

POLARIZATION DEPENDENCE IN GEOMETRICAL AND PHYSICAL
OPTICS INVERSE SCATTERING - AN OVERVIEW

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1 - Introduction

The primary objective of this paper is to discuss the relationship between the geometrical characteristics of a target and the elements of its scattering matrix and its Mueller matrix. The results are then applied to inverse scattering. The analysis is performed at high frequencies using Geometrical and Physical Optics and Geometrical Theory of Diffraction.

2 - Smooth convex target

At high frequencies, when the local principal radii of curvature of a smooth convex target are large compared to the wavelength, the diffracted field can be split into a specular point contribution and creeping wave contributions.

Specular point contribution : extended geometrical optics approach

In the illuminated region in space, away from the shadow boundary, the incident field gives rise to an extended reflected field not strictly limited to the GO reflected field, which can be represented by an asymptotic expansion in powers of $1/k$ (k = wave number) known as the Luneberg-Kline expansion. The first two terms of this expansion may be written in the form :

$$\vec{E}^D(P) = (\bar{R} + \frac{1}{ik} \bar{C}) \cdot \vec{E}^i(Q) \sqrt{\rho_1 \rho_2} \frac{e^{ik|\vec{QP}|}}{|\vec{QP}|} \quad (1)$$

where \bar{R} is the GO reflection dyadic, \bar{C} the second order diffraction dyadic, $\vec{E}^i(Q)$ the incident field at the reflection point Q and $\rho_{1,2}$ the principal radii of curvature at Q of the reflected wavefront.

If \vec{e}^i is the vector amplitude of the incident field, the polarization dependence of the extended GO field is contained in the vector \vec{e}^D defined by :

$$\vec{E}^D = \bar{R} \cdot \vec{e}^i + \frac{1}{ik} \bar{C} \cdot \vec{e}^i \quad (2)$$

It is shown that the first term is independent of the geometry of the target whereas the second term is curvature dependent.

Explicit expressions for the second order term of the reflected field at the surface will be given. They have first been published by Meckelburg [1]. For monostatic diffraction they depend locally on the difference between the principal curvatures of the surface. This property has been established earlier by Bennett [2] via the space-time integral equation approach and applied to inverse scattering by Foo, Chaudhuri and Boerner [3]. For bistatic diffraction the exact expression obtained in (1) is more complex. But for small bistatic angles it leads to a formula identical to that obtained by a different approach in [4].

Creeping wave contribution

It is well known that for a perfectly conducting target, the normal and tangential components of the incident field at the shadow boundary travel independently along the geodesic and that no coupling between these two components occurs, at least in the leading term of the asymptotic expansion. Moreover, the decay coefficients of the creeping wave modes for the tangential component are much higher than for the normal component. As a consequence, the contribution to the diffracted field coming from the tangential component is negligible, especially in the monostatic case due to the length of the

geodesic path. This property dominates the polarization dependence of the creeping waves in the monostatic case on a smooth convex body illuminated by a local plane wave.

The paths corresponding to the dominant contributions depend on the polarization of the incident field. For an oblong body for instance, the shortest geodesic path gives the dominant contribution to the diffracted field when the polarization of the incident field is perpendicular to the axis of the body whereas the longest geodesic path gives the dominant contribution when the polarization is parallel to this axis. The amplitude of the diffracted field associated with creeping waves is therefore not very sensitive to polarization. However, the relative phase is highly sensitive to polarization and contains information on the shape of the body through the creeping wave path length.

For the bistatic case, the polarization of a creeping wave diffracted field depend on the angle between the normals $\hat{n}(Q)$ and $\hat{n}(Q')$ to the surface at the extremities Q and Q' of the geodesic which is directly related to the torsion of the geodesic between Q and Q' . This last parameter gives also an information on the shape of the body especially when its variation with the aspect angles is measured.

3 - Target with edges

The GTD solution for the far field resulting from the diffraction of a plane wave by a curved perfectly conducting wedge is given by [5] :

$$\vec{E}^d(P) = \frac{1}{\sqrt{k}} \overline{\overline{D}} \cdot \vec{E}^i(Q) \sqrt{\rho} \frac{e^{ik|\vec{QP}|}}{|\vec{QP}|} \quad (3)$$

where Q is the diffraction point on the edge and $\vec{E}^i(Q)$ is the incident field at that point. The other notations in (3) have the following meaning :

ρ : radius of curvature at Q of the diffracted wavefront in the plane of diffraction
 $\overline{\overline{D}}$: diffraction dyadic

The general expression of $\overline{\overline{D}}$ in ray fixed coordinates is :

$$\overline{\overline{D}} = -\hat{\phi} \hat{\phi}' D_h - \hat{\beta} \hat{\beta}' D_s \quad (4)$$

where $\hat{\phi}$, $\hat{\phi}'$, $\hat{\beta}$, $\hat{\beta}'$ are unit vectors associated with spherical coordinates as shown on figure 1 and where D_s , D_h are the diffraction coefficients for soft and hard boundary conditions.

For our purpose, it is not necessary to know the explicit expressions of D_s and D_h which can be found in [5]. If we write :

$$D_h = \frac{D_h + D_s}{2} + \frac{D_h - D_s}{2} \quad (5)$$

$$D_s = \frac{D_h + D_s}{2} - \frac{D_h - D_s}{2}$$

and substitute these expressions in (3), we get for the polarization dependent term $\overline{\overline{D}} \cdot \vec{e}^i$:

$$\overline{\overline{D}} \cdot \vec{e}^i = \left\{ -\frac{1}{2}(D_h + D_s) (\hat{\phi} \hat{\phi}' + \hat{\beta} \hat{\beta}') - \frac{1}{2}(D_h - D_s) (\hat{\phi} \hat{\phi}' - \hat{\beta} \hat{\beta}') \right\} \cdot \vec{e}^i \quad (6)$$

All quantities in the bracket on the right hand side of (6) are fixed for a given ray path. Moreover, the polarization of the incident field is

completely defined by its projections a and b on $\hat{\phi}'$ and $\hat{\beta}'$:

$$\vec{e}^i = a\hat{\phi}' + b\hat{\beta}' \quad (7)$$

If we substitute (7) into (6) we get :

$$\vec{D} \cdot \vec{e}^i = -\frac{1}{2}(D_h + D_s) (a\hat{\phi} + b\hat{\beta}) - \frac{1}{2}(D_h - D_s) (a\hat{\phi} - b\hat{\beta}) \quad (8)$$

We see that the vector $(a\hat{\phi} + b\hat{\beta})$ has the same polarization ellipse as the incident field. The ellipse is rotated by an angle which is equal to the angle between the edge-fixed plane of incidence and the plane of diffraction. For monostatic diffraction, these two planes are the same. Hence the vector $a\hat{\phi} + b\hat{\beta}$ is identical to \vec{e}^i . We call this term the symmetric part. It is insensitive to the polarization of the incident field. The other term is the non symmetric part. It is sensitive to the polarization of the incident field and is proportional to the difference between the edge diffraction coefficients. If a target has more than one edge, several singly diffracted rays reach the observer. For an observer at large distance from the target all incident and diffracted rays in monostatic diffraction are parallel and have therefore the same ray fixed coordinate system. Hence, the symmetric part of the total diffracted field is equal to the sum of the symmetric parts of the fields diffracted by each edge and is therefore also insensitive to polarization.

4 - Inverse scattering techniques

Formulas (5) show that the polarization dependence of a wedge diffracted field is similar to that of the second order term of the Luneberg-Kline expansion of the extended reflected field. Both phenomena are therefore well described by the Stokes matrix (Mueller matrix) [6]. Indeed we have shown that for monostatic diffraction, the edge diffracted field can be split into a symmetric part which is insensitive to polarization and a non symmetric part which contains the polarization dependence. The same procedure can be applied to the second term of the asymptotic expansion of the extended reflected field and more generally to the field diffracted by higher order singularities (curvature discontinuity). Since the symmetric part of the scattering matrix appears on the diagonal of the Mueller matrix whereas the non symmetric part appears in the coefficients D and C [6], a relationship appears between the geometrical characteristics of a target and the elements of its Mueller matrix. It will be shown that this relationship leads to new inverse scattering techniques.

References

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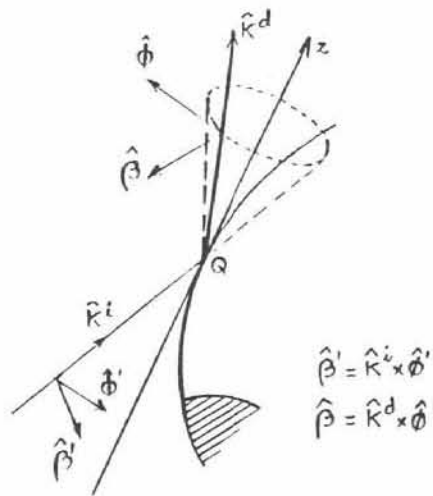


Figure 1 : Ray-fixed coordinates for edge diffraction