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Number of Signals Estimation Algorithm Using Multi Beam Forming and the Cyclic Shift Operation

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1. Introduction

Recently, direction of arrival (DOA) estimator has been used to search for the origin of a wave. For this objective, the MUSIC algorithm [1] which precisely estimates the DOA of multiple waves has been used. In this algorithm, an estimation of the number of signals is necessary to distinguish the signals from noise. The number of signals can be estimated by using the eigen values of the correlation matrix of the received signals, because the eigen values are related to the noise power and the signal number. The eigen values of the signals are larger than those of only the noise. However, this type of estimation assumes an equality in the received noise powers between the element antennas. In real systems, the received noise powers of element antennas are not equal because of hardware differences. Therefore, it is not easy to estimate the number of signals by using only the eigen values of the correlation matrix. To conquer this problem, AIC and MDL methods [2] [3] using statistics were proposed. However, since these algorithms need large degrees of freedom, they cannot precisely estimate the number of signals in situations where there are not enough elements. In this paper, we propose a new estimation algorithm for estimating the number of signals using multi-beam forming and the cyclic shift operation. In this algorithm, the noise powers of each element antenna are equalized by using multi beam signals. Then, in order to suppress the correlations among the noise signals in the multi beam signals, a cyclic shift operation is used to the element of the beam signals vector. This algorithm is simply designed and makes a precise estimation of the number of signals. Simulation results showed that the distribution of the eigen values is effectively suppressed by the proposed algorithm.

2. Principle for Number of Signals Estimation in MUSIC Algorithm

The MUSIC algorithm estimates the DOA of multiple signals using the eigen vectors of the correlation matrix for the noise, which is orthogonal to the steering vector of each incident signal. Therefore, the MUSIC algorithm needs the number of signals information to distinguish the eigen values of noise from the ones of a signal. The number of signals is estimated by the eigen values of the correlation matrix. Here, the received signal vector X, which has the received signals $x_m(t)$ of element antenna #m, the incident signal vector U, which has the k-th incident signals $u_k(t)$, and the noise vector, which includes the noise signals, are defined as follows:

$$X = \left[x_1(t) \quad x_m(t) \quad \cdots \quad x_M(t) \right]^T \tag{1}$$

$$U = \left[u_1(t) \quad u_k(t) \quad \cdots \quad u_K(t) \right]^T \tag{2}$$

$$N = \left[n_1(t) \quad n_m(t) \quad \cdots \quad n_M(t) \right]^T \tag{3}$$

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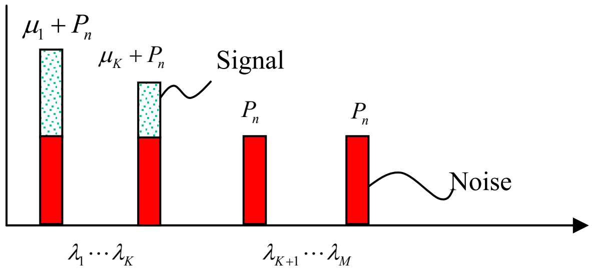

Figure 1: Eigen Value Model of Signals and Noise.

M is the number of element antennas, and K is the number of incident signals. When matrix A, which has a steering vector $\boldsymbol{a}_k$ of the k-th signal, is defined in Eq. (4), the received signal vector X is written as Eq. (5).

$$A = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_k & \cdots & \boldsymbol{a}_K \end{bmatrix} \tag{4}$$

$$X = AU + N \tag{5}$$

Correlation matrix R is calculated by Eq. (6) using the correlation matrix of the signal component R_u in Eq. (7) and the relation in Eq. (5). Here, each noise power of all the element antennas is assumed to be same, P_n. In Eq. (6), the first term is the signal component and has rank K. The second term is the noise component and has a full rank. So, the resulting eigen values are the summation of the K non-zero eigen values related to the signal and the M eigen values related to the noise power. Figure 1 shows the eigen value model. From this figure, we can see that the K eigen values related to the number of signals are larger than the other eigen values of only the noise. This characteristic makes it possible to find the number of signals. However, in practical systems, since the noise powers are different among the element antennas, the eigen values of noise are uneven, which causes an estimation failure for the number of signals.

$$R = E\left[XX^H\right] = AR_u A^H + P_n I \tag{6}$$

$$R_u = E\left[UU^H\right] \tag{7}$$

3. Number of Signals Estimation Algorithm Using Multi Beam Forming and the Cyclic Shift Operation

In this section, we propose a new estimation algorithm for estimating the number of signals using multi beam forming and cyclic shift operation. At first, the multi beam forming shown in Figure 2 is used to make the noise powers in each beam signal $y_l(t)$ $(l = 1, 2, \cdots, M)$ even. For this purpose, we used beam forming weights that have amplitudes that satisfy condition (8).

$$\left|w_{1,m}\right| = \left|w_{l,m}\right| = \cdots = \left|w_{M,m}\right| \qquad (m = 1, 2, \cdots, M) \tag{8}$$

where $w_{l,m}$ is the weight multiplied by the m-th received signal $x_m(t)$ to generate the l-th beam signal $y_l(t)$. The use of FFT weights is desirable for the beam forming because they have the same amplitude and their beams are independent of each other. In addition, their calculation cost is low. The noise power components among the beam signals are the same in condition (8) as those shown in Eq. (9). Here, b_l is the noise power component of the l-th beam signal.

$$b_l = \sum_{m=1}^{M} \left|w_{l,m}\right|^2 p_m \tag{9}$$

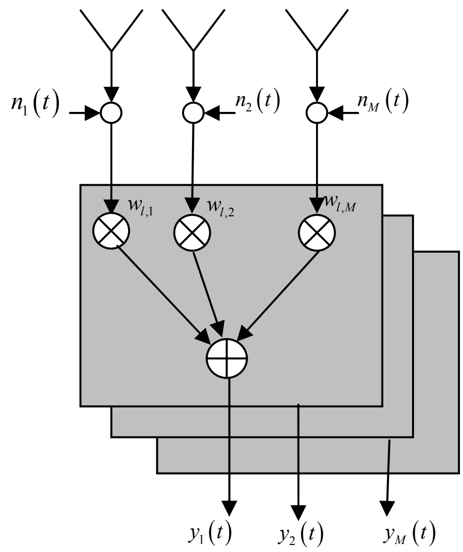

Figure 2: Configuration of Multi Beam Forming.

By using the multi beam signals for the calculation of the correlation matrix, the noise component of each signal can be made the same. However, the unevenness in eigen values is not suppressed owing to the correlation among the noise components of each beam signal. So, we use a cyclic shift operation, which cyclically shifts the samples of the beam signals and suppresses the correlation of the noise components among the beam signals. The cyclic shift operation is shown in Eqs. (10) and (11). Equation (10) shows the beam signal vector Y , which includes the time sample. By cyclically shifting the time sample, Eq. (10) changes and a new beam signal vector Y' is given, as shown in Eq. (11). The shift sample is decided by the signal bandwidth. If the ratio of the sampling frequency to the bandwidth of the noise is B_r , the B_r sample shift can completely suppress the correlation of the noise component. So, we can easily estimate the number of signals by finding the eigen values of the correlation matrix of the cyclic shifted beam signal vector Y', because the eigen values of the noise are all even.

$$Y = \left[y_1(t) \quad y_2(t) \quad \cdots \quad y_M(t) \right]^T = \begin{bmatrix} y_1(0) & y_2(0) & \cdots & y_M(0) \\ y_1(1) & y_2(1) & \cdots & y_M(1) \\ \vdots & \vdots & \ddots & \vdots \\ y_1(T-1) & y_2(T-1) & \cdots & y_M(T-1) \end{bmatrix}^T \qquad (t = 0,1,\cdots,T-1) \tag{10}$$

$\downarrow$**Cyclic Shift**

$$Y' = \left[y_1(t \bmod T) \quad y_2((t-1) \bmod T) \quad \cdots \quad y_M((t-M+1) \bmod T) \right]^T = \begin{bmatrix} y_1(0) & y_2(T-1) & \cdots & y_M(T-M+1) \\ y_1(1) & y_2(0) & \cdots & y_M(T-M+2) \\ \vdots & \begin{matrix} y_2(1) \\ \vdots \end{matrix} & \cdots & \vdots \\ y_1(T-1) & y_2(T-2) & \cdots & y_M(T-M) \end{bmatrix}^T \qquad (t = 0,1,\cdots,T-1) \tag{11}$$

Figures 3, 4, and 5 show the eigen values for the simulation of the element space, the multi-beam forming without a cyclic shift operation (CS), and the multi-beam forming with a CS, respectively. The simulation conditions are shown in Fig. 5. We used five circular array antennas, and the number of signals was assumed to be one. There is the noise distribution ratio of a maximum five. We define the distribution d as the ratio of the 2nd eigen value $\lambda 2$ and the 5th eigen value $\lambda 5$, which shows the unevenness in eigen values of the noise. From Fig. 3, we can see that the unevenness in eigen values of the noise and distribution d is 3.3. The distribution d= 3.7 in Fig. 4 of the multi-beam forming without CS operation is almost the same as that in Fig. 3. Thus, only multi beam forming processing cannot suppress the unevenness in eigen values. The multi-beam forming with CS operation shown in Fig. 5 shows that the unevenness in eigen values of the noise is effectively suppressed and the distribution is 1.2. From these results, we can see that multi-beam forming and CS operation can effectively suppress the noise unevenness and then the number of signals can be easily and precisely estimated.

4. Conclusion

In this paper, we proposed a new estimation algorithm for estimating the number of signals using multi beam forming and a cyclic shift operation. This algorithm evens out the noise power in each element antenna by using multi beam forming. The cyclic shift operation suppresses the correlations among the noise signals in multi beam signals. This algorithm is simply designed and can precisely estimate the number of signals. Simulation results confirmed that the proposed algorithm effectively suppresses the distribution of the eigen values of the noise and then the number of signals can be easily and precisely estimated.

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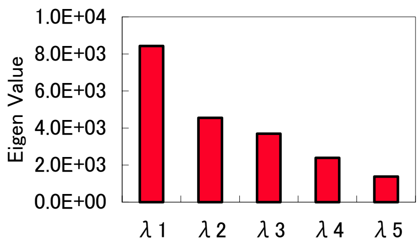

Figure 3: Eigen values for Element Space (d= 3.3).

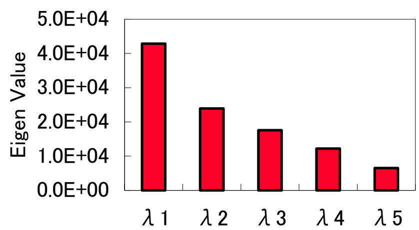

Figure 4: Eigen values for Multi Beam Forming without CS (d= 3.7).

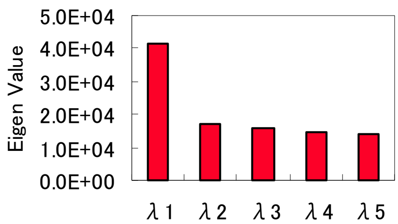

Figure 5: Eigenvalues for Multi-beam forming with CS (d= 1.2).

Conditions: Five-element circular array, Number of Signals: 1, S/N= 10 dB, DOA= 60deg, and Noise powers of elements= 1:2:3:4:5.