

Improvement of Array Calibration Method for DOA Estimation by Using Weighted Reference Signals

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1. Introduction

Recent development of wireless communications is remarkable as observed in the increased users of cellular phones, and simultaneously, various kinds of radio waves make the radio environments much complicated. Therefore, it is important to understand the radio wave propagation structures to keep high quality of communications. For the purpose, it is most effective to estimate the signal parameters(e.g., DOA: directions of arrival) of individual incoming waves in the wireless systems. It is well known that the DOA estimators using array antennas have many advantages such as high-resolution direction finding [1]. Also, the performance of the estimators depends on the fine error calibration of the array antennas [2], [3], and the errors mainly come from the mutual coupling effects among array elements, location errors of antenna elements, and unequal gain and phase responses at receivers including antenna elements and cables [3].

To reduce those array errors, many calibration methods have been studied [2], [3]. In this paper, similar to [3], we introduce the weighting function into the array calibration method using reference(pilot) signals whose DOAs are known. Furthermore, we investigate the optimum weighting function and improved performance of the calibration method through computer simulation.

2. Array Calibration Method for DOA Estimation

Using the K -element array to which we apply the calibration, we receive individually N narrowband reference signals whose DOAs: $\theta_1, \dots, \theta_N$ are known. Then, the array input vectors are expressed as

$$\mathbf{x}_i(t) = \mathbf{M}\mathbf{a}(\theta_i)s_i(t) + \mathbf{n}_i(t) \quad (i = 1, 2, \dots, N) \tag{1}$$

where $\mathbf{a}(\theta_i)$ and $s_i(t)$ are the array response vector without array errors and the complex amplitude, respectively, of the i th reference signal. Also, $\mathbf{n}_i(t)$ is the internal additive noise vector in receiving the i th reference signal, and \mathbf{M} is the $K \times K$ array error matrix including effects such as inter-element mutual coupling. Estimating the matrix $\text{vec}\mathbf{M}$ accurately is the purpose of calibration. The correlation(covariance) matrices of the input vectors $\mathbf{x}_1(t), \dots, \mathbf{x}_N(t)$ are obtained as follows.

$$\mathbf{R}_i = E[\mathbf{x}_i(t)\mathbf{x}_i^H(t)] = \sigma_{s_i}^2\mathbf{a}_m(\theta_i)\mathbf{a}_m^H(\theta_i) + \sigma_n^2\mathbf{I} \quad (i = 1, 2, \dots, N) \tag{2}$$

where

$$\mathbf{a}_m(\theta_i) = \mathbf{M}\mathbf{a}(\theta_i) \quad (i = 1, 2, \dots, N) \tag{3}$$

and $\sigma_{s_i}^2$ and σ_n^2 are input powers of the i th reference signal and internal noise, respectively. The dominant eigenvalue λ_i and the corresponding normalized eigenvector \mathbf{e}_i of the matrix \mathbf{R}_i are given by

$$\lambda_i = \sigma_{s_i}^2\mathbf{a}_m^H(\theta_i)\mathbf{a}_m(\theta_i) + \sigma_n^2 \tag{4}$$

$$\mathbf{e}_i = \mathbf{a}_m(\theta_i)/\|\mathbf{a}_m(\theta_i)\| \equiv \mathbf{a}_e(\theta_i) \tag{5}$$

because there are the following relations

$$\mathbf{R}_i\mathbf{a}_m(\theta_i) = \{\sigma_{s_i}^2\mathbf{a}_m^H(\theta_i)\mathbf{a}_m(\theta_i) + \sigma_n^2\}\mathbf{a}_m(\theta_i) \quad (i = 1, 2, \dots, N) \tag{6}$$

Table 1: Simulation conditions.

array configuration	4-element uniform linear array
element spacing	half wavelength
antenna element	half wavelength vertical dipole
number of snapshots for DOA estimation	50

Table 2: Radio environment.

number of waves	1
DOA from array broadside	-80 deg to 80 deg
SNR	20dB

As seen from the above expressions, $\mathbf{a}_m(\theta_i)$ is obtained from the eigenvector corresponding to the dominant eigenvalue of the correlation matrix \mathbf{R}_i . Using eq.(5) and the relation

$$\mathbf{a}_m(\theta_i) = c_i \mathbf{a}_e(\theta_i) \quad (c_i: \text{complex constants}) \quad (7)$$

we find the \mathbf{M} and c_i which minimize the following cost function.

$$J = \sum_{i=1}^N w_i \|\mathbf{M} \mathbf{a}(\theta_i) - c_i \mathbf{a}_e(\theta_i)\|^2 \quad (8)$$

where $w_i (i = 1, 2, \dots, N)$ are real-valued weights for the reference signals. Furthermore, eq.(8) leads to

$$J = \|(\mathbf{M} \mathbf{A} - \mathbf{A}_e \mathbf{A}) \mathbf{W}^{1/2}\|_F^2 \quad (9)$$

with

$$\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_N)] \quad (10)$$

$$\mathbf{A}_e = [\mathbf{a}_e(\theta_1), \dots, \mathbf{a}_e(\theta_N)] \quad (11)$$

$$\mathbf{A} = \text{diag}\{c_1, \dots, c_N\} \quad (12)$$

$$\mathbf{W} = \text{diag}\{w_1, \dots, w_N\} \quad (13)$$

where $\|\cdot\|_F$ denotes the Frobenius norm. The function J is minimized with respect to both \mathbf{M} and \mathbf{A} . The solution of \mathbf{M} which is denoted by $\hat{\mathbf{M}}$ is the calibration matrix desired for DOA estimation.

By giving the weight matrix \mathbf{W} effectively, it is expected that we can obtain the calibration matrix $\hat{\mathbf{M}}$ suitable for the DOA estimation algorithm.

3. Performance Analysis by Computer Simulation

Under conditions shown in Tables 1–4, the computer simulation is carried out to clarify the performance. The number of snapshots in Table 3 is for obtaining the matrices $\mathbf{R}_1, \dots, \mathbf{R}_N$. As the array error, we have here the mutual coupling among the antenna elements and antenna element location errors which are given in Table 4. The sign '+' in the antenna location error of Table 4 means the error in the direction from element 1 to element 4. Also, the mutual coupling effects are computed using ICT(Improved Circuit Theory) [4].

The DOA estimation process in this paper is as follows:

Table 3: Setup of reference signals.

number of reference signals	15
DOA from array broadside	-70, -60, ..., 60, 70 [deg]
SNR	30dB
number of snapshots	100

Table 4: Location errors of antenna elements.

	element 1	element 2	element 3	element 4
error	0	+0.1 λ	-0.1 λ	+0.05 λ

(λ : wavelength)

Step 1: The DOA estimation is carried out using the Beamformer method [1].

Step 2: The adequate weight matrix \mathbf{W} is determined, and then the calibration matrix $\hat{\mathbf{M}}$ is computed.

Step 3: To estimate DOAs more accurately, the MUSIC algorithm [1] is applied to the calibrated correlation matrix $\hat{\mathbf{M}}^{-1} \mathbf{R}_{xx} (\hat{\mathbf{M}}^H)^{-1}$ where \mathbf{R}_{xx} is the correlation matrix of array inputs for DOA estimation.

For simplicity, the weighting function is set to be rectangular, and so the value of each weight w_i is 1 or α ($0 < \alpha < 1$). An example of weighting function with the rectangle width of 5 is illustrated in Fig.1. Here, the rectangle width means the number of reference signals with the weight equal to 1, and θ_p in Fig.1 represents the estimated DOA in **Step 1** above. Figures 2–4 show the RMSE(Root Mean Square Error) obtained from independent trials of 100 in **Step 3** versus the DOA of the incident wave for the cases where the values of rectangle width are 3, 5, and 7, respectively. In those figures, α is equal to 10^{-3} , 10^{-4} , or 10^{-5} . Also, the stochastic Cramer-Rao bound(CRB) [5] is shown.

From the figures, it is observed that there seem to be the optimum value of α and the optimum rectangle width. In this simulation, the case of the rectangle width of 3 is seen to outperform the other cases because the sensitivity of choice of α is relatively low.

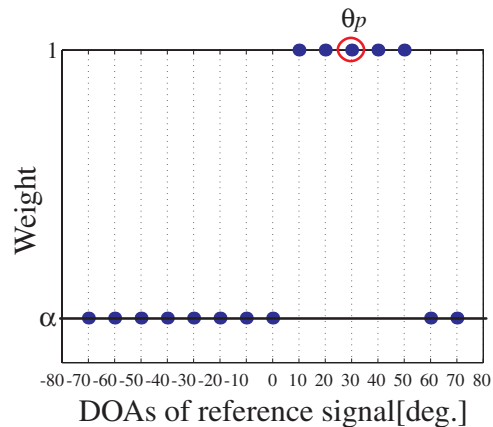


Figure 1: Example of rectangular weighting ($\theta_p = 30\text{deg}$).

4. Conclusion

In this paper, we have investigated the effect of the rectangular weighting on the array calibration using reference signals via computer simulation of DOA estimation. As a result, it is shown that there are the optimum rectangle width and height. In the case of 4-element array with half wavelength inter-element spacing, the rectangular weighting with the width of 3(i.e., 20deg) and $\alpha = 10^{-5} \sim 10^{-3}$ gives good performance. As the future work, we have to examine the performance when multiple waves are incident on the various types of array.

References

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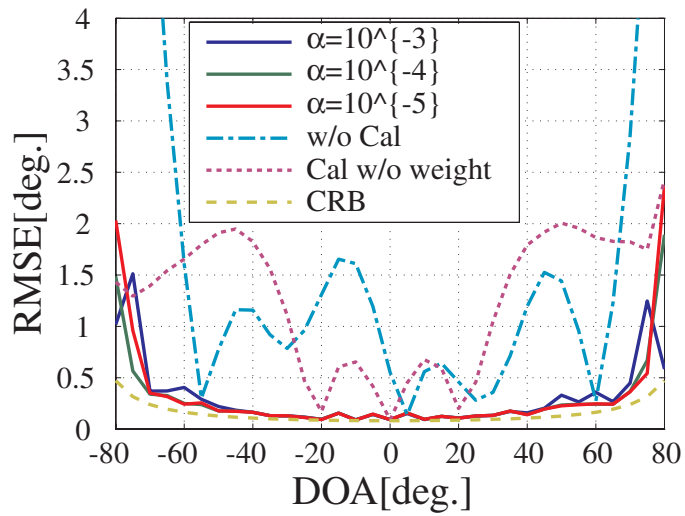


Figure 2: DOA estimation results in the case of rectangle width of 3.

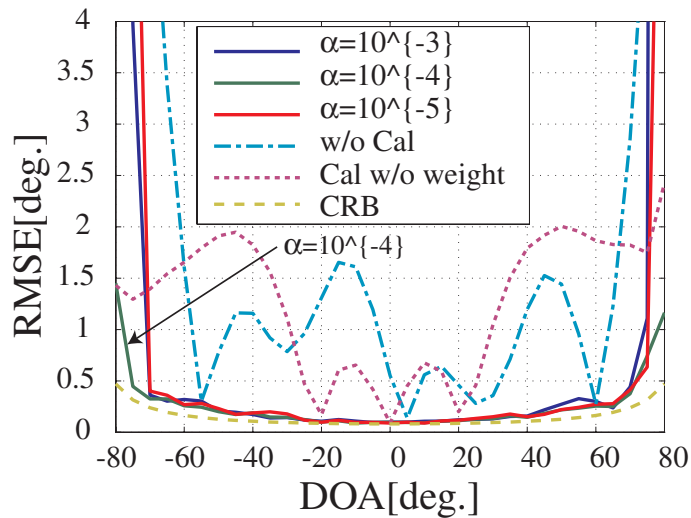


Figure 3: DOA estimation results in the case of rectangle width of 5.

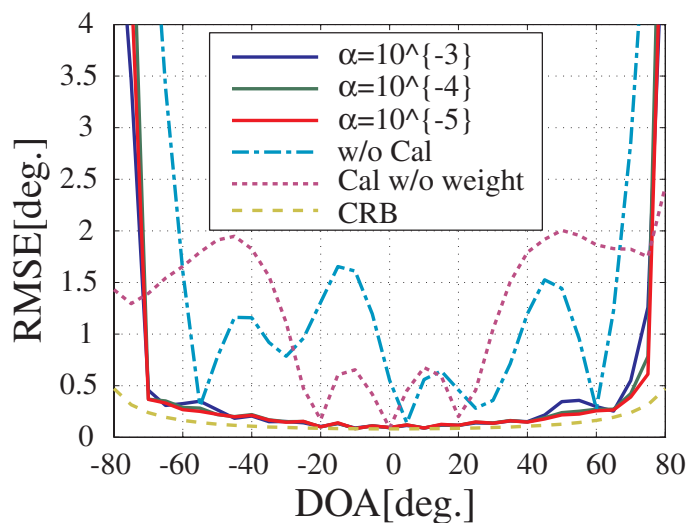


Figure 4: DOA estimation results in the case of rectangle width of 7.