2D3-1

# High Resolution DOA Estimation Using Phase Information Of MUSIC Spectrum

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## 1. Introduction

Direction-Of-Arrival (DOA) estimation is one of fundamental but significant techniques for high-speed mobile communication to know wave propagation environment [1]. "Superresolution" DOA estimation methods such as MUSIC [2], Root-MUSIC [3] and ESPRIT [4] methods are paid so much attention because of their brilliant properties in estimating DOAs of incident signals. Those methods achieve very high accuracy in estimating DOAs in a good propagation environment, but would fail to estimate DOAs in severe environments like low Signal-to-Noise Ratio (SNR), small number of snapshots, or when incident waves are coming from close angles.

Here we try to further improve the DOA estimation accuracy of search-based MUSIC algorithm by introducing a new concept of using phase characteristics in DOA estimation. As is well-known, MUSIC spectrum is calculated based on the absolute value of the inner product between array response vector and noise eigenvector, means that MUSIC employs only the amplitude characteristics and does not use any phase characteristics. Recalling that phase characteristics plays an important role in signal and image processing, we expect that DOA estimation accuracy could be further improved using phase characteristics in addition to MUSIC spectrum.

In this paper, we develop a novel approach to DOA estimation mainly based on phase information of noise eigenbeams. The accuracy of DOA estimation is evaluated through some simulations in comparison with some representative algorithms, and we found that the proposed method achieve very high accuracy in severe environments like low SNR or incident waves coming from close angles.

## 2. Preliminaries

### 2.1 Signal and Array Model

Assume that $L$ uncorrelated narrowband incident waves $s_\ell(t)$, $(\ell = 1, 2, \cdots, L)$ arrive at $N$-element array antenna from the direction of $\theta_\ell$, respectively. We assume AWGN process and the incident waves are uncorrelated to each other and also to noise signals. The complex input signal vector $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_K(t)]^T$, where $x_k$ is the input signal at $n$-th array element, is written as

$$\boldsymbol{x}(t) = \sum_{\ell=1}^{L} s_\ell(t)\boldsymbol{a}(\theta_\ell) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{a}(\theta_\ell)$ denotes array response vector for $\ell$-th wave. In (1), array response matrix $\boldsymbol{A}$, incident signal vector $\boldsymbol{s}(t)$, and noise signal vector $\boldsymbol{n}(t)$ are respectively given by

$$\begin{aligned}
\boldsymbol{A} &= [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \ldots, \boldsymbol{a}(\theta_L)], \\
\boldsymbol{a}(\theta_\ell) &= [a_1(\theta_\ell), a_2(\theta_\ell), \ldots, a_N(\theta_\ell)]^T, \\
\boldsymbol{s}(t) &= [s_1(t), s_2(t), \ldots, s_L(t)]^T, \\
\boldsymbol{n}(t) &= [n_1(t), n_2(t), \ldots, n_N(t)]^T.
\end{aligned}$$

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The covariance matrix $\boldsymbol{R}_{xx}$ of the array input vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{R}_{xx} = E\left[\boldsymbol{x}(t)\boldsymbol{x}^H(t)\right] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \sigma^2\boldsymbol{I}_N$$

where $E[\cdot]$ denotes the expectation value, $\boldsymbol{S} = E[\boldsymbol{s}(t)\boldsymbol{s}^H(t)]$, $\sigma^2$ is the noise power, and $\boldsymbol{I}$ is the identity matrix.

### 2. 2 MUSIC method

MUSIC method first separates the input signal space into signal- and noise-subspaces, and uses the noise-subspaces to estimate DOAs. MUSIC spectrum $P_{MU}(\theta)$ is written as

$$P_{MU}(\theta) = \boldsymbol{a}^H(\theta)\boldsymbol{a}(\theta) \Bigg/ \left(\sum_{n=L+1}^{N} \left|\boldsymbol{e}_n^H\boldsymbol{a}(\theta)\right|^2\right) = \frac{\boldsymbol{a}^H(\theta)\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{a}(\theta)}, \tag{2}$$

where $\boldsymbol{e}_n, (n = L+1, \ldots, K)$ denote the noise eigenvectors, and $\boldsymbol{E}_N = [\boldsymbol{e}_{L+1}, \boldsymbol{e}_{L+2}, \ldots, \boldsymbol{e}_N]$.

Finding $L$ peaks of MUSIC spectrum, the estimated DOAs are given as the corresponding angles of $L$ peaks. Figure 1(a) shows an example MUSIC spectrum where two waves from $\theta_1 = 0$[deg] and $\theta_2 = 30$[deg] are received by 6-element half-wavelength ULA when SNR=0[dB] and 100 snapshots.

## 3. Proposed DOA Estimation Method

The proposed DOA estimation method is described in this section.

Let $Q_n(\theta)$ and $\tilde{Q}_n(\theta)$ respectively denote the phase characteristics of $\boldsymbol{e}_n^H\boldsymbol{a}(\theta)$ and its phase unwrapping version, i.e.,

$$Q_n(\theta) = \arg\{\boldsymbol{e}_n^H\boldsymbol{a}(\theta)\} \tag{3}$$

$$\tilde{Q}_n(\theta) = \mathrm{unwrap}\{Q_n(\theta)\} \tag{4}$$

where $\arg\{\cdot\}$ and $\mathrm{unwrap}\{\cdot\}$ denote the phase characteristics and the phase-unwrapping, respectively. Figure 1(b) shows example phase characterisics where two waves are coming from 0[deg] and 30[deg]. We see from Fig.1(b) that the gradient of $\tilde{Q}_n(\theta)$ of 4 becomes large at the angles of DOAs.

Here we define the differentiation $\tilde{Q}'_n(\theta) = d\tilde{Q}_n(\theta)/d\theta$, where Fig. 1(b) shows its example. We see from Fig. 1(c) that the differentiation $\tilde{Q}'_n(\theta)$ makes sharp peaks at the angles of DOAs. Note that the differentiation $\tilde{Q}'_n(\theta)$ as the parallel average of the absolute values of $\tilde{Q}'_n(\theta)$, i.e.,

$$\tilde{Q}'(\theta) = 1 \Bigg/ \sum_{n=L+1}^{K} \frac{1}{|\tilde{Q}'_n(\theta)|} \tag{5}$$

Multiplying the MUSIC spectrum $P_{MU}(\theta)$ of (2) to (5), the proposed phase-based spectrum $P_{PHA}(\theta)$ is given by $P_{PHA}(\theta) = \tilde{Q}'(\theta)P_{MU}(\theta)$. Similarly to MUSIC method, DOA estimation could be done by the peak search of the derived spectrum. Figure 1(d) shows the example spectrum of $P_{PHA}(\theta)$.

The phase-based spectrum $P_{PHA}(\theta)$ could preserve the following two properties: high-resolution sharp peaks of $\tilde{Q}'(\theta)$, and low noise-floor level in $P_{MU}(\theta)$. We expect that the DOA estimation accuracy could be enhanced using phase characteristics added to MUSIC spectrums.

## 4. Simulation

The accuracy of DOA estimation is evaluated through some simulations in this section. Specifications of the simulations are summarized in Table 1. DOA estimation accuracy is evaluated by RMSE (Root Mean Square Error) averaged for two waves:

$$\mathrm{RMSE} = \frac{1}{2}\sum_{\ell=1}^{2}\sqrt{\sum_{i=1}^{N}(\hat{\theta}_{\ell,i} - \theta_\ell)^2}$$

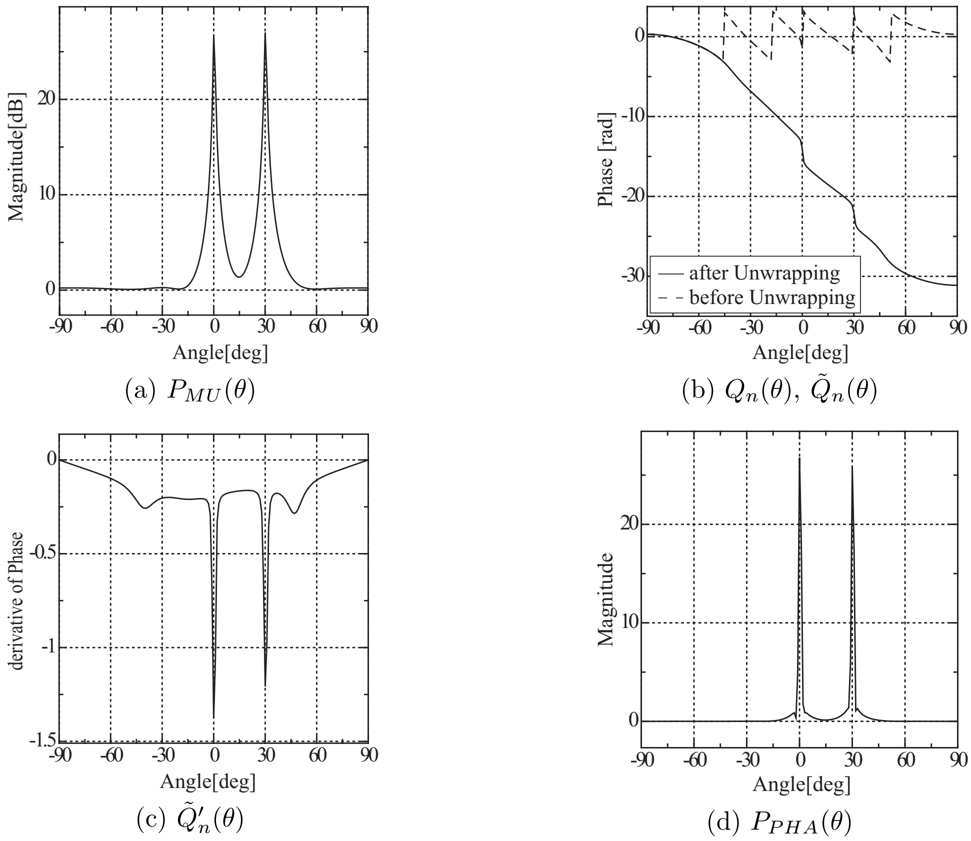



Figure 1: Example spectrums: (a) MUSIC spectrum, (b) phase characteristics and its unwrapped version, (c) unwrapped and differentiated phase characteristics, and (d) proposed phase-based spectrum.

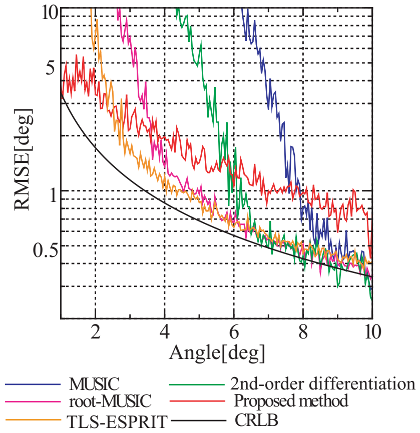
where $\hat{\theta}_{\ell,i}$ denotes the estimated DOA of $\ell$-th incident wave at $i$-th trial, and $N$ is the number of trials.

Table 1: Basic Specifications of Simulations

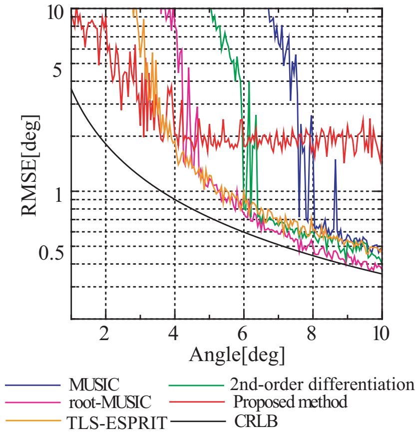
| | |
|---|---|
| array configuration | ULA |
| array element interval | half-wavelength |
| # of array elements | 6 or 8 |
| # of incident waves | 2 waves |
| carrier frequency | 2.0 GHz |
| modulation type | QPSK |
| # of snapshots | 100 |
| # of trials | 100 |

Figure 2(a)-(d) compares the RMSE of the proposed method with MUSIC method [2], Root-MUSIC [3], TLS-ESPRIT [4], second-order differential of MUSIC [5] and also with CRLB (Cramer-Rao Lower Bound). Figure 2(a)-(b) shows close-angle dependency where the horizontal axis denotes the DOA of second wave $\theta_2$, and Fig.2(c)-(d) shows the SNR dependency. As seen from Fig.2(a)-(d), the proposed method gives smaller RMSEs than the other methods in severe environments like low SNR or incident waves coming from close angles. Note that the proposed method is applicable to arbitrary array configuration while Root-MUSIC is only for ULA.

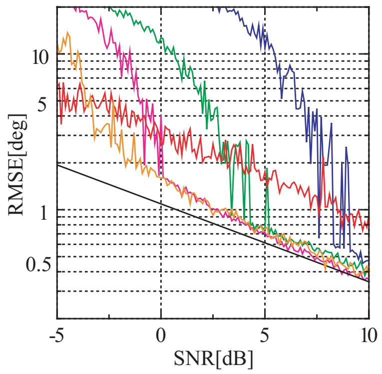
One drawback is that the RMSE of the proposed method does not come close to that of the other methods in good environments like high SNR or two waves with large angle difference. This is because the proposed phase-based spectrum was too sharp and sometimes the parallel averaging derives bad results like that. The way of synthesizing each eigenbeams should be replaced with a different way, it remains as one of future studies.



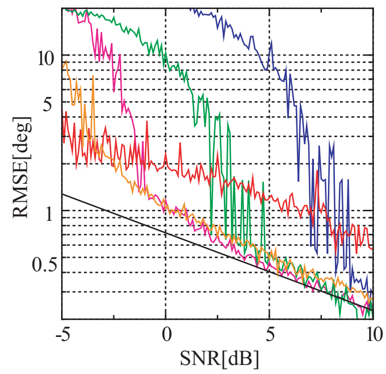

(a) $\theta_1 = 0$[deg], SNR=7[dB], 6-elements ULA




(b) $\theta_1 = 0$[deg], SNR=0[dB], 8-elements ULA




(c) $\theta_1 = 0$[deg], $\theta_2 = 7$[deg], 6-elements ULA




(d) $\theta_1 = 0$[deg], $\theta_2 = 5$[deg], 8-elements ULA

Figure 2: Simulation Results: (a)(b) close-angle dependency of RMSE where two waves are coming from close directions, and (c)(d) SNR dependency of RMSE

# 5. Concluding Remarks

This paper presented a novel DOA estimation formula using phase information of MUSIC spectrum. We developed a new approach to DOA estimation mainly based on phase information of noise eigenbeams. The proposed method achieves very high accuracy in severe environments like low SNR or small number of snapshots.

The accuracy in good environments should be improved by introducing different spectrum synthesizing method or something else, which will be considered as one of future studies.

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