MMSE ANALYSIS OF SPACE-TIME EQUALIZER IN DS/CDMA SYSTEMS

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Abstract

Theoretical results using the minimum mean-squared error (MMSE) criterion are developed, which show that an integrated space-time receiver can offer significant performance improvement relative to the corresponding time-domain-only counterpart. Using the recursive least squares (RLS) algorithm, the MSE convergence towards the optimum Wiener solution is also presented.

1. Introduction

CDMA for mobile communications has received considerable interest in recent years. Much of this work has addressed the near-far problem in which strong interfering signals can overwhelm a weak desired signal in the detection process. In addition, the effect of multipath as well as multiple-access interference caused by other users sharing the same bandwidth significantly affects the quality of the received signal. Adaptive antenna array [1], particularly space-time processing techniques, can provide one of the most effective solutions to these problems. The results in this paper illustrate that the space-time (ST) receiver outperforms the corresponding time-domain-only (TDO) counterpart [2,3] in terms of MMSE and capacity.

2. System description

The receiver employs an array of M antennas, which allows it to exploit the space domain information in the received signals, and at the same time, each antenna incorporates an adaptive filter operated in a manner similar to an adaptive equalizer, with the aim to further suppress interference in the time domain. After the down-conversion the total received signal at each antenna element is chip-matched filtered (CMF) and sampled at the chip rate. Each antenna element is then followed by an N-tap delay line (TDL), with the tap delay equal to the chip period, where N is the period of spreading sequence. The received signal samples from a complete symbol interval are then shifted into the TDL for each iteration. Thus, the M TDLs linearly process $M\times N$ samples from M CMFs of the array. The outputs of TDLs are then summed and sampled at the bit rate. During the training period, the error signal ε is formed as the difference between the soft decision and the desired user's data bit. Once the MSE is at an acceptable level, the training is terminated and the data transmission begins. At this time, the error signal is formed as the difference between the soft and hard decisions. The weights of the TDLs are adapted to minimize the MSE according to an adaptive algorithm.

3. Theoretical analysis of MMSE

The spreading waveform of the kth user is assumed to be periodic with the period $T_b = NT_c$ where T_b is the bit interval, and T_c is the chip interval. The spreading code $\boldsymbol{a}_k$ of the kth user can be defined as $\boldsymbol{a}_k = (a_k(0), a_k(1), ..., a_k(N-1))^T$, and the total received sample signal vector of the ith bit interval at the mth antenna element with $m = 1, 2, \ldots, M$ is represented by $\boldsymbol{r}_m^{(i)} = [r_m(iN), r_m(iN+1), ..., r_m(iN+N-1)]^T$.

The demodulation of the ith bit $b_1(i)$ of the 1st user is now considered, in which the total received signal vector $\boldsymbol{r}_m^{(i)} \in \mathbb{C}^{N\times 1}$ at the mth antenna element can be expressed in a vector form as

$$\boldsymbol{r}_m^{(i)} = \sum_{k=1}^{K}\sum_{l=1}^{L}\sqrt{P_{k,l}}\cos(\theta_{k,l})[b_k(i)\boldsymbol{a}_{k,l}^{(i)} + b_k(i-1)\boldsymbol{a}_{k,l}^{(i-1)}]\exp(j(m-1)\frac{2\pi d\sin\phi_{k,l}}{\lambda}) + \boldsymbol{n}_m^{(i)} \quad (1)$$

where $$\boldsymbol{a}_{k,l}^{(i)} = \frac{\delta_{k,l}}{T_c} \boldsymbol{f}_k (N - p_{k,l} - 1) + (1 - \frac{\delta_{k,l}}{T_c}) \boldsymbol{f}_k (N - p_{k,l}), \quad (2)$$

$$\boldsymbol{a}_{k,l}^{(i-1)} = \frac{\delta_{k,l}}{T_c} \boldsymbol{g}_k (N - p_{k,l} - 1) + (1 - \frac{\delta_{k,l}}{T_c}) \boldsymbol{g}_k (N - p_{k,l}), \quad (3)$$

$$\boldsymbol{f}_k (e) = [0, 0, ..., a_k (0), ..., a_k (e-1)]^T , \quad (4)$$

$$\boldsymbol{g}_k (e) = [a_k (e), a_k (e+1), ..., a_k (N-1), 0, 0, ..., 0]^T . \quad (5)$$

In the above equations, $\boldsymbol{a}_{k,l}^{(i)}$, $\boldsymbol{a}_{k,l}^{(i-1)}$, $\boldsymbol{f}_k(e)$, and $\boldsymbol{g}_k(e)$ are N-dimensional vectors, $b_k(i)$ is the current data bit, and $b_k(i-1)$ is the previous data bit of the kth user. $\theta_{k,l}$ and $P_{k,l}$ are the phase, and received power of the lth path of the kth user, respectively. L is the number of paths per user, $\tau_{k,l}$ is the delay of the lth path of the kth user, $\tau_{k,l} = p_{k,l}T_c + \delta_{k,l}$, where $p_{k,l}$ is an integer, and $0 \le \delta_{k,l} < T_c$; d, λ, and $\phi_{k,l}$ are the element spacing, free-space wavelength, and direction of arrival (DOA) of the lth path of the kth user, respectively. The vector $\boldsymbol{n}_m^{(i)}$ in the mth element is a zero-mean Gaussian random vector uncorrelated in both time and space with variance σ^2, i.e., $E[n_i(t)n_j(s)] = \sigma^2\delta(t-s)\delta(i-j)$, where $\sigma^2 = (2E_b / N_o N)^{-1}$ with E_b / N_o representing the data-bit energy to one-sided noise power spectral density. It is assumed that the signal is narrowband, which means that the propagation time across the array is small enough so that the time delay at each element can be considered as a phase shift. It is also assumed that the receiver is synchronised to the desired signal (k = 1) with the strongest path (l = 1), and the carrier phase $\theta_{1,1}$ of the desired signal is perfectly tracked, i.e., $\theta_{1,1}$ = 0 and $\tau_{1,1}$ = 0, and therefore $\boldsymbol{a}_{1,1}^{(i-1)}$ is a zero vector. The total received signal vector $\boldsymbol{r}_m^{(i)}$ at the mth antenna element during the ith bit interval can be written as

$$\boldsymbol{r}_m^{(i)} = \sum_{k=1}^{K}\sum_{l=1}^{L}[d_{k,l}(i)\boldsymbol{a}_{k,l}^{(i)} + d_{k,l}(i-1)\boldsymbol{a}_{k,l}^{(i-1)}]\exp(j(m-1)\varphi_{k,l}) + \boldsymbol{n}_m^{(i)} , \quad (6)$$

where $\varphi_{k,l} = \dfrac{2\pi d \sin\phi_{k,l}}{\lambda}$, and $d_{k,l}(j) = \sqrt{P_{k,l}}\cos(\theta_{k,l})b_k(j)$, $j = i, i-1$. The observation vector $\boldsymbol{r}(i) \in \mathbb{C}^{MN\times 1}$ at time iT is then generated by concatenating the received signal vectors $\boldsymbol{r}_m^{(i)}$ defined in (6) for M array elements as

$$\boldsymbol{r}(i) \equiv [\boldsymbol{r}_1^{(i)^T}, \boldsymbol{r}_2^{(i)^T}, ..., \boldsymbol{r}_M^{(i)^T}]^T , \quad (7)$$

where $$\boldsymbol{r}_m^{(i)} = \boldsymbol{A}\Phi^{m-1}\boldsymbol{d}^{(i)} + \boldsymbol{n}_m^{(i)} ,\ m = 1, 2, .., M. \quad (8)$$

In the above equation, the matrix $\boldsymbol{A} \in \mathbb{R}^{N\times 2KL}$ is written by concatenating the vectors $\boldsymbol{A_k}$ for each k as

$$\boldsymbol{A} = [\boldsymbol{A}_1 , \boldsymbol{A}_2 , \dots, \boldsymbol{A}_K], \quad (9)$$

where $\boldsymbol{A_k} \in \mathbb{R}^{N\times 2L}$ for each k defined by $\boldsymbol{A_k} \equiv [\boldsymbol{a}_{k,1}^{(i)}, \boldsymbol{a}_{k,1}^{(i-1)}, \boldsymbol{a}_{k,2}^{(i)}, \boldsymbol{a}_{k,2}^{(i-1)}, ..., \boldsymbol{a}_{k,L}^{(i)}, \boldsymbol{a}_{k,L}^{(i-1)}]$. Similarly, the vector $\boldsymbol{d}^{(i)} \in \mathbb{R}^{2KL\times 1}$ is formed by concatenating the vectors $\boldsymbol{d}_k \in \mathbb{R}^{2L\times 1}$ for each k as

$$\boldsymbol{d}^{(i)} = [\boldsymbol{d}_1^T , \boldsymbol{d}_2^T , \dots, \boldsymbol{d}_K^T]^T , \quad (10)$$

where $\boldsymbol{d}_k \equiv [d_{k,1}^{(i)}, d_{k,1}^{(i-1)}, d_{k,2}^{(i)}, d_{k,2}^{(i-1)}, ..., d_{k,L}^{(i)}, d_{k,L}^{(i-1)}]^T$.

The matrix $\Phi \in \mathbb{C}^{2KL\times 2KL}$ is expressed in terms of sub-matrices $\Phi_{(k)}$ for each k as

$$\Phi = \begin{bmatrix} \Phi_{(1)} & & & \\ & \Phi_{(2)} & & \\ & & \ddots & \\ & & & \Phi_{(K)} \end{bmatrix} \ (11), \quad \text{where } \Phi_{(k)} = \begin{bmatrix} e^{j\varphi_{k,1}}\boldsymbol{I}_{(2\times 2)} & & & \\ & e^{j\varphi_{k,2}}\boldsymbol{I}_{(2\times 2)} & & \\ & & \ddots & \\ & & & e^{j\varphi_{k,L}}\boldsymbol{I}_{(2\times 2)} \end{bmatrix}, \quad (12)$$

and $\Phi_{(k)} \in \mathbb{C}^{2L\times 2L}$, k = 1, 2, …, K . The vector $\boldsymbol{r}(i)$ can then be written in a matrix form as

$$\boldsymbol{r}(i) = \Omega\boldsymbol{d}^{(i)} + \boldsymbol{n}^{(i)} , \quad (13)$$

where

$$\Omega = [\boldsymbol{A}^T \,|\, (\boldsymbol{A}\Phi)^T \,|\, (\boldsymbol{A}\Phi^2)^T \,|\ldots|\, (\boldsymbol{A}\Phi^{M-1})^T\,]^T \quad \in \mathbb{C}^{MN\times 2KL} \tag{14}$$

is formed by writing $(\boldsymbol{A}\Phi^m)^T$ into the columns of a matrix, and

$$\boldsymbol{n}^{(i)} \equiv [\boldsymbol{n}_1^{(i)^T}, \boldsymbol{n}_2^{(i)^T}, \ldots, \boldsymbol{n}_M^{(i)^T}]^T \in \mathbb{R}^{(MN\times 1)} . \tag{15}$$

The correlation matrix $\boldsymbol{R}$ of the observation vector $\boldsymbol{r}(i)$ can be shown to equal

$$\boldsymbol{R} = E[\boldsymbol{r}(i)\boldsymbol{r}(i)^H] = \Omega \boldsymbol{D} \Omega^H + \boldsymbol{E} . \tag{16}$$

In the above equations, $E[.]$, $[.]^T$, and $[.]^H$ denote the expectation, transpose, and complex-conjugate transpose, respectively. It is assumed that the adjacent data bits are independent of each other, and furthermore, that desired data are independent of the interference and noise. In (16), the correlation matrix $\boldsymbol{D} \in \mathbb{R}^{(2KL\times 2KL)}$ of the vector $\boldsymbol{d}^{(i)}$ can then be expressed by

$$\boldsymbol{D} = E[\boldsymbol{d}^{(i)}\boldsymbol{d}^{(i)^T}] = \begin{bmatrix} \boldsymbol{D}_1 & & & \\ & \boldsymbol{D}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{D}_K \end{bmatrix}, \tag{17}$$

with $\boldsymbol{D}_k \in \mathbb{R}^{2L\times 2L}$ and given by

$$\boldsymbol{D}_k = \begin{bmatrix} P_{k,1}\cos^2\theta_{k,1}\boldsymbol{I}_{(2\times 2)} & (P_{k,1}P_{k,2})^{1/2}\cos\theta_{k,1}\cos\theta_{k,2}\boldsymbol{I}_{(2\times 2)} & \cdots & (P_{k,1}P_{k,L})^{1/2}\cos\theta_{k,1}\cos\theta_{k,L}\boldsymbol{I}_{(2\times 2)} \\ (P_{k,1}P_{k,2})^{1/2}\cos\theta_{k,1}\cos\theta_{k,2}\boldsymbol{I}_{(2\times 2)} & P_{k,2}\cos^2\theta_{k,2}\boldsymbol{I}_{(2\times 2)} & \cdots & (P_{k,2}P_{k,L})^{1/2}\cos\theta_{k,2}\cos\theta_{k,L}\boldsymbol{I}_{(2\times 2)} \\ \vdots & \vdots & \ddots & \vdots \\ (P_{k,1}P_{k,L})^{1/2}\cos\theta_{k,1}\cos\theta_{k,L}\boldsymbol{I}_{(2\times 2)} & (P_{k,2}P_{k,L})^{1/2}\cos\theta_{k,2}\cos\theta_{k,L}\boldsymbol{I}_{(2\times 2)} & \cdots & P_{k,L}\cos^2\theta_{k,L}\boldsymbol{I}_{(2\times 2)} \end{bmatrix} \tag{18}$$

and

$$\boldsymbol{E} = E[\boldsymbol{n}^{(i)}\boldsymbol{n}^{(i)^T}] = \sigma^2 \boldsymbol{I}_{(MN\times MN)} . \tag{19}$$

The cross-correlation between the desired signal $b_1(i)$ and observation vector $\boldsymbol{r}(i)$ can be shown to equal

$$\boldsymbol{q} \equiv E[b_1(i)\boldsymbol{r}(i)] = \sum_{l=1}^{L} \sqrt{P_{1,l}}\cos\theta_{1,l}[\boldsymbol{a}_{1,l}^{(i)^T}, e^{j\varphi_{1,l}}\boldsymbol{a}_{1,l}^{(i)^T}, e^{j2\varphi_{1,l}}\boldsymbol{a}_{1,l}^{(i)^T}, \ldots, e^{j(M-1)\varphi_{1,l}}\boldsymbol{a}_{1,l}^{(i)^T}]^T \tag{20}$$

Given that $\boldsymbol{R}$ has an inverse, it follows that the optimum choice for the weight vector that gives the minimum of $E[\varepsilon^2(i)]$, i.e., the MMSE J_{min}, must satisfy

$$\boldsymbol{w}_{opt} = \boldsymbol{R}^{-1}\boldsymbol{q} \tag{21}$$

which is referred to as the Wiener-Hopf equation or the optimum Wiener solution. The resulting MMSE J_{min} is found to be [4]

$$J_{min} = \overline{|\varepsilon|^2_{min}} = 1 - \boldsymbol{q}^H \boldsymbol{w}_{opt} = 1 - \boldsymbol{q}^H \boldsymbol{R}^{-1}\boldsymbol{q} . \tag{22}$$

When the DOAs of the desired signal and interference as well as the signal power are unknown, the computation of the optimum weight vector $\boldsymbol{w}_{opt}$ is not possible, and the RLS algorithm [4] can be used to approximate this solution.

4. Numerical results

In this section, the MMSE performance of the ST structure is presented and compared to the TDO structure using (22). The code sequences are chosen arbitrarily from a Gold set of length $N = 31$. The element spacing of uniform linear array is one-half wavelength, and the desired signal is received at an E_b/N_o of 20dB. There are 10 users, and the multipath channel has 3 paths for each user with DOAs uniformly distributed in [-90°, 90°]. The path delays are uniformly distributed within 8 chips, and the results are averaged over 100 runs. In Figs. 1,3, and 4, the interference-to-desired signal power ratio P_k/P_1 ($k \neq 1$) is set to 10dB for strongest paths, and is uniformly distributed within 10dB for the remaining paths.

Fig. 1 demonstrates that a better MSE convergence with considerably reduced MSE is obtained by the ST architecture ($M = 3$) compared with the TDO counterpart ($M = 1$). The results also show that the MSE produced by the RLS algorithm converges to the MMSE J_{min} attained by the

Wiener solution for both cases (M = 1, and M = 3). Fig. 2 shows that the ST structure can also provide a better near-far resistance in the sense that the performance without power control is almost identical to the performance with perfect power control. The results in Figs. 3 and 4 indicate that MMSE J_{min} against the number of users and E_b/N_o can be further reduced by increasing the number of antenna elements M in the array.

5. Conclusions

In this paper, the simulation and analytical results clearly demonstrate that, compared to the TDO receiver, the ST architecture has been shown to provide significantly better performance and higher capacity. The results also illustrate that not only has the MMSE been significantly reduced, but the near-far resistance can also be achieved.

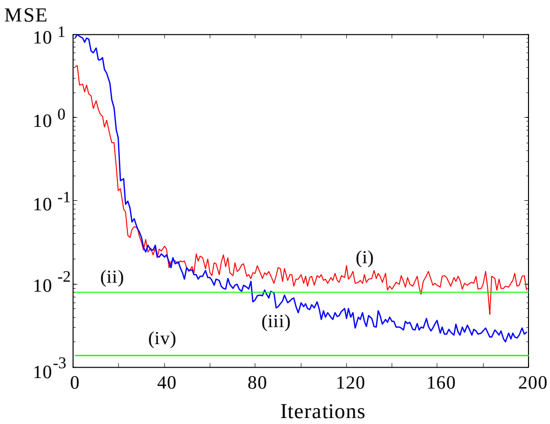

Fig. 1. *MSE convergence to MMSE J_{min}.*
Time-domain-only (M = 1):
(i) learning curve, (ii) MMSE J_{min}
Integrated space-time (M = 3):
(iii) learning curve, (iv) MMSE J_{min}

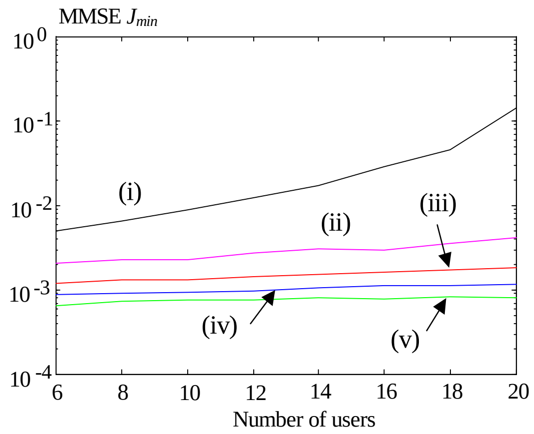

Fig. 3. *MMSE J_{min} vs number of users*
(i) M = 1, (ii) M = 2, (iii) M = 3, (iv) M = 4,
(v) M = 5

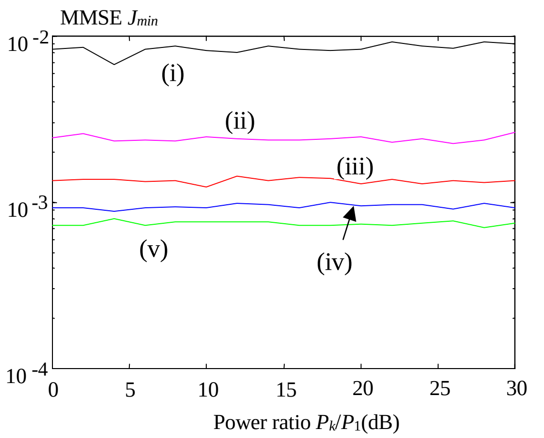

Fig. 2. *Near-far resistance*
(i) M = 1, (ii) M = 2, (iii) M = 3, (iv) M = 4,
(v) M = 5

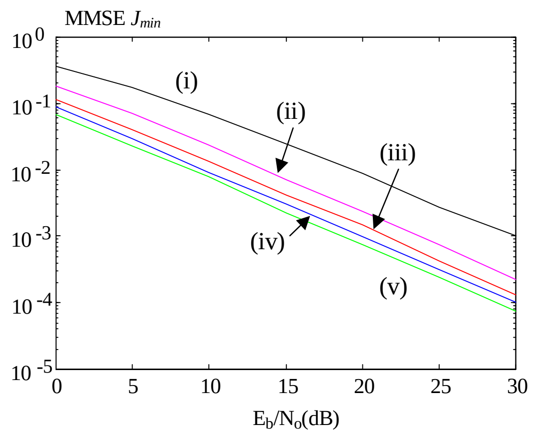

Fig. 4. *MMSE J_{min} vs E_b/N_o.*
(i) M = 1, (ii) M = 2, (iii) M = 3, (iv) M = 4,
(v) M = 5

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