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Analysis of Mueller Matrices for Optical Waves Scattered from a Random Medium with Random Rough Surfaces and Discrete Particles

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Abstract

We present the analysis of Mueller matrices for optical waves scattered from a random medium with random rough surfaces and discrete particles. The analysis is based on a model developed for a slab medium which contains Gaussian statistical rough surfaces and randomly distributed spherical particles. The refractive indices of the surrounding media are different from the background refractive index of the slab medium. This model can be applied to random media such as vegetation, terrain and biological tissue, etc. at the appropriate frequency. Small surface roughness is considered in this model. Kirchhoff rough surface scattering theory is used to calculate the scattering of optical waves from the rough surface. Vector radiative transfer theory is used to evaluate the volume scattering due to the discrete particles. The scattered wave is computed for an arbitrarily polarized incident wave for optical thicknesses $\tau = 0.4$ to 5, mean size parameters $\langle ka \rangle = 0.3$ to 1 and for various surface roughnesses. The scattered waves are expressed in terms of the modified Stokes vectors and they are found to be partially polarized. The Stokes vectors are used to construct the Mueller matrices. Polarization signatures are constructed from the Mueller matrices of the reflection side at the backscattering direction. The Mueller matrices are found to have some symmetrical properties, and there are eight nonvanishing elements.

Introduction

Many objects and matter which exist in nature have irregular shapes and forms. For example, the ocean surface, desert and terrain have random rough surfaces. However, atmospheric clouds, rain and biological particles in tissue consist of randomly distributed particles. The propagation of optical waves are, in general, influenced by the shapes and forms of these scattering objects. An understanding of how optical waves are affected by media containing such objects may lead to methods of improving techniques for laser imaging and of providing information for interpreting remote sensing measurements.

Recent interest has been focused on measuring the Mueller matrices for random media such as nonspherical particles [1], ocean water [2], and biological particles [3]. On the other hand, little has been done on the theoretical calculations of Mueller matrices except for some limited cases. Bickel [4] calculated the Mueller matrices for a single sphere by the Rayleigh, Rayleigh-Gan approximations and Mie solution; Kattawar [5] calculated the Mueller matrix for dielectric cubes by solving an integral equation; Hofer [6] employed the extended boundary condition, also called the T-matrix method, to compute the Mueller matrices for spheroids. However, the multiple scattering effect is not included in any of the above calculations.

In this paper, the vector radiative transfer equation [7] is solved for the scattered specific intensities in terms of the Stokes vectors and hence the Mueller matrices are constructed. Kirchhoff rough surface scattering theory [8,9] is used to derive the reflectivity and transmittivity matrices which govern the interaction between the surface and volume scattering at the rough surface boundary. The Mueller matrices contain the multiple scattering effect governed by the vector radiative transfer theory.

Volume and Surface Scattering

Consider a polarized plane wave with incident angle θ_i which impinges upon a slab medium containing both random rough surfaces and discrete spherical particles as shown in Fig. 1. The incident plane wave, $\bar{\mathbf{I}}_{in}$, is expressed in terms of the modified Stokes vector. The radiative transfer equation is expressed in terms of the unknown specific intensity $\bar{\mathbf{I}}(\tau, \hat{\mathbf{s}})$ as follows,

$$\mu \frac{d\bar{\mathbf{I}}(\tau, \hat{\mathbf{s}})}{d\tau} = -\bar{\mathbf{I}}(\tau, \hat{\mathbf{s}}) + \int_{4\pi} \mathbf{P}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\, \bar{\mathbf{I}}(\tau, \hat{\mathbf{s}}') d\hat{\mathbf{s}}' + \bar{\mathbf{I}}_i(\tau, \hat{\mathbf{s}}), \qquad (1)$$

where $\mu = \cos\theta, d\hat{\mathbf{s}}' = d\mu' d\phi'$ the differential solid angle,

$$\bar{\mathbf{I}}_i(\tau, \hat{\mathbf{s}}) = \int_{4\pi} \mathbf{P}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \left\{ \bar{\mathbf{I}}^{+}_{ri}(\tau, \hat{\mathbf{s}}') + \bar{\mathbf{I}}^{-}_{ri}(\tau, \hat{\mathbf{s}}') \right\} d\hat{\mathbf{s}}', \qquad \bar{\mathbf{I}}^{\pm}_{ri} = \bar{\mathbf{I}}^{\pm}_1 + \bar{\mathbf{I}}^{\pm}_2 + \bar{\mathbf{I}}^{\pm}_3 + \cdots \qquad (2)$$

and the incident plane wave is given by

$$\bar{\mathbf{I}}_{in} = \bar{\mathbf{I}}_o \delta(\mu - \mu_i)\delta(\phi), \qquad \bar{\mathbf{I}}_o = \left[I_1^{in}, I_2^{in}, U^{in}, V^{in} \right]^T. \qquad (3)$$

Here $\mathbf{P}(\hat{s}, \hat{s}')$ is the phase matrix which governs the scattering of the specific intensity from direction $\hat{s}'$ into $\hat{s}$. The elements of the phase matrix are expressed in terms of the amplitude scattering functions of the particle. The source term, $\bar{\mathbf{I}}_i(\tau, \hat{\mathbf{s}})$, comes from the redistribution of the reduced intensities, $\bar{\mathbf{I}}^{+}_{ri}(\tau, \hat{\mathbf{s}})$ and $\bar{\mathbf{I}}^{-}_{ri}(\tau, \hat{\mathbf{s}})$, inside the slab medium. The boundary conditions are expressed in terms of the reflectivity matrices $\mathbf{R}^c_{pq}$ and $\mathbf{R}^i_{pq}$ in integral forms. $\mathbf{R}^c_{pq}$ and $\mathbf{R}^i_{pq}$ are the coherent and incoherent reflectivity matrices for media pq, and they are derived by using the Kirchhoff rough surface scattering theory.

Calculations and Discussions of Mueller Matrices

Numerical techniques are employed in solving the vector radiative transfer equation (1) subject to the appropriate boundary conditions. The procedure can be summarized as follows: (i) Using the Fourier series expansion in terms of ϕ and $(\phi' - \phi)$ to extract the azimuth angle dependence from the vector radiative equation and the boundary conditions, and to separate them into a number of independent equations containing the Fourier components on ϕ. (ii) Using the Gaussian quadrature technique to discretize the continuum of the directions $0 \leq \theta \leq \pi$, where θ is the observation (scattering) zenith angle. (iii) Using the eigenanalysis to solve the remaining system of first-order ordinary differential equations subject to the boundary conditions.

The volume scattering due to the discrete spherical particles is calculated in terms of the Stokes vectors by solving the vector radiative transfer equation for four independent polarized incident waves: vertical, horizontal, 45-degree and left-hand circularly polarized waves. The total scattered waves are given by the sum of the surface and volume scattering. Figure 2 shows an example of the Mueller matrix on the reflection side of a random medium. The random medium consists of: $n_1 = 1$, $n_2 = 1.33$, $n_3 = 1$, $n_o = 1.4 + i10^{-3}$, $\theta_1 = 0$, $h = 0.05$, $l = 2$ and $\langle ka \rangle = 1$. The Mueller matrices are found to have some symmetrical properties and there are eight nonvanishing elements.

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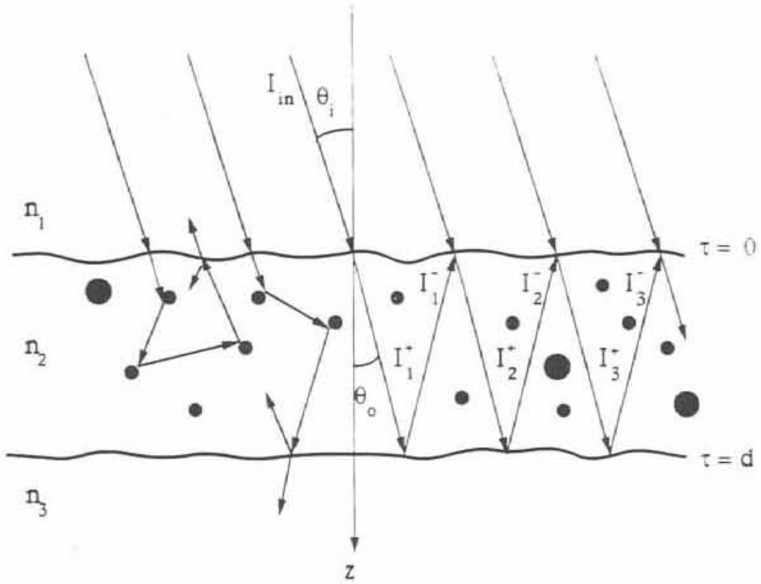

Figure 1. Geometry of a random medium with random rough surfaces and discrete particles. Here n_1, n_2 and n_3 are the background refractive indices; n_o is the refractive index of the spherical particles. The incident plane wave is expressed in terms of the modified Stokes vector and with an angle of incidence θ_i. Optical thickness is denoted by d. Surface rms height and correlation length are given by h and l.

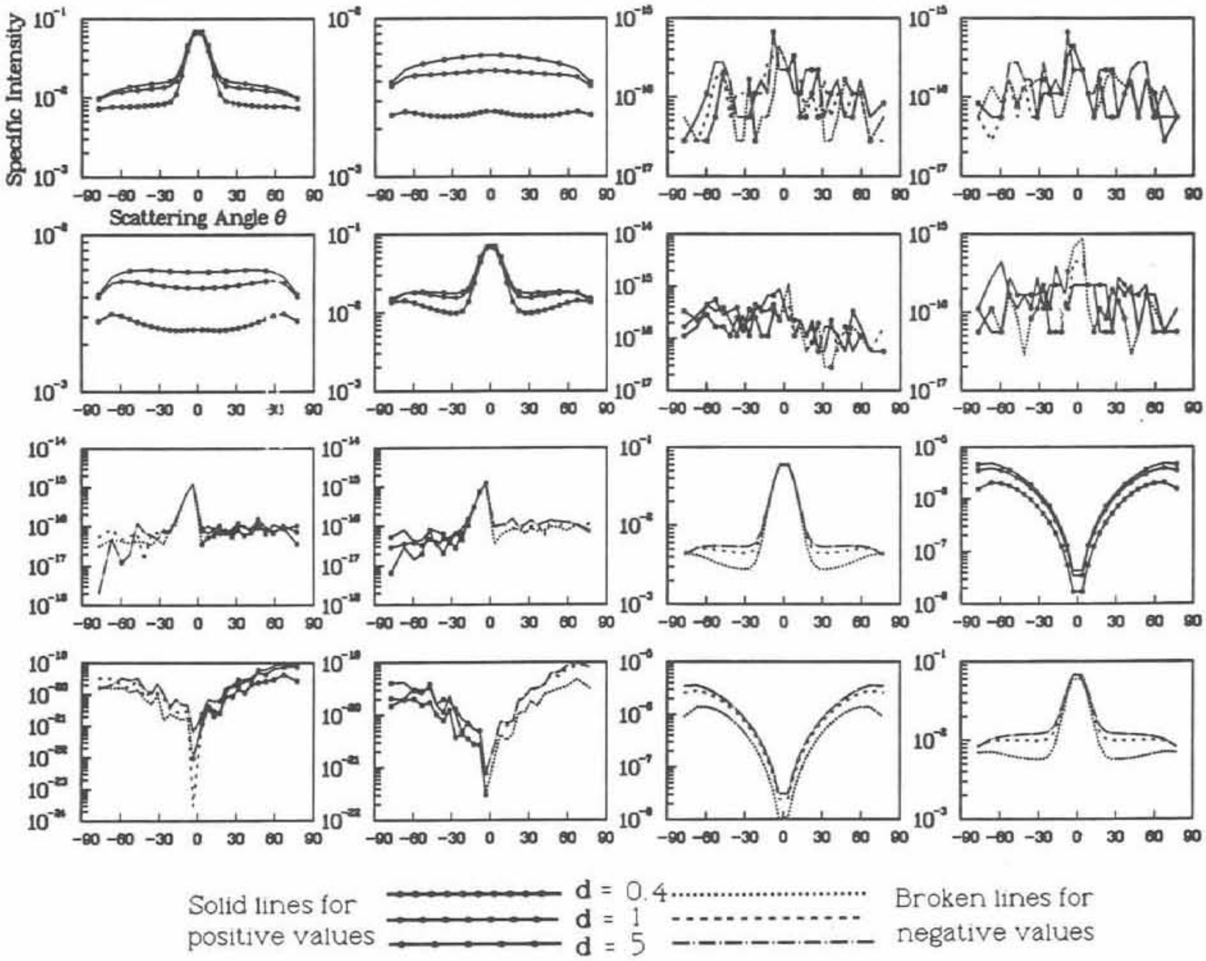

Figure 2. Mueller matrices of the reflection side for various optical thickness.