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CALIBRATION-FREE DOA ESTIMATION BY FREQUENCY DIFFERENTIAL ARRAY ANTENNA

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1 Introduction

DOA (Direction Of Arrival) estimation with array antenna has many usage such as estimation interference waves in urban area, or estimating the position of BCON [1]. When we estimate DOA, we must consider performance degradation factor such as mutual coupling and phase and amplitude error between channels [2]. In order to estimate DOA accurately, we need to calibrate these performance degradation factor [3] [4]. In this paper, we propose calibration-free DOA estimation system which uses 2-signal correlation matrices of array antenna with large element spacing to reduce mutual coupling. The results of simulations and experiments verify that the proposed technique can estimate DOA without calibration when phase and amplitude error between channels are enough small to neglect. We apply MUSIC algorithm for simulations and experiments, and the estimation results are ebaluated with RMSE (Root Mean Square Error).

2 FREQUENCY DIFFERENTIAL ARRAY ANTENNA

In this section, we explain frequency differential array antenna. This technique may apply to all types of array antenna. We explain with linear array antenna in order to simplify the discussion. We assume L incident waves and K elements linear array whose element spacing is d. The received signal of array antenna is described as follows,

$$\boldsymbol{X}(t) = \boldsymbol{A}(\theta)\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{1}$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}(\theta_1) & \boldsymbol{a}(\theta_2) & \cdots & \boldsymbol{a}(\theta_\ell) \end{bmatrix} \tag{2}$$

$$\boldsymbol{a}_k(\theta) = exp\{-j\frac{2\pi}{\lambda}d(k-1)sin\theta\} \quad (k = 1, 2, \cdots, K) \tag{3}$$

$$= exp\{-j2\pi\frac{f}{c}d(k-1)sin\theta\} \tag{4}$$

where c is the light verocity, $\boldsymbol{S}$ and $\boldsymbol{N}$ denote signal component and noise component, respectively. $\boldsymbol{a}$ is the modevector for the component of direction matrix $\boldsymbol{A}$ and are expressed with wavelength(λ) or frequency(f). The correlation matrix of the received signal is described as follows,

$$\boldsymbol{R}_{xx} = E[\boldsymbol{X}(t)\boldsymbol{X}^H(t)] \tag{5}$$

$$= \boldsymbol{A}E[\boldsymbol{S}(t)\boldsymbol{S}^H(t)]\boldsymbol{A}^H + \sigma^2\boldsymbol{I} \tag{6}$$

$$= \sum_{\ell=1}^{L} P_\ell \boldsymbol{a}_\ell \boldsymbol{a}_\ell^H + \sigma^2\boldsymbol{I} \tag{7}$$

where E[$\cdot$], superscript H means ensemble average and complex transpose, respectively. P_ℓ, σ^2 denotes the power of ℓ-th incident angle and that of noise, respectively and $\boldsymbol{I}$ is the unit matrix. We use 2 received signals of array antenna whose element spacing is different. When we received

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2 frequencies instead of 2 element spacings, the ratio of the element in each received correlation matrix is described as follows,

$$(\boldsymbol{R}_{xx})_{Dif(i,j)} = \frac{(\boldsymbol{R}_{xx})_{1(i,j)}}{\boldsymbol{R}_{xx})_{2(i,j)}} \tag{8}$$

$$= \frac{\sum_{\ell=1}^{L} P_{1,\ell}\boldsymbol{a}_{1,i}(\theta_\ell)\boldsymbol{a}_{1,j}^H(\theta_\ell) + \sigma_1^2\boldsymbol{\delta}_{i,j}}{\sum_{\ell=1}^{L} P_{2,\ell}\boldsymbol{a}_{2,i}(\theta_\ell)\boldsymbol{a}_{2,j}^H(\theta_\ell) + \sigma_2^2\boldsymbol{\delta}_{i,j}} \tag{9}$$

where $\boldsymbol{\delta}_{i,j}$ is Cronecker's delta. When the SNR is enough large, we approximate eq. 9 by neglecting the noise factor as follow.

$$(\boldsymbol{R}_{xx})_{Dif(i,j)} \cong \frac{\sum_{\ell=1}^{L} P_{1,\ell}\boldsymbol{a}_{1,i}(\theta_\ell)\boldsymbol{a}_{1,j}^H(\theta_\ell)}{\sum_{\ell=1}^{L} P_{2,\ell}\boldsymbol{a}_{2,i}(\theta_\ell)\boldsymbol{a}_{2,j}^H(\theta_\ell)} \tag{10}$$

Assuming a single incident wave, eq. 10 is described as follows

$$(\boldsymbol{R}_{xx})_{Dif} = \frac{P_1}{P_2}\boldsymbol{a}_{Dif}\boldsymbol{a}_{Dif}^H \tag{11}$$

$$\boldsymbol{a}_{Dif,k} = \frac{\boldsymbol{a}_{1,k}}{\boldsymbol{a}_{2,k}} = exp\{-j\frac{2\pi}{\lambda}(d_1 - d_2)(k-1)sin\theta\} \tag{12}$$

$$= exp\{-j\frac{2\pi d}{c}(f_1 - f_2)(k-1)sin\theta\} \tag{13}$$

The eq. 11 is the correlation matrix of array antenna with element spacing is $d(f_1 - f_2)/c$ and the signal power of P_1/P_2. Figure 1 is the outline of frequency differential array antenna.

Now we consider performance degradation factor such as mutual coupling between antenna elements and phase and amplitude error between receiver channels. Mutual coupling is occured by coupling between antenna elements and evaluated by S parameters of array antenna. The standard array antenna has narrow element spacing less than $\lambda/2$ and strong mutual coupling. On the other hand, we can set the element spacing enough large to ignore mutual coupling because the virtual element spacing is just given as the difference of 2 element spacings or frequencies. Phase and amplitude error between receiver channels Γ should be considered if two frequencies are not close enough to have the deviation, however it can be neglected for very close frequency allocation. Γ is a diagonal matrix and we may approximate as follows when Γ can be regarded as the same value for matrix f_1 and f_2.

$$(\boldsymbol{R}_{xx})_{Dif(i,j)} = \frac{\gamma_i\gamma_i^H a_{1,i}a_{1,j}^H}{\gamma_i\gamma_i^H a_{2,i}a_{2,j}^H} = \frac{a_{1,i}a_{1,j}^H}{a_{2,i}a_{2,j}^H} \tag{14}$$

$$\boldsymbol{\Gamma} = diag\{\gamma_1, \gamma_2, \cdots, \gamma_K\} \tag{15}$$

3 SIMULATION

In this section, we show the results of simulation. We used 3 elements linear array with element spacing of 30cm. We set $f_1 = 2000$MHz and $f_2 = 1750$MHz, then the virtual element spacing is 0.25λ. The input SNR is 20dB and the snapshot number is 2000. Γ is assumed to be constant for f_1 and f_2, and the incident wave is a sinusoidal wave. Figure 2 shows MUSIC spectrum of frequency differential array and conventional linear array with element spacing of 0.25λ. The proposed array can estimate DOA accurately though the convential linear array can not estimate DOA at all because of the influence of Γ. Figure 3 shows SNR characteristics. RMSE is calculated by the average of 100 trials. RMSE decreases for high SNR and many snapshots. RMSE is less than $0.1°$ with SNR 20dB and 2000 snapshots. Figure 4 shows the estimation error when we change the frequency difference from 10MHz to 300MHz. It is equivalent to change

Table 1: Experiment Specifications

d	30cm	element number	3
f_1	2035MHz	Intermediate Frequency	10MHz
f_2	1745MHz	Sampling Frequency	40MHz
d/λ	0.29λ	Wave Source	Sinusoidal
Power	0dBm	snapshots	2000

the element spacing from 0.01λ to 0.3λ. When the frequency difference is small, the estimation accuracy decreases. The frequency difference of 250MHz (0.25λ) is needed in order to keep RMSE less than $0.1°$ with 3 elements linear array. Actually, channel characteristics are changed by the frequency difference. We need small frequency difference to minimize the deviation of Γ, then we need large element spacing in order to use near frequencies.

4 EXPERIMENT

In this section, we show the results of experiment in anechoic chamber. The specifications of experiment are shown in Table 1, and we used the array antenna shown in Fig. 5. Figure 6 shows that we have not obtained good spectrum for each incident wave and the RMSE was $2.35°$. This is caused by the deviation of Γ between f_1 and f_2. Figure 7 shows MUSIC spectrum after the calibration using the received signal from $0°$ and its estimation error is shown in Fig. 8. The RMSE after the calibration was $1.58°$. These figures show that frequency differential array can estimate DOA when Γ is constant. In this experiment, the reference signal is used, however it can be replaced by the calibration circuit between receiver channels.

5 CONCLUSION

In this paper, we proposed new DOA estimation system by the frequency differential array antenna using the ratio of two correlation matrices of array antenna. We verified the propose technique by simulations and experiments. Frequency differential array antenna can estimate DOA with high accuracy provided with very small deviation between receiver channels. Frequency differential array antenna is expected to estimate DOA without calibration when the frequency difference is enough small. We will carry out the experiment without calibration and 2D DOA estimation in the future.

References

[1] N.Kikuma, "ADAPTIVE SIGNAL PROCESSING with Array Antenna," Sciene and Technology Publishing Company, Japan 1999.

[2] R.S.Adve and T.K.Sarkar, "Compensation for the effects on the effects of mutual coupling on direct datadomain adaptive algorithms," In:IEEE Transactions on Volume 48, Issue 1, Jan 2000 Page(s):86-94

[3] T.Arai, R.Hara, H.Yamada, and Y,Yamaguchi, "Calibration Technique for Superresolution Array," In:IEICE Trans, A.P.2002-28,May.2002.

[4] H.Matsuno and H.Arai, "Calibration-free DOA estimation by the differential array antenna," ISAP2006 TD-2.

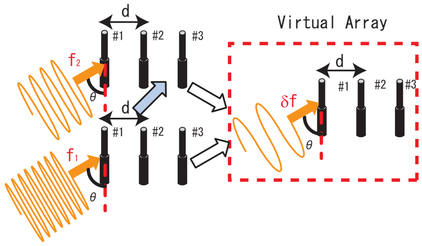

Figure 1: Image of frequency differential array

Figure 5: array antenna for experiment

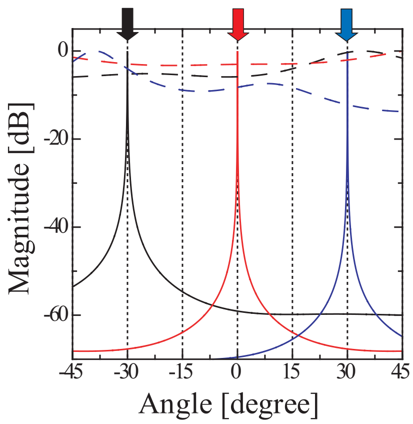

Figure 2: MUSIC spectrum

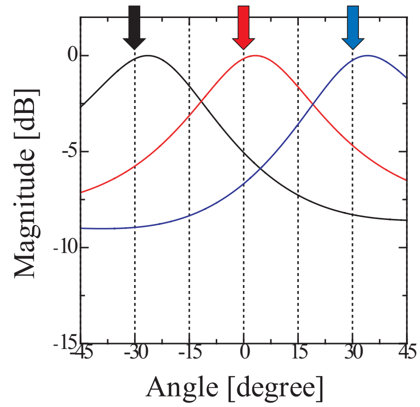

Figure 6: Experiment result

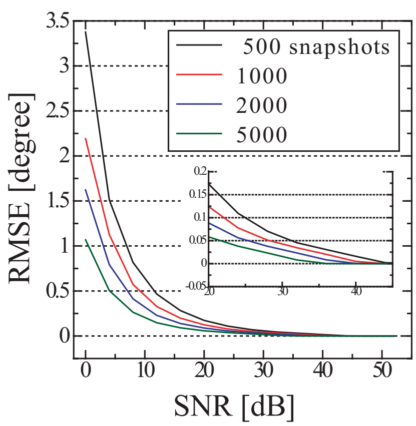

Figure 3: SNR characteristics

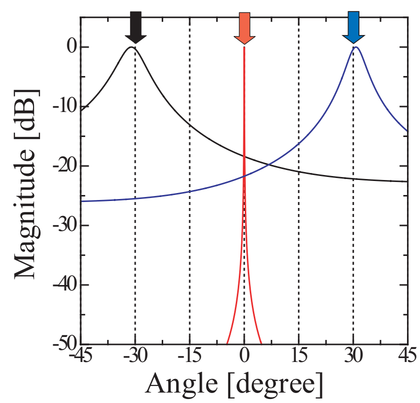

Figure 7: Frequency differetial array

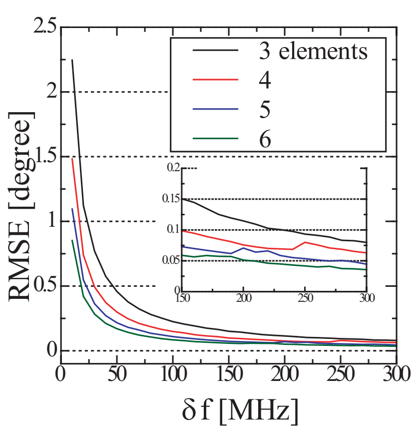

Figure 4: Apperture and estimation error

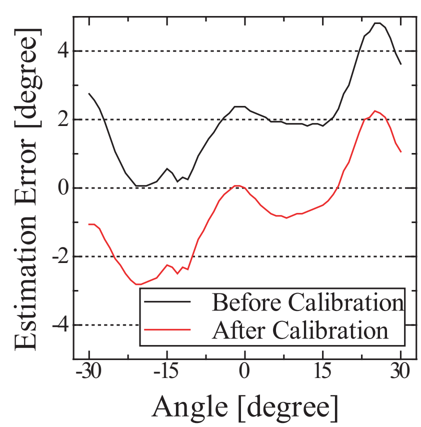

Figure 8: Frequency differetial array