

# Optimization of Broadband Beamformer with Specified Nulls and Beam Steering Capabilities

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## Abstract

This paper presents a solution of a broadband antenna array optimization problem. The beamforming problem allows an array designer to steer the mainbeam with a specified frequency response in any direction without using steering delays and to place specified nulls anywhere in the antenna pattern. Simulation results are presented to show the effectiveness of the scheme.

## 1 Introduction

In processing broadband signals induced on an array of sensors a tap delay line (TDL) filter in front of each sensor is used and the filter coefficients are estimated using some constrained beamforming problem [1]. These beamforming problems allow specification of frequency response of a presteered processor in a given direction referred to as the look direction. The presteering is done by inserting steering delays between the sensors and the TDL filter to compensate the effect of misalignment between the array geometry and the look direction. There are implementation difficulties in presteering the array in arbitrary direction particularly when the look direction is changing. Apart from specifying the response in the look direction, sometimes it is desirable to place nulls in an antenna array pattern. Broad nulls in a sector are useful to suppress slow moving interferences whereas point nulls are useful in suppressing periodically transmitting smart jammers. There are pattern synthesis techniques available in literature see for example [2–5]. However the pattern obtained using these methods is not optimized against unwanted interferences in unknown directions.

This paper presents the solution of a beamforming problem such that the processor is not only able to cancel interferences in unknown direction but also allows specified nulls in antenna pattern along with the specified response in the look direction without using steering delays.

## 2 Problem Formulation

Consider a time domain broadband antenna array processor consisting of  $L$  isotropic sensors followed by a TDL filter with  $N$  taps as shown in Figure 1.

It follows from the figure that the mean output power of the processor for a given  $\underline{W}$  is given by

$$P(\underline{W}) = \underline{W}^T R \underline{W} \quad (1)$$

where

$$\underline{W} = [ w_1^T, w_2^T, \dots, w_N^T ]^T, \quad (2)$$

$$R = E [ \underline{X}(t) \underline{X}^T(t) ], \quad (3)$$

and

$$\underline{X}(t) = [ \underline{x}^T(t), \underline{x}^T(t-T), \dots, \underline{x}^T(t-(N-1)T) ]^T. \quad (4)$$

The problem is to find a weight vector  $\hat{\underline{W}}$  such that the mean output power of the processor is minimized with the constraints that the processor has a specified response in the look direction over the band of interest and has specified nulls placed in the antenna pattern at a given number of places.

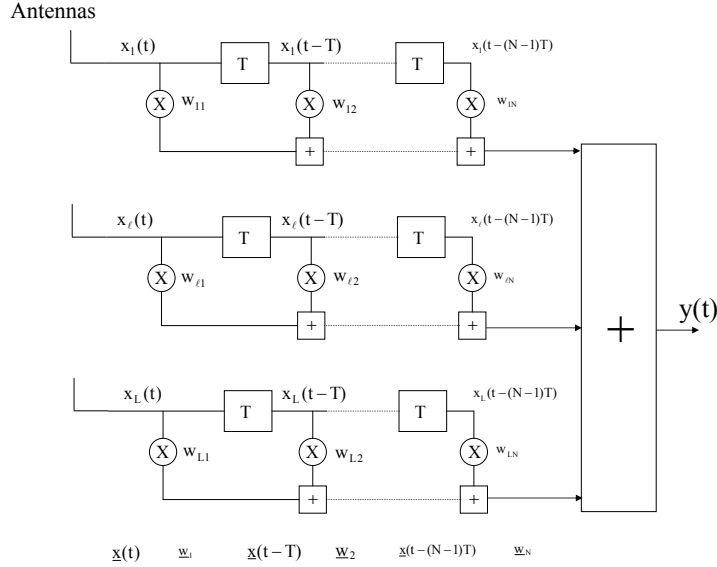


Figure 1: Broadband time domain beamformer structure.

### 3 Proposed Method

In this section a method to estimate  $\hat{W}$  is presented. The method exploits the relationship between the time domain processor as shown in Figure 1 and a frequency domain processor where broadband signals are transformed into narrowband signals using DFT and these signals are processed using narrowband structure at frequency bin using weights  $\underline{h}(k)$ ,  $k = 0, 1, \dots, N-1$  [6].

For the identical outputs in the two cases the following relationship holds [5]

$$\underline{h}^*(k) = DFT \{ \underline{w}_m \}, \quad \begin{array}{l} k=0, 1, \dots, N-1 \\ m=1, 2, \dots, N \end{array} \quad (5)$$

$$\text{and for odd } N \quad \underline{h}(N-k) = \underline{h}^*(k) \quad k=0, 1, \dots, \frac{N-1}{2}. \quad (6)$$

$$\text{Furthermore} \quad \underline{h}^H(k) \underline{S}_0(k) = \tilde{f}_k, \quad k=0, 1, \dots, \frac{N-1}{2} \quad (7)$$

where  $\underline{S}_0(k)$  is the steering vector in the look direction for the  $k$ th bin,

$$\tilde{f}_k = DFT \{ f_m \}, \quad \begin{array}{l} m=1, 2, \dots, N \\ k=0, 1, \dots, N-1 \end{array} \quad (8)$$

and  $f_m$ ,  $m = 1, 2, \dots, N$  specify the frequency response of the broadband beamformer in the look direction.

In this paper weights of the narrowband processor over the band of interest are obtained by minimizing the mean output power of the processor at each bin subjected to look direction and null direction constraints, that, is

$$\text{Minimize} \quad \underline{h}^H(k) R_f(k) \underline{h}(k) \quad (9)$$

$$\underline{h}(k)$$

$$\text{Subject to} \quad A^H(k) \underline{h}(k) = \underline{e}(k) \quad (10)$$

where

$$A(k) = [ \underline{S}_0(k), \underline{S}_1(k), \dots, \underline{S}_M(k) ], \quad (11)$$

$$\underline{e}(k) = [ \tilde{f}_k^*, \tilde{f}_{1k}^*, \dots, \tilde{f}_{1M}^* ]^T, \quad (12)$$

$\underline{S}_i(k)$ ,  $i = 1, \dots, M$  are  $M$  steering vectors at the  $k$ th bin associated with  $M$  directions where nulls are required

$$\tilde{f}_{ik} = DFT \{ f_{im} \}, \quad \begin{array}{l} m=1, 2, \dots, N \\ i=1, 2, \dots, M \end{array} \quad k=0, 1, \dots, N-1 \quad (13)$$

$f_{im}$ ,  $m = 1, 2, \dots, N$  specify the frequency response in the null direction,

$$(R_f(k))_{\ell i} = \underline{e}^H(k) (R_{\ell i}) \underline{e}(k), \quad \begin{array}{l} k=0, 1, \dots, N-1 \\ \ell, i=1, 2, \dots, L \end{array} \quad (14)$$

$$\underline{e}(k) = \left[ 1, e^{-j(\frac{2\pi}{N})k}, \dots, e^{-j(\frac{2\pi}{N})(N-1)k} \right]^T \quad k=0, 1, \dots, N-1 \quad (15)$$

and  $(R_{\ell i})$  denotes an  $N \times N$  matrix denoting the correlation between signals following elements  $\ell$  and  $i$ .

Solving the optimization problem using the Lagrange multiplier method the solution is given by

$$\hat{\underline{h}}(k) = R_f^{-1}(k)A(k) \left( A^H(k)R_f^{-1}(k)A(k) \right)^{-1} \underline{e}(k) \quad (16)$$

and the weights  $\hat{\underline{W}}$  of the broadband processor are estimated as follows, assuming that frequency bins  $k = k_1, k_1 + 1 \dots k_2$  correspond to the desired frequency band:

**Step1:** Calculate  $\hat{\underline{h}}(k)$   $k = 0, \dots, \frac{N-1}{2}$  using

$$\hat{\underline{h}}(k) = \begin{cases} R_f^{-1}(k)A(k) \left( A^H(k)R_f^{-1}(k)A(k) \right)^{-1} \underline{e}(k) & k = k_1, k_1+1, \dots, k_2 \\ \frac{S_0(k)\tilde{f}_k^*}{L} & \text{Otherwise} \end{cases} \quad (17)$$

Note that the  $\hat{\underline{h}}(k)$  outside the band of interest are taken to be the conventional weights satisfying the desired constraints.

**Step2:** Calculate

$$\hat{\underline{h}}(N-k) = \hat{\underline{h}}^*(k) \quad k = \frac{N+1}{2}, \dots, N-1. \quad (18)$$

**Step3:** Estimate  $\hat{\underline{W}}$  using

$$\hat{\underline{w}}_m = IDFT \left\{ \hat{\underline{h}}^*(k) \right\} \quad \begin{matrix} m = 1, 2, \dots, N \\ k = 0, 1, \dots, N-1. \end{matrix} \quad (19)$$

## 4 Examples and Discussion

Two examples presented here to show the effectiveness of the proposed method. A 20 element linear array with inter element spacing of a half wavelength at the highest desired frequency is used in the presence of a directional interference of power 10 dB above the signal level and the white noise power 10 dB below the signal level. A tap delay line filter of 125 taps with tap spacing  $T$  is used. The desired normalized frequency is assumed from (0.22 – 0.46) corresponding to frequency bins  $28 \leq k \leq 59$ . The normalization is carried out with the sampling frequency  $f_s = \frac{1}{T}$ . The desired frequency response of the filter in the look direction is a bandpass filter over the band of interest.  $f_{im}, m = 1, 2, \dots, 7$  are selected by designing a notch filter with the attenuation level of 0.001. The Sector nulling is accomplished by placing 5 uniformly spaced constraints over the sector interval. Point nulls are also placed at various locations for each example.

Figures 2 and 3 show the power pattern of the beamformer when the weights are estimated using the method described in the previous section. For Figure 2 the look direction and the interference direction respectively makes an angle of  $80^\circ$  and  $110^\circ$  with the line of the array whereas for Figure 3 these angles are  $110^\circ$  and  $160^\circ$  respectively. In both cases a broad null is placed in the sector of  $40^\circ$  to  $60^\circ$  and two point nulls are placed as shown in these figures.

One observes from Figure 2 and Figure 3 that the beamformer has unity response in the look direction and is able to cancel the directional sources by reducing the response below about  $-40$  dB in both examples. The broad null and the point nulls for each example are also shown in these figures. The response of the processor in these direction is about  $-40$  dB.

Figure 4 shows the frequency response of the beamformer in the look direction for Example 2. It shows that the processor has the desired response over the band of interest.

Figure 5 shows the frequency response of the beamformer in the direction of  $160^\circ$  where the interference is assumed to be present for Example 2. The reduced response of the beamformer over all the frequency band of interest is clearly visible in the figure. Similar response of the the beamformer in all other null directions were obtained and not shown here.

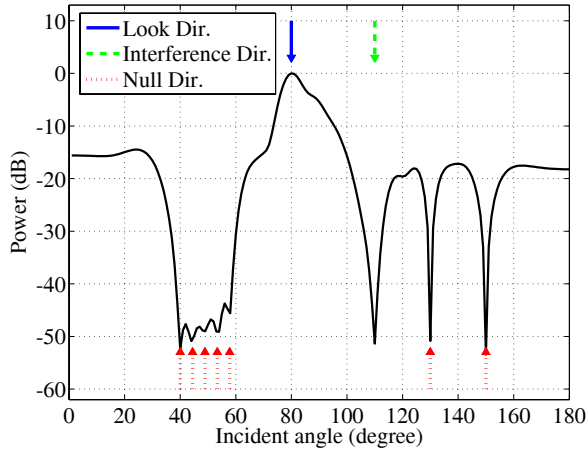


Figure 2: Power pattern of the beamformer for Example 1.

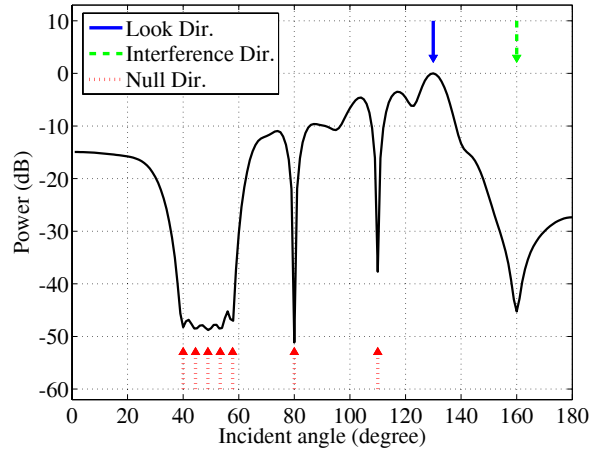


Figure 3: Power pattern of the beamformer for Example 2.

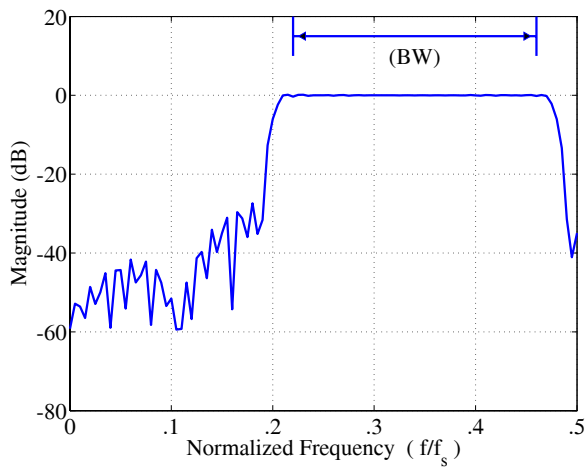


Figure 4: Frequency Response of the beamformer in the look direction for Example 2.

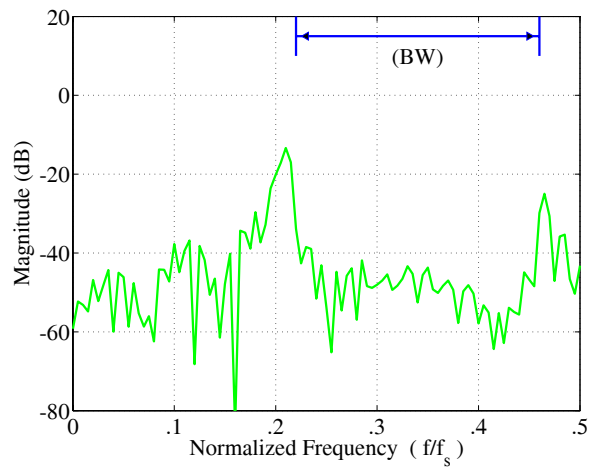


Figure 5: Frequency Response of the beamformer in the interference direction for Example 2.

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