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OPTIMUM ANTENNA POLARIZATIONS FOR POLARIMETRIC CONTRAST ENHANCEMENT

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1. Introduction

Radar polarimetry continues to be of great interest and practical importance owing to numerous applications in remote sensing [1-4]. An imaging radar polarimeter measures the polarization properties of a target and then provides a vector description of the resulting scattered wave [4]. In radar target discrimination and imaging in a clutter environment, it is very important to enhance a desired echo and to suppress a clutter by selecting the polarization states of transmitting and receiving antennas [5],[6].

This paper discusses the optimization of polarimetric contrast between two time-dependent objects. We present an efficient algorithm for finding optimum antenna polarizations based on a three-stage procedure [1]. Note that a partially polarized wave can be decomposed into a completely polarized wave and a completely unpolarized wave [7-9]. Then the ratio of two completely polarized powers in the scattered wave is optimized as a function of transmitted Stokes vector. The power received from the unpolarized portion of the scattered wave does not depend on the polarization characteristics of the receiving antenna [8]. Thus one can find the optimum receiver polarization by minimizing the power received from the polarized portion of the clutter. It should be remarked that the derivation of the optimum transmitter polarization can be decoupled from the receiving antenna parameters. The optimization procedure is illustrated with a numerical example. The optimal Stokes vectors of the transmitting and receiving antennas and the completely polarized power in the received echo are given to show the validity of the algorithm. In the present analysis, the time-averaged Mueller matrices for the target and clutter are assumed to be known *a priori* by measurements.

2. Problem Formulation

Now consider a bistatic scattering geometry. It is assumed that the transmitting and receiving antennas are located at the origins of local Cartesian coordinate systems (H,V,N) and (h,v,n), respectively. When a time-varying object is illuminated by a plane monochromatic wave, the scattered wave becomes partially polarized. The Stokes vector of the scattered wave is expressed as [8],[9]

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \langle |E_h|^2 + |E_v|^2 \rangle \\ \langle |E_h|^2 - |E_v|^2 \rangle \\ 2 \langle \mathrm{Re}(E_h E_v^*) \rangle \\ 2 \langle \mathrm{Im}(E_h E_v^*) \rangle \end{bmatrix}, \tag{1}$$

where the angle bracket indicates a time average operation, Re and Im denote the real part and the imaginary part, and the asterisk indicates complex conjugation. The relationship among the elements of **S** is given by [8]

$$S_0 \geq \sqrt{S_1^2 + S_2^2 + S_3^2}\,. \tag{2}$$

For a completely polarized wave, the sign of time averaging is omitted from Eq. (1) and the equality sign holds in Eq. (2).

As is well known [7-9], **S** is uniquely represented as a sum of the two Stokes vectors of a completely polarized wave and a completely unpolarized wave. Then one can write **S** as

$$\mathbf{S} = \mathbf{S}^{(1)} + \mathbf{S}^{(2)}$$

$$= \begin{bmatrix} \sqrt{S_1^2 + S_2^2 + S_3^2} \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} + \begin{bmatrix} S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{3}$$

Here the superscript (1) refers to the polarized portion and (2) to the unpolarized portion. Note that $S_0^{(1)} \left(= \sqrt{S_1^2 + S_2^2 + S_3^2}\right)$ denotes the scattered power of the polarized part. The degree of polarization of the scattered wave is defined as $p = S_0^{(1)}/S_0$ [8].

Let us indicate by **T** and **R** the Stokes vectors describing the transmitter and receiver polarizations, respectively. **T** and **R** are similarly expressed as Eq. (1) in terms of the coordinate systems (H,V,N) and (h,v,n), respectively. The depolarization properties of the object are described as [7]

$$\mathbf{S} = [M]\,\mathbf{T}\,, \tag{4}$$

where $[M] = (m_{ij})$ $(i,j=0,1,2,3)$ is a time-averaged Mueller matrix.

Consider the enhancement of a signal-to-clutter power ratio between two time-dependent objects. We assume that the time-averaged Mueller matrices for objects A (target) and B (clutter) are known *a priori* by measurements. From Eqs. (3) and (4), the ratio of the two completely polarized powers in the scattered wave may be written as

$$r_s = \frac{S_{A0}^{(1)}}{S_{B0}^{(1)}} = \sqrt{\frac{\mathbf{T}^t [\widehat{M}_A]^t [\widehat{M}_A] \mathbf{T}}{\mathbf{T}^t [\widehat{M}_B]^t [\widehat{M}_B] \mathbf{T}}}\,, \tag{5}$$

where $[\widehat{M}] = (m_{ij})$ $(i=1,2,3;\, j=0,1,2,3)$, and the superscript t denotes the transpose. The transmitted power is now normalized to unity and thus

$$T_0 = \sqrt{T_1^2 + T_2^2 + T_3^2} = 1\,. \tag{6}$$

Noting Eq. (6), r_s becomes a function of T_1 and T_2. The optimal transmitted Stokes vector can be obtained by solving the simultaneous equations $\partial r_s / \partial T_1 = 0$ and $\partial r_s / \partial T_2 = 0$. Therefore, the derivation of the optimum transmitter polarization can be decoupled from the receiver parameters.

The optimal Stokes vector of the receiving antenna is found by minimizing the power received from the polarized portion of the clutter [8]. Then we have

$$
\mathbf{R} = \begin{bmatrix} R_0 \\ R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} 1 \\ (1-|F_R|^2)/(1+|F_R|^2) \\ 2\,\mathrm{Re}(F_R)/(1+|F_R|^2) \\ -2\,\mathrm{Im}(F_R)/(1+|F_R|^2) \end{bmatrix}, \tag{7}
$$

where

$$
F_R = -\frac{S_{B0}^{(1)} + S_{B1}}{S_{B2} - j\,S_{B3}} . \tag{8}
$$

3. Numerical Results

Numerical results are presented for the following time-averaged Mueller matrices for the target and clutter :

$$
[M_A] = \begin{bmatrix} 0.915 & 0.028 & 0.061 & -0.040 \\ -0.701 & 0.737 & -0.403 & -0.583 \\ 0.135 & -0.339 & 0.808 & -0.665 \\ -0.214 & 0.547 & -0.220 & -0.819 \end{bmatrix}, \tag{9}
$$

$$
[M_B] = \begin{bmatrix} 0.824 & -0.015 & 0.003 & -0.062 \\ 0.158 & -0.621 & 0.256 & -0.147 \\ -0.530 & 0.303 & -0.698 & 0.386 \\ 0.461 & -0.289 & 0.512 & -0.702 \end{bmatrix}. \tag{10}
$$

These matrices satisfy the Fry-Kattawar inequalities and the Barakat condition, which are necessary constraints on the Mueller matrix elements [7].

Table 1 shows the optimal Stokes vectors of the transmitting and receiving antennas together with the corresponding values of r_s. It is confirmed that the maximum (minimum) value of r_s is larger (smaller) than those obtained for some commonly used polarizations.

Figure 1 presents the completely polarized power $P_{echo}^{(1)}$ in the received echo as a function of receiver polarization. In Fig. 1, ψ_R and χ_R denote the polarization ellipse orientation angle and the ellipticity angle of the receiving antenna, respectively.

4. Conclusion

A method for finding optimum antenna polarizations for polarimetric contrast enhancement has been presented. The derivation of the optimum transmitter polarization can be decoupled from receiving antenna parameters using a three-stage procedure. We have illustrated the optimization procedure with a numerical example to show the validity of the algorithm. Since the proposed method is based on the Stokes vector and the Mueller matrix, it can also be applied to the completely polarized case.

References

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Table 1 Optimal Stokes vectors of transmitting and receiving antennas.

Maximal Solution		Minimal Solution	
$T_0 = 1.000$	$R_0 = 1.000$	$T_0 = 1.000$	$R_0 = 1.000$
$T_1 = -0.241$	$R_1 = -0.540$	$T_1 = 0.948$	$R_1 = 0.574$
$T_2 = -0.970$	$R_2 = -0.830$	$T_2 = 0.309$	$R_2 = 0.678$
$T_3 = 0.028$	$R_3 = 0.140$	$T_3 = 0.077$	$R_3 = 0.460$
$r_{s,\,max} = 7.708$		$r_{s,\,min} = 0.386$	

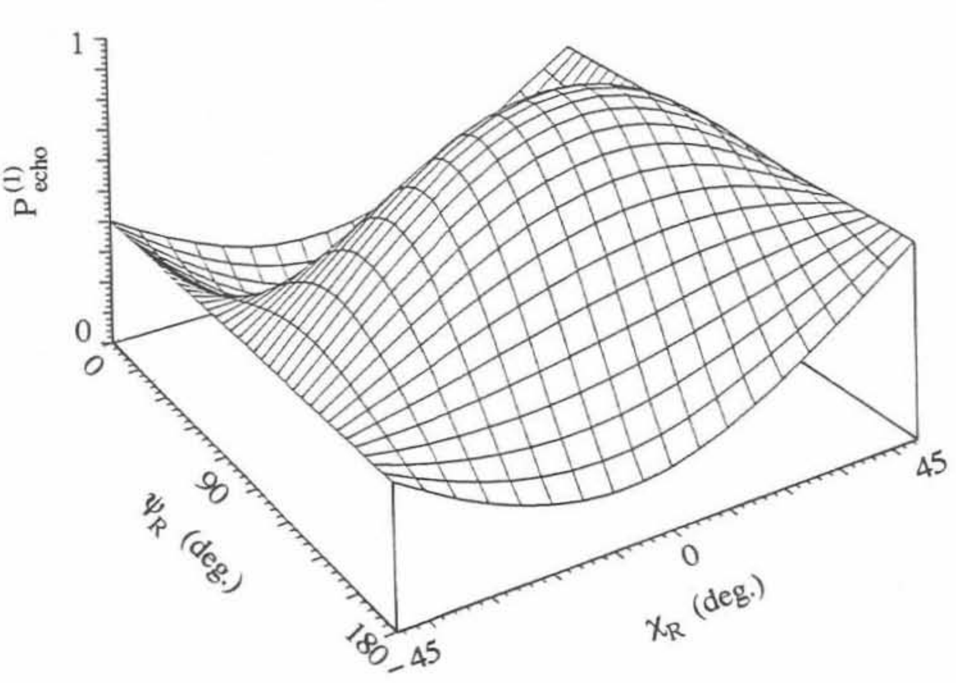

Fig. 1 Completely polarized power in the received echo.