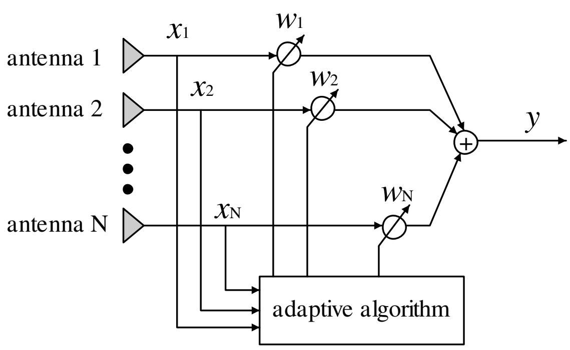

Fig. 1: Adaptive array antenna system with N antenna elements.

The output of a narrowband beamformer is given by

$$y(k) = \boldsymbol{w}^* \boldsymbol{x}(k)$$

where k is the time index, $\boldsymbol{x}(k) = [x_1(k), \ldots, x_N(k)]^T \in \mathbb{C}^{N\times 1}$ is the complex vector of array observations, and $\boldsymbol{w} = [w_1, \ldots, w_N]^T \in \mathbb{C}^{N\times 1}$ is the complex vector of beamformer weights.

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In general, when some of the directions of the desired signal and interference are known, these signals are expressed equality constraints as follows [5]:

$$\boldsymbol{w}^* C = \boldsymbol{h}$$

where M is the number of the constraints, $C \in \mathbb{C}^{N\times M}$ is a coefficient matrix and $\boldsymbol{h} \in \mathbb{C}^{M\times 1}$ is a constant vector.

The purpose of the adaptive array antenna system is to maintain the constraints on the desired signal and interference and to minimize the output interference-plus-noise power. We can express such an optimization problem as

$$\min_{\boldsymbol{w}} \frac{1}{2}\boldsymbol{w}^* R\boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{w}^* C = \boldsymbol{h}, \tag{1}$$

where $R = E[\boldsymbol{x}\boldsymbol{x}^*]$ is the covariance matrix of $\boldsymbol{x}$.

In the conventional method, the following recursive rule is derived by using the steepest descent method and Lagrange's method of undetermined multipliers [5]:

$$\boldsymbol{w}(k+1) = P[\boldsymbol{w}(k) - \mu\boldsymbol{x}(k)\bar{y}(k)] + \boldsymbol{f}, \tag{2}$$

where $\mu > 0$ is a scalar step parameter and

$$\begin{aligned} \boldsymbol{w}(0) &= \boldsymbol{f} \\ P &= I - C(C^*C)^{-1}C^* \\ \boldsymbol{f} &= C(C^*C)^{-1}\bar{\boldsymbol{h}}. \end{aligned}$$

By the recursive rule (2), the weight $\boldsymbol{w}$ is updated in the subspace related to the constraints.

3 Robust adaptive beamforming

If we can estimate R in some methods, we can apply the minimization problem (1) to the Rosen's gradient projection method [6]. As a result, we obtain the following recursive rule:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) - P\Theta P\boldsymbol{x}(k)\bar{y}(k), \tag{3}$$

where $\Theta \in \mathbb{C}^{N\times N}$ is positive semidefinite. The obtained recursive rule with $\Theta = \mu I$ turns out to be consistent with the conventional one. Therefore, the recursive rule (3) is more general than the conventional recursive rule.

Now, we make a moderate assumption that $\boldsymbol{w}$ is updated more slowly than $\boldsymbol{x}$ as in other adaptive algorithms [4]. Under this assumption, the convergence condition of the algorithm with (3) is obtained as follows:

Theorem 1 *The algorithm with (3) converges if and only if*

$$|\lambda_i(I - S^* P\Theta PRPS)| < 1 \text{ for all } i, \tag{4}$$

where $\lambda_i(A)$ denotes the eigenvalue of a matrix A and $S \in \mathbb{C}^{N\times(N-M)}$ is a matrix consisting eigenvectors corresponding to $N - M$ nonzero eigenvalues of P.

Since it is generally impossible to have the complete knowledge on R, we consider R with uncertainty as follows

$$R = \hat{R} + \Delta,$$

where $\hat{R}$ is a nominal autocorrelation matrix and Δ is an uncertainty belonging to the following set:

$$\boldsymbol{\Delta}(\gamma) := \{\Delta \in \mathbb{C}^{N\times N} : \Delta = \Delta^*, \|\Delta\| \leq \gamma\}.$$

Here, $\|A\|$ denotes the largest singular value of a matrix A. Note that the size of uncertainty is determined by the parameter $\gamma > 0$.

Moreover, it is difficult to globally solve (4) because (4) is nonconvex w.r.t. Θ. Hence, using the relation of $|\lambda_i(A)| \leq \|A\|$ for any square matrices, we seek Θ such that ρ is minimized subject to

$$\|I - S^* P \Theta P (\hat{R} + \Delta) P S\| \leq \rho. \tag{5}$$

Applying some matrix manipulations [9] to (5), we obtain the following theorem.

Theorem 2 *Suppose that $\boldsymbol{w}$ is updated more slowly than $\boldsymbol{x}$. Then, the algorithm with (3) converges if there exist an Hermitian positive semidefinite matrix Θ, real scalars τ and $\rho^2 \leq 1$ such that*

$$\begin{bmatrix} \rho^2 I - \tau\gamma^2 S^* P^* P S & (I - (S^* P \Theta P \hat{R} P S))^* & 0 \\ I - (S^* P \Theta P \hat{R} P S) & I & -S^* P \Theta P \\ 0 & -P^* \Theta^* P^* S & \tau I \end{bmatrix} > 0. \tag{6}$$

Inequality (6) is a linear matrix inequality (LMI) and is globally solvable because of its convexity.

In order to satisfy the assumption of slow updating of $\boldsymbol{w}(k)$, we reduce the values of $\|P\Theta P\|$ so that $\boldsymbol{w}(k)$ is moderately updated since the following inequality holds from (3):

$$\begin{aligned} \|\boldsymbol{w}(k+1) - \boldsymbol{w}(k)\| &= \|-P\Theta P \boldsymbol{x}(k) \bar{y}(k)\| \\ &\leq \|P\Theta P\| \|\boldsymbol{x}(k) \bar{y}(k)\|. \end{aligned}$$

To this end, we can use the following relation:

$$\|P\Theta P\| \leq \varepsilon \leftrightarrow (P\Theta P)^*(P\Theta P) \leq \varepsilon^2 I \leftrightarrow \begin{bmatrix} \varepsilon^2 I & (P\Theta P)^* \\ P\Theta P & I \end{bmatrix} \geq 0. \tag{7}$$

To sum up, we can design Θ by solving the following optimization problem: for given design parameters γ and ε, minimize ρ^2 subject to LMIs (6) and (7). This optimization problem is called a semidefinite programming problem which is efficiently solvable by using recently developed solvers [3].

4 Numerical example

Consider an adaptive array antenna system with 4 antenna elements. The desired signal $s(k)$ and the interference $u(k)$ achieve the antenna in the directions $\theta_s = 30°$ and $\theta_u = 45°$, respectively, from the broadside direction. Suppose that the interval between the array elements is the half of the wavelength ($\lambda/2$) of the carrier wave. $s(k)$ and $u(k)$ are randomly generated with $\phi_s(k), \phi_u(k) \in [0, 2\pi)$ by $s(k) = \exp(j\phi_s(k))$, $u(k) = \exp(j\phi_u(k))$, and Gaussian noises with zero-mean and 0.01-variance are added to the array elements as internal noises. All computations are carried out on MATLAB.

To compare robustness of the proposed and conventional methods, we first obtain Θ for the design parameters $\gamma = 1$, $\varepsilon = 0.58$ in the proposed method, and set $\mu = 0.01$ in the conventional method. When the antenna elements have no uncertainty, the performances, i.e., the signal to interference-plus-noise ratio (SINR), by the two methods are almost the same as shown in the left side of Fig. 2.

Next we consider the case where the second antenna element is moved by 0.075λ far from the first antenna element. The right side of Fig. 2 shows the performances by the two methods with the parameters mentioned above. We see that the proposed method is robust for the variation of the antenna element, while the conventional method is much degraded.

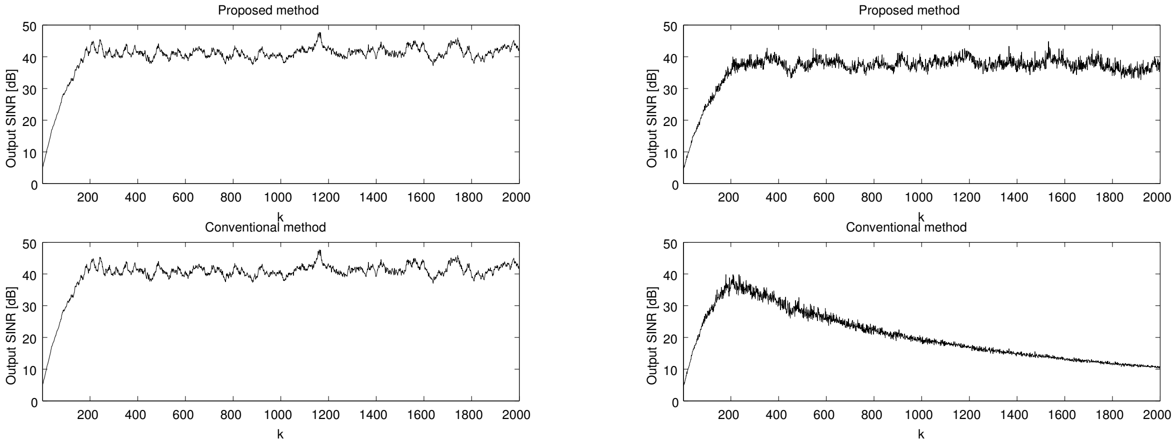

Fig. 2: Output SINR; without variation (left side); with variation (right side).

5 Conclusion

In this paper, we have proposed an algorithm for robust adaptive beamforming by applying the design method in [9]. As a result, we have shown that the proposed algorithm gives better performance than the conventional algorithm from the viewpoint of robustness.

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