Scattering of Electromagnetic Waves by Inhomogeneous Dielectric Gratings Loaded with Three Perfectly Conducting Strips

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Abstract

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with three perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method. This method can be applied to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials .

Numerical results are given for the transmitted scattered characteristics for the case of frequency for TE cases. The effects of the strip gratings comparison with that of the finite thickness on the transmitted power are discussed.

1. Introduction

Recently, the refractive index can easily be controlled to make the periodic structures such as optoelectronic devices, photonic bandgap crystals , frequency selective devices, and other applications by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials[1]-[4] .

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with three perfectly conducting strips[10] using the combination of improved Fourier series expansion method[5]-[7] and point matching method[8]-[9].

Numerical results are given for the transmitted scattered characteristics for the case of frequency for TE cases. The effects of the strip gratings comparison with that of the finite thickness[11] on the transmitted power are discussed.

2. Method of Analysis

We consider inhomogeneous dielectric gratings loaded with three perfectly conducting strips shown in Fig.1. The grating is uniform in the y-direction and the permittivity $\varepsilon(x,z)$ is an arbitrary periodic function of z with period p. The permeability is assumed to be μ_0 . The time dependence is $\exp(-i\omega t)$ and suppressed throughout.

In the formulation, the TE wave is discussed. When the TE wave (the electric field has only the y-component) is assumed to be incident from $x>0$ at the angle θ_0 ,

$$E_y^{(i)} = e^{ik_1(z\sin\theta_0 - x\cos\theta_0)} \bullet k_1 @\omega\sqrt{\varepsilon_1\mu_0} \tag{1}$$

the electric fields in the regions $S_1\,(x\le 0)$, $S_2\,(0<x<D)$, and $S_3\,(x\ge D)$ are expressed[10] as

$\underline{S_1(x\le 0)}$:

$$E_y^{(1)} = E_y^{(i)} + e^{ik_1 z\sin\theta_0}\sum_{n=-N}^{N} b_n^{(1)} e^{i\left(-k_n^{(1)}x + 2\pi nz/p\right)} \tag{2}$$

$\underline{S_2(0<x<D)}$:

$$E_y^{(2,1)} = \sum_{v=1}^{2N+1}\left[A_v^{(1)} e^{ih_v^{(1)}x} + B_v^{(1)} e^{-ih_v^{(1)}(x-d)}\right] f_v^{(l)}(z) \tag{3}$$

$$E_y^{(2,2)} = \sum_{v=1}^{2N+1}\left[A_v^{(2)} e^{ih(2)_v(x-d)} + B_v^{(2)} e^{-ih_v^{(2)}(x-D)}\right] f_v^{(1)}(z) \tag{4}$$

$$f_v^{(l)}(z) @ e^{ik_1\sin\theta_0 z}\sum_{m=-N}^{N} u_m^{(v,l)} e^{i2\pi mz/p} \quad ,l=1,\,2$$

$\underline{S_3(x\ge D)}$:

$$E_y^{(3)} = e^{ik_1 z\sin\theta_0}\sum_{n=-N}^{N} c_n^{(3)} e^{i\left\{k_n^{(3)}(x-D) + 2\pi nz/p\right\}} \tag{5}$$

$$H_z^{(j)} = \{i\omega\mu_0\}^{-1}\,\partial E_y^{(j)}/\partial x \ ,\ (j=1 \quad 3` \tag{6}$$

where λ is the wavelength in free space, he wavelength in free space, $b_n^{(1)}, A_v^{(1)}, B_v^{(1)}, A_v^{(2)}, B_v^{(2)}$, and $C_n^{(3)}$ are unknown coefficients to be determined from boundary conditions. $k_n^{(j)}$ $(j=1,\,3)$ is propagation constants in the x direction, and , $h_v^{(k)}$,$u_n^{(v,l)}(l=1.2)$,the propagation constant and eigenvectors, are satisfy the following eigenvalue equation in regard to h [5]

$$,\quad \mathbf{U} \doteq h^2\mathbf{U} \qquad (7)$$

where,

$$\mathbf{U}^{(v,l)} \equiv \left[u_{-N}^{(v,l)}, \cdots, u_0^{(v,l)}, \cdots u_N^{(v,l)}\right]^T, T\ :\ \text{transpose},$$

$$\mathbf{A} \equiv \left[a_{m,n}^{(l)}\right],$$

$$a_{n,m}^{(l)} \equiv k_1{}^2\xi_{n,m}^{(l)} - (2\pi n/p + k_1\sin\theta_0)^2,$$

$$\xi_{n,m}^{(l)} \equiv \frac{1}{p}\int_0^p \{\frac{\varepsilon_2^{(l)}(z)}{\varepsilon_0}\} e^{i2\pi(n-m)z/p}dz,\ m,n = (-N,\cdots,0,\cdots,N)$$

We obtain the matrix form combination of metallic region C and the dielectric region $\overline{C}$ using boundary condition $Z_j = (j-1)p/[(2N+1)]$; $j = 1 : (2N+1)$ at the matching points on $x =$ 0, *and D*. Boundary condition using Point Matching are as follows:

$$Z_j \in C_1\ ;\ [E_z^{(1)} = 0,\ E_z^{(2,1)} = 0]_{x=0} \qquad (8)$$

$$Z_j \in \overline{C_1}\ ;\ [E_y^{(1)} = E_y^{(2,1)}]_{x=0},\ [H_z^{(1)} = H_z^{(2,1)}]_{x=0} \qquad (9)$$

$$Z_j \in C_3\ ;\ [E_z^{(2,2)} = 0,\ E_z^{(3)} = 0]_{x=D} \qquad (10)$$

$$Z_j \in \overline{C_3}\ ;\ [E_y^{(2,2)} = E_y^{(3)}]_{x=D},\ [H_z^{(2,2)} = H_z^{(3)}]_{x=D} \qquad (11)$$

In the boundary condition at Eq.(8), and Eq.(10), it is satisfied in all matching points by using the orthogonality properties of $\{e^{i2\pi nz/p}\}$, we get following equation in Eq.(9), and Eq.(11) in regard to $A_v^{(1)}, B_v^{(1)}, A_v^{(2)}, and\ B_v^{(2)}$

$$\mathbf{Q}_1\mathbf{A}^{(1)} + \mathbf{Q}_2\mathbf{B}^{(1)} = \mathbf{F}$$

(12)

$$\mathbf{Q}_3\mathbf{A}^{(2)} + \mathbf{Q}_4\mathbf{B}^{(2)} = \mathbf{0} \qquad (13)$$

where, $\mathbf{F} \equiv \left[0(Z_k \in C_1)\ ,2k_0^{(1)}(Z_k \in \overline{C_1})\right]^T$

$$\mathbf{A}^{(k)} \equiv \left[A_1^{(k)}, A_2^{(k)}, \cdots, A_{2N+1}^{(k)}\right]^T, k = 1,2$$

$$\mathbf{B}^{(k)} \equiv \left[B_1^{(k)}, B_2^{(k)}, \cdots, B_{2N+1}^{(k)}\right]^T, k = 1,2$$

$$\mathbf{Q}_1 \equiv (\text{Ç}_2^{(1)} + \text{Ç}_1^{(1)}\mathbf{D}^{(1)})\ ,\mathbf{Q}_2 \equiv (\text{Ç}_2^{(1)} - \text{Ç}_1^{(1)}\mathbf{D}^{(1)})$$

$$\mathbf{Q}_3 \equiv (\text{Ç}_2^{(2)}\mathbf{D}^{(2)} - \text{Ç}_3^{(2)}), \mathbf{Q}_4 \equiv (\text{Ç}_2^{(2)}\mathbf{D}^{(2)} + \text{Ç}_3^{(2)})$$

$$n = (-N,\cdots,0\ ,\cdots,N)\ , v = 1 \sim (2N_f + 1)$$

$$\mathbf{H}_1^{(l)} = \begin{bmatrix} \vdots & \vdots & \vdots & & \vdots\vdots \\ e^{-iNz_j} & \cdots & e^{i0z_j} & \cdots & e^{iNz_j} \\ \vdots & \vdots & \vdots & & \vdots\vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \begin{matrix} \left.\right\} Z_j \in C_1 \\ \left.\right\} Z_j \in \overline{C}_1 \end{matrix} \cdot \mathbf{U}^{(l)}$$

$$\mathbf{H}_2^{(l)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots\vdots \\ 0 & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots\vdots \\ k_{-N}^{(l)}e^{-iNz_j} & & k_0^{(l)}e^{i0z_j} & k_N^{(l)}e^{iNz_j} \end{bmatrix} \begin{matrix} \left.\right\} Z_j \in C_1 \\ \left.\right\} Z_j \in \overline{C}_1 \end{matrix} \cdot \mathbf{U}^{(l)}$$

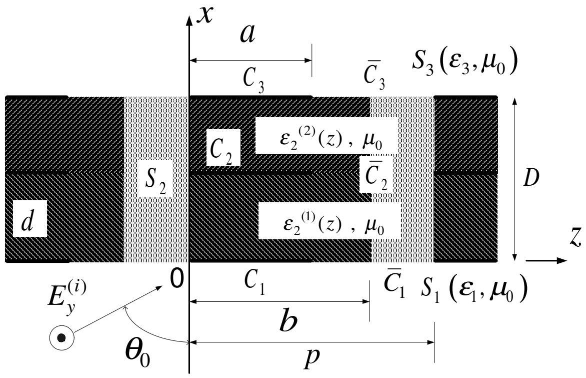


Fig.1 . Structure of inhomogeneous dielectric gratings loaded with three perfectly conducting strips.

$$\mathbf{H}_3^{(l)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots\vdots \\ 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots\vdots \\ e^{-iNz_j} & \cdots & e^{i0z_j} & \cdots & e^{iNz_l} \end{bmatrix} \begin{matrix} \left.\right\} z_j \in c \\ \left.\right\} z_j \in \overline{c} \end{matrix} \cdot \mathbf{U}^{(l)}$$

$$\mathbf{U}^{(l)} \equiv \left[u_n^{(v,l)}\right],\quad \mathbf{D}^{(l)} \equiv \left[\eta_1 e^{i\eta_2 h_v^{(2,l)}\eta_3}\cdot\delta_{(n+N+1),v}\right],\quad l = 1,2$$

$$\eta_1 \equiv \begin{cases} 1 & ;\ Z_j \in C \\ h_v^{(2,l)}; & Z_j \in \overline{C} \end{cases}\ \eta_2 \equiv \begin{cases} 0\ ;\ Z_j \in C \\ 1\ ;\ Z_j \in \overline{C} \end{cases}, \eta_3 \equiv \begin{cases} d\ ; l = 1 \\ D - d\ ; l = 2 \end{cases}$$

Boundary condition on $x = d$ are as follows:

$$Z_j \in C_2\ ;\ [E_y^{(2,1)} = 0,\ E_y^{(2,2)} = 0]_{x=d} \qquad (14)$$

$$Z_j \in \overline{C_2}\ ;\ [E_y^{(2,1)} = E_y^{(2,2)}]_{x=d},\ [H_z^{(2,1)} = H_z^{(2,2)}]_{x=d} \qquad (15)$$

In the boundary condition at Eq.(14), it is satisfied in all matching points by using the orthogonality properties of $\{e^{i2\pi nz/p}\}$, we get following equation in $A_v^{(1)}, B_v^{(1)}, A_v^{(2)}, and\ B_v^{(2)}$

$$\mathbf{R}_1\mathbf{A}^{(1)} + \mathbf{R}_2\mathbf{B}^{(1)} = \mathbf{R}_3\mathbf{A}^{(2)} + \mathbf{R}_4\mathbf{B}^{(2)} \qquad (16)$$

$$where,\ \mathbf{R}_1 \equiv \mathbf{U}^{(1)}\mathbf{D}'^{(1)},\ \mathbf{R}_2 \equiv \mathbf{U}^{(1)},\ \mathbf{R}_3 \equiv \mathbf{U}^{(2)},\ \mathbf{R}_4 \equiv \mathbf{U}^{(2)}\mathbf{D}'^{(2)}$$

We get following matrix form combined with Eq.(14) and Eq.(15).

$$\mathbf{S}_1\mathbf{A}^{(1)} + \mathbf{S}_2\mathbf{B}^{(1)} = \mathbf{S}_3\mathbf{A}^{(2)} + \mathbf{S}_4\mathbf{B}^{(2)} \qquad (17)$$

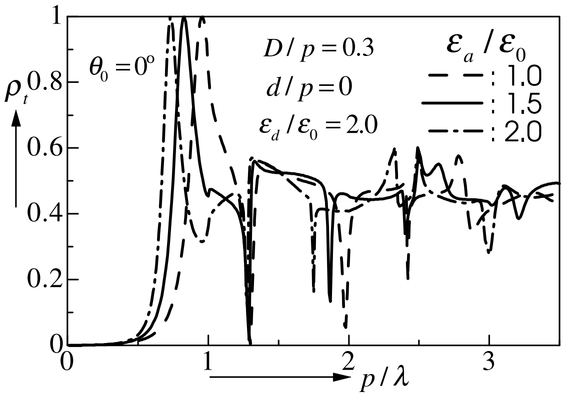
where,

$$\mathbf{D}'^{(1)} \equiv [e^{ih_v^{(2,1)}d}\delta_{(n+N+1),v}],\ \mathbf{D}'^{(2)} \equiv [e^{ih_v^{(2,2)}(D-d)}\delta_{(n+N+1),v}]$$

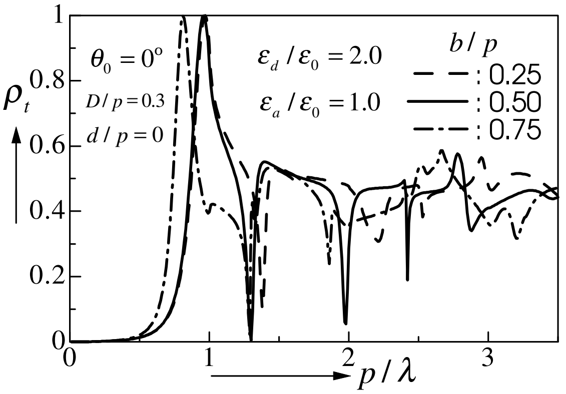
$$\mathbf{S}_1 \equiv (\text{Ç}_1^{(1)}\mathbf{D}'^{(1)} + \text{Ç}_3^{(1)}\mathbf{D}''^{(1)})\ ,\mathbf{S}_2 \equiv (\text{Ç}_1^{(1)} - \text{Ç}_1^{(1)}\mathbf{D}'''^{(2)})$$

$$\mathbf{S}_3 \equiv \text{Ç}_3^{(2)}\mathbf{D}''^{(2)},\ \mathbf{S}_4 \equiv -\text{Ç}_3^{(2)}\mathbf{D}'''^{(2)})$$

$$\mathbf{D}''^{(1)} \equiv [h_v^{(2,1)}\delta_{(n+N+1),v}],\ \mathbf{D}'''^{(1)} \equiv [h_v^{(2,1)}e^{ih_v^{(2,1)}d}\delta_{(n+N+1),v}]$$

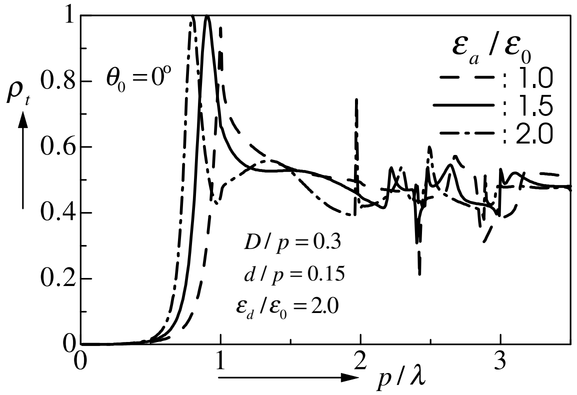


(a) $a/b = 1.0$

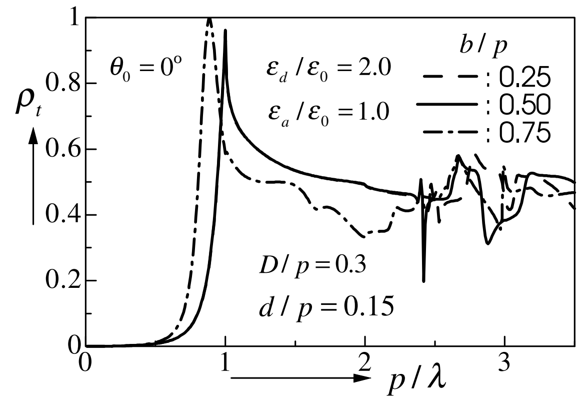


(b) $\varepsilon_a/\varepsilon_0 = 1.0$

Fig.2 ρ_t vs. p/λ for the case of $d/p = 0$



(a) $a/b = 1.0$



(b) $\varepsilon_a/\varepsilon_0 = 1.0$

Fig.3 ρ_t vs. p/λ for the case of $d/p = 0.15$

$$\mathbf{D}'^{(3)} @ h_\nu^{(2,2)} \delta_{(n+N+1),\nu} \quad \mathbf{D}'^{(2)} @ h_\nu^{(2,2)} e^{ih_\nu^{(2,2)}(D-d)} \delta_{(n+N+1),\nu}$$

By using matrix relationship between $\mathbf{A}^{(1)}$, $\mathbf{B}^{(1)}$ $\mathbf{A}^{(2)}, \mathbf{B}^{(2)}$, we get the following homogeneous matrix equation in regard to $A_\nu^{(2)}$ $(\nu = 1 \sim 2N+1)$.

$$\mathbf{W} \cdot \mathbf{A}^{(2)} = \mathbf{F} \tag{18}$$

where $\mathbf{W} @ \left[\mathbf{Q}_1\mathbf{P}_1 + \mathbf{Q}_2\mathbf{P}_3 - (\mathbf{Q}_1\mathbf{P}_2 + \mathbf{Q}_2\mathbf{P}_4)\mathbf{Q}_4^{-1}\mathbf{Q}_3\right]$

$$\mathbf{P}_1 @ (\mathbf{R}_2^{-1}\mathbf{R}_1 - \mathbf{S}_2^{-1}\mathbf{S}_1)^{-1}(\mathbf{R}_2^{-1}\mathbf{R}_3 - \mathbf{S}_2^{-1}\mathbf{S}_3),$$
$$\mathbf{P}_2 @ (\mathbf{R}_2^{-1}\mathbf{R}_1 - \mathbf{S}_2^{-1}\mathbf{S}_1)^{-1}(\mathbf{R}_2^{-1}\mathbf{R}_4 - \mathbf{S}_2^{-1}\mathbf{S}_4),$$
$$\mathbf{P}_3 @ (\mathbf{R}_1^{-1}\mathbf{R}_2 - \mathbf{S}_1^{-1}\mathbf{S}_2)^{-1}(\mathbf{R}_1^{-1}\mathbf{R}_3 - \mathbf{S}_1^{-1}\mathbf{S}_3),$$
$$\mathbf{P}_4 @ (\mathbf{R}_1^{-1}\mathbf{R}_2 - \mathbf{S}_1^{-1}\mathbf{S}_2)^{-1}(\mathbf{R}_1^{-1}\mathbf{R}_4 - \mathbf{S}_1^{-1}\mathbf{S}_4)$$

The mode power transmission coefficients ρ_t is given by

$$\rho_t @ \sum_{n=-N}^{N} \mathrm{Re}\left[k_n^{(3)}\right] |c_n^{(3)}|^2 \tag{19}$$

where• $C_n @ \sum_{n=-N}^{N} \left[A_\nu^{(2)} e^{ih_\nu^{(1)}(D-d)} + B_\nu^{(2)}\right] U_n^{(\nu)}$.

3. Numerical Analysis

We consider the following structure of inhomogeneous gratings:

$$\varepsilon_2^{(1)}(x,z) = \varepsilon_2^{(2)}(x,z) @ \begin{cases} \varepsilon_d : (0 \le x \le b) \\ \varepsilon_a : (b < x < p) \end{cases} \tag{20}$$

The values of parameters chosen are $\varepsilon_1 = \varepsilon_3 = \varepsilon_0$, $a/p = 0.5, \varepsilon_d/\varepsilon_0 = 2.0$, $\theta_0 = 0$, and $D/p = 0.3$.

The relative error are less than about 0.1% and the energy error is less than about 10^{-3} for TE waves when we computed with $N = 15$ at $a/b = 1$ and $p/\lambda = 1.5$.

First, we consider the two strip gratings for the case of $d/p = 0$.

Figures 2(a) and 2(b) show ρ_t for various values of normalized frequency (p/λ) for $\varepsilon_a/\varepsilon_0 = 1$, 1.5, and 2 at $a/b = 1, b/p = 0.25$, 0.5, and 0.75 at $\varepsilon_a/\varepsilon_0 = 1.0$.

From in Figs.2(a) ,the maximum of coupling resonance $\rho_t \cong 1$ moves toward smaller (p/λ) as $\varepsilon_a/\varepsilon_0$ increases, and the effect of the minimum of coupling resonance $\rho_t \cong 0$ is more significant at $(p/\lambda \approx 1.3)$.

From in Figs.2(b) ,the maximum of coupling resonance $\rho_t \cong 1$ moves toward smaller (p/λ) as b/p increases, and the effect of b/p is more significant at $b/p \le 0.5$.

Next, we consider the three strip gratings for the case of $d/p = 0.15$.

Figures 3(a) and 3(b) show ρ_t for various values of normalized frequency (p/λ) with the same parameters as in Fig.2. We note that the characteristic tendencies for the effect of the d/p are approximately same at $p/\lambda < 1.0$, but for about $1.0 < p/\lambda > 1.8$, the effect of the $\varepsilon_a/\varepsilon_0$ and b/p is more significant for the minimum of coupling resonance at $(p/\lambda \approx 1.3)$ comparing with Fig.2(a).There is no resonance in Fig.3(a) and Fig.3(b).

Figures 4 shows ρ_t for the various values of normalized frequency (p/λ) for $d/p = 0$, 0.15 comparison of the perfectly conducting strips of finite thickness $(\varepsilon_d/\varepsilon_0 = \infty)$ for the same condition Fig.3.

From in Figs.4, the effect of d/p is seen clearly at the $(p/\lambda) < 2.0$ .Therefore, our method can be applied to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

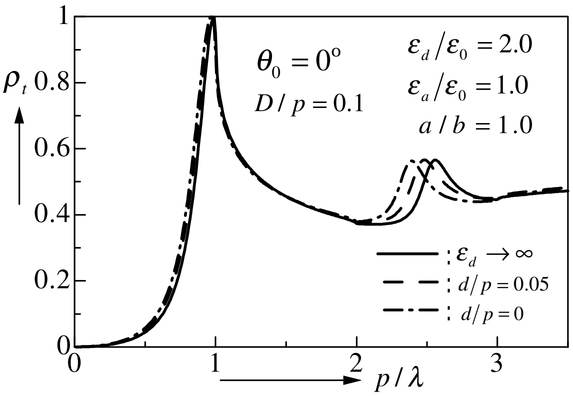


Fig.4 Comparison of the strips with perfectly conducting strips of finite thickness.

4. Conclusion

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with three perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method.

Numerical results are given for the transmitted scattered characteristics for the case of frequency for TE cases. The effects of the strip gratings comparison with that of finite thickness on the transmitted power are discussed.

This method can be applied to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

Reference

[1]T.Tamir.,M. Jiang and K.M.Leung. ; URSI International Symposium on Electromagnetic Theory, IN-T2.25,pp 332-334(2001).

[2]H.Jia and K.Yasumoto: IEEE Antennas and Waves Propagation Lett.Vol.3,pp.41-44(2004).

[3]S.D Gedney and R.Mittra • Analysis of the electromagnetic scattering by thick grating using a combined FEM/MoM technique,IEEE Trans., AP-39, 11. pp. 1605-1614(1991).

[4]H.C. Chu, S.K. Jeng, and C.H. Chen•Reflection and Transmission Characte-ristics of Single-Layer Periodic Composite Structures for TE Case , IEEETrans.AP-45,No.7, pp.1065-1070(1997).

[5]T.Yamasaki, T. Hinata, and T.Hosono:Electromagnetic field analysis of planar gratings with periodic distribution of dielectric constant, IECE Trans. in Japan., Vol.J69-B, No.1. pp.125-132, 1985 (in Japanese).

[6]T.Yamasaki, T.Hosono and J.A.Kong: Propagation Characteristics of Dielectric Waveguides with Periodic Surface-Relief,Trans. IEICE. Japan, Vol.E74, 9, pp.2839-2847(1991).

[7]T.Yamasaki, H.Tanaka, T.Hinata and T.Hosono: Analysis of electromagnetic fields in inhomogeneous dielectric gratings with periodic surface relief, Radio Science,Vol.31,No.6, pp.1931-1939 (1996).

[8]T.Hinata and T.Hosono • On the Scattering of Electromagnetic Wave by Plane Grating Placed in Homogeneous Medium –Mathematical Foundation of Point Matching Method and Numerical Analysis- ,IEICE Trans .in Japan , Vol.J59-B, No.12, pp.571-578 (in Japanese).

[9] T. Yamasaki, T.Hinata and T.Hosono•Scattering of Electromagnetic Waves by Plane Grating with Reflector ,IEICE Trans .in Japan , Vol.J61-B, No.11, pp.953-961 (in Japanese)..

[10]T.Yamasaki, T. Ujiie, T. Hinata • Scattering of lectromagnetic Waves by Inhomogeneous ielectric Gratings loaded with PerfectlyConducting Strip, Spring ConferenceIEICE in Japan,CS-1-5•pp.S45-46(2006) (in Japanese).

[11]T.Hinata, M.Iida and T. Hosono • Scattering of Electromagnetic Wave by a Plane Grating of Finite Thickness.,Vol.58-B,No.3,pp.122-124(1975)(in panese)..