Solution of Combined Field Volume Integral Equation using Adaptive Integral Method

Wei-Bin EWE [1], Er-Ping LI [1] and Le-Wei LI [2]

[1] *Engineering Software & Applications, Institute of High Performance Computing*
Singapore 117528
email: ewewb@ihpc.a-star.edu.sg

[2] *Dept. Electrical & Computer Engineering, National University of Singapore*
Singapore 119260

Abstract

This paper presents a fast solution to the electromagnetic scattering by large-scale 3D dielectric bodies of arbitrary permittivity and permeability. The scattering problem is characterized by using combined field volume integral equation (CFVIE). The CFVIE is formulated in the volume of the scatterers by considering the total electric and magnetics fields as the sum of the incident wave and the radiated wave of the equivalent electric and magnetic volume currents. The resultant CFVIE is discretized and solved by using the method of moments (MoM). For large-scale scattering problems, adaptive integral method (AIM) is then applied in the MoM in order to reduce the memory requirement and accelerate the matrix-vector multiplication in the iterative solver. The resultant method has a memory requirement of $O(N)$ and a computational complexity of $O(N \log N)$ respectively, where N denotes the number of unknowns.

1. Introduction

The electromagnetic analysis of dielectric structures is important as it is widely use in many engineering applications. An understanding of the scattering by dielectric structures is imperative in order to take account of its interference and coupling effect with other components. Volume integral equation (VIE) is among one of the methods that has been widely used to analyze electromagnetic problems involving dielectric materials, especially inhomogeneous dielectric materials. In the past, most of the attention have been focused on solving the electromagnetic problems of non-magnetic problems ($\mu = \mu_0$), which the VIE only needs to consider the total electric field in the inhomogeneous region [1–3]. However, magnetic materials ($\mu \neq \mu_0$) are also been considered in engineering applications, for example, magnetic coating materials. Hence the existing VIE can be extended to analyze scattering problems of object made of magnetic material or complex material properties, *i.e.* arbitrary permittivity and permeability.

The VIE for pure dielectric object can be denoted as electric field volume integral equation (EFVIE) as the equation consider only the electric fields due to impressed sources and equivalent electric currents in the object. Likewise, we denote the VIE for pure magnetic object as magnetic field volume integral equation (MFVIE) as it only requires the magnetic fields due to the impressed sources and equivalent magnetic currents in the object. For a complex material object with mixed dielectric and magnetic properties, the combined field volume integral equation is used (CFVIE) as both electric and magnetic fields inside the object are involved in the formulation. In this paper, the CFVIE is presented for the analysis of scattering by objects with arbitrary permittivity and permeability. The CFVIE is then discretized and solved by using method of moments to obtain the two unknown electric and magnetic currents.

However, it is well known that the traditional MoM cannot handle electrically large objects due to its huge memory requirement and computational complexity. The CFVIE further burden the MoM as the equation contains two set of unknowns. To alleviate this problem, we can resort to the adaptive integral method (AIM) [4–7]. The AIM is applied to reduce the memory requirement for matrix storage and also to accelerate the matrix-vector multiplication in iterative solver. It is noted that all the past efforts in AIM are only focus on solving the EFVIE for dielectric objects. However in this paper, we will apply the AIM to solve the CFVIE of electrical large inhomogeneous complex material object. In the following sections, we will first give the formulation of CFVIE for scattering problems of complex material object and follow by the MoM procedures to solve the CFVIE numerically. Then we will describe the use of AIM in MoM to solve large-scale electromagnetic problems. Finally we will present numerical examples to demonstrate the accuracy of our method.

2. Formulation

A. Combined Field Volume Integral Equation

Consider an arbitrarily shaped 3-D complex material scatterer, which consists of inhomogeneous dielectric and magnetic materials. The object is embedded in an isotropic homogeneous background medium with permittivity ϵ_b and permeability μ_b. The scatterer is illuminated by an incident wave ($\boldsymbol{E}^{inc}, \boldsymbol{H}^{inc}$), which is excited by impressed sources in the background medium. The scatterer is assumed to have permeability $\mu(\boldsymbol{r})$ and permittivity $\epsilon(\boldsymbol{r})$ at location $\boldsymbol{r}$.

By invoking the volume equivalence principle, the dielectric and magnetic materials can be removed and replaced by equivalent volume electric current densities J_V and equivalent volume magnetic current densities K_V, respectively. Throughout this paper, the subscripts e and h are used to denote any variables associated with electric or magnetic sources. By considering the total electric field $\boldsymbol{E}$ in the inhomogeneous material region as the sum of the incident electric field and the scattered electric field due to the J_V and K_V, the EFVIE can be obtained

$$\boldsymbol{E}^{inc} = \frac{\boldsymbol{D}}{\epsilon(\boldsymbol{r})} + \eta_b \mathcal{L}(\boldsymbol{J}_V) + \mathcal{M}(\boldsymbol{K}_V) \tag{1}$$

where $\boldsymbol{D} = \epsilon(\boldsymbol{r})\boldsymbol{E}$ is the electric flux density and $\eta_b = \sqrt{\mu_b/\epsilon_b}$ is the intrinsic impedance of the background medium. The operator $\mathcal{L}$ and $\mathcal{M}$ are defined as

$$\mathcal{L}(\boldsymbol{X}) = jk_b \int_V \boldsymbol{X}G + \frac{1}{k_b^2}\boldsymbol{\nabla\nabla}\cdot(\boldsymbol{X}G)\, dV' \tag{2a}$$

$$\mathcal{M}(\boldsymbol{X}) = \boldsymbol{\nabla}\times\int_V \boldsymbol{X}G\, dV' \tag{2b}$$

where $k_b = \omega\sqrt{\mu_b\epsilon_b}$ is the wavenumber of the background medium and $G = \exp(-jk_b|\boldsymbol{r}-\boldsymbol{r}'|)/(4\pi|\boldsymbol{r}-\boldsymbol{r}'|)$ is the 3-D scalar Green's function in the background medium. Similarly, by considering the total magnetic field $\boldsymbol{H}$ in the inhomogeneous material as the sum of the incident magnetic field and the scattered magnetic field due to the J_V and K_V, we can write the MFVIE as the following

$$\boldsymbol{H}^{inc} = \frac{\boldsymbol{B}}{\mu(\boldsymbol{r})} + \frac{1}{\eta_b}\mathcal{L}(\boldsymbol{K}_V) - \mathcal{M}(\boldsymbol{J}_V) \tag{3}$$

where $\boldsymbol{B} = \mu(\boldsymbol{r})\boldsymbol{H}$ is the magnetic flux density. It is also noted that the equivalent current densities are related to the electric and magnetic flux densities through

$$\boldsymbol{J}_V = j\omega\kappa_e(\boldsymbol{r})\boldsymbol{D}(\boldsymbol{r}) \tag{4a}$$

$$\boldsymbol{K}_V = j\omega\kappa_h(\boldsymbol{r})\boldsymbol{B}(\boldsymbol{r}), \tag{4b}$$

where κ_e and κ_h are the respective contrast ratios of the permittivity and permeability, given as

$$\kappa_e(\boldsymbol{r}) = \frac{\epsilon(\boldsymbol{r}) - \epsilon_b}{\epsilon(\boldsymbol{r})} \tag{5a}$$

$$\kappa_h(\boldsymbol{r}) = \frac{\mu(\boldsymbol{r}) - \mu_b}{\mu(\boldsymbol{r})}. \tag{5b}$$

B. Method of Moments

To solve the resultant CFVIE using the Method of Moments (MoM), the equivalent current densities are not directly used as the unknown quantities. Instead, the electric and magnetic flux densities are used as the continuity of the normal components of both flux densities can be ensured by using proper basis functions.

The volume of the complex material objects are discretized by using tetrahedral elements. Tetrahedral elements are used because of their flexibility to model arbitrarily shaped 3-D object. The dielectric and magnetic properties in each individual tetrahedral element are assumed constant, which is convenient to model the arbitrary material properties. For tetrahedral elements, the suitable basis functions are the SWG basis functions [1]. The features of SWG basis functions, such as the continuity of the electric flux density normal to the interior face, make them suitable to be implemented in the volume integral equation. The electric and magnetic flux densities are then expanded using the SWG basis functions $\boldsymbol{f}_n$ as

$$\boldsymbol{D}(\boldsymbol{r}) = \sum_{n=1}^{N_e} D_n \boldsymbol{f}_n \tag{6a}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \eta_b \sum_{n=1}^{N_h} B_n \boldsymbol{f}_n \tag{6b}$$

where the D_n and B_n denote the coefficients to be determined, and the $\boldsymbol{f}_n$ denotes the n-th basis function, which is defined on two attached tetrahedrons associated with the n-th face. For the n-th face located at the exterior boundary of the object, an auxiliary tetrahedron is introduced in the exterior region where the free vertex of the auxiliary tetrahedron coincide with the center of the n-th face [1]. And from (4), the $\boldsymbol{J}_V$ and $\boldsymbol{K}_V$ can be expressed as

$$\boldsymbol{J}_V = j\omega \sum_{n=1}^{N_e} \kappa_e(\boldsymbol{r})\, D_n \boldsymbol{f}_n \tag{7a}$$

$$\boldsymbol{K}_V = j\omega\eta_b \sum_{n=1}^{N_h} \kappa_h(\boldsymbol{r})\, B_n \boldsymbol{f}_n. \tag{7b}$$

Subsequently, we substitute (7) into (1) and (3), and test the (1) with $\boldsymbol{f}_m$ and (3) with $\eta_b\boldsymbol{f}_m$. This results in a linear system consists of $N_e + N_h$ independent equations as given below

$$\left\langle \boldsymbol{f}_m, \boldsymbol{E}^{inc}\right\rangle = \sum_{n=1}^{N_e} D_n \left[\left\langle \boldsymbol{f}_m, \frac{\boldsymbol{f}_n}{\epsilon(\boldsymbol{r})}\right\rangle + \left\langle \boldsymbol{f}_m, \eta_b\mathcal{L}(\kappa_e\boldsymbol{f}_n)\right\rangle\right] + \sum_{n=1}^{N_h} \eta_b B_n \left[\ \langle \boldsymbol{f}_m, \mathcal{M}(\kappa_h\boldsymbol{f}_n)\rangle\right], \quad m = 1, 2, ..., N_e \tag{8a}$$

$$\left\langle \eta_b\boldsymbol{f}_m, \boldsymbol{H}^{inc}\right\rangle = -\sum_{n=1}^{N_e} \eta_b D_n \left[\ \langle \boldsymbol{f}_m, \mathcal{M}(\kappa_e\boldsymbol{f}_n)\rangle\right] + \sum_{n=1}^{N_h} \eta_b^2 B_n \left[\left\langle \boldsymbol{f}_m, \frac{\boldsymbol{f_n}}{\mu(\boldsymbol{r})}\right\rangle + \left\langle \boldsymbol{f}_m, \frac{1}{\eta_b}\mathcal{L}(\kappa_h\boldsymbol{f}_n)\right\rangle\right], \quad m = N_e+1, N_e+2, ..., N_e+N_h. \tag{8b}$$

Alternatively, (8) also can be written in a matrix form as

$$\begin{pmatrix} \overline{\boldsymbol{Z}}^{EE} & \overline{\boldsymbol{Z}}^{EH} \\ \overline{\boldsymbol{Z}}^{HE} & \overline{\boldsymbol{Z}}^{HH} \end{pmatrix} \begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{B} \end{pmatrix} = \begin{pmatrix} \boldsymbol{E}^{inc} \\ \boldsymbol{H}^{inc} \end{pmatrix}. \tag{9}$$

The sub-matrices $\overline{\boldsymbol{Z}}^{EE}$ and $\overline{\boldsymbol{Z}}^{EH}$ represent the contributions of the equivalent electric and magnetic sources, respectively, to the electric field. While the sub-matrices $\overline{\boldsymbol{Z}}^{HE}$ and $\overline{\boldsymbol{Z}}^{HH}$ denote the contributions of the equivalent electric and magnetic sources, respectively, to the magnetic field.

C. Adaptive Integral Method

If the MoM matrix equation is solved by using an iterative solver, the memory requirement for matrix storage and computational complexity for computing the matrix vector multiplication are both $O(N^2)$ for a N-unknown problem. Hence for a large N problem, the computational complexity and memory requirement are prohibitively high. In order to apply MoM to solve large scale electromagnetic problems, the adaptive integral method (AIM) can be used to alleviate the stringent computing requirements.

The AIM is used to reduce the memory requirement for matrix storage and to accelerate the matrix-vector multiplication. The basic idea of AIM is to split the matrix-vector multiplication into two parts:

$$\boldsymbol{Z}\boldsymbol{I} = \boldsymbol{Z}^{near}\boldsymbol{I} + \boldsymbol{Z}^{far}\boldsymbol{I} \tag{10}$$

where $\boldsymbol{Z}^{near}\boldsymbol{I}$ and $\boldsymbol{Z}^{far}\boldsymbol{I}$ represent the near-zone interaction and far-zone interaction, respectively. The far-zone interaction is approximated using fast Fourier Transform (FFT) and the near-zone interaction is computed directly using MoM. To employ AIM, the object is first placed in a rectangular grid and then recursively sub-divided into smaller grids. Then the element current densities are projected the surrounding grid points, which can be achieved by matching the multipole expanded at the center of element [4]. The projection of the current densities can be represented by matrix $\boldsymbol{\Lambda}$. Thus the far-zone interaction can be written as

$$\boldsymbol{Z}^{far}\boldsymbol{I} = \boldsymbol{\Lambda}\boldsymbol{g}\boldsymbol{\Lambda}^T\boldsymbol{I} \tag{11}$$

where g is the Green's function matrix. The matrix g is Toeplitz and this enable the use of FFT to compute (11) efficiently. Hence matrix-vector multiplication in (10) can be represented as

$$\boldsymbol{Z}\boldsymbol{I} = \boldsymbol{Z}^{near}\boldsymbol{I} + \boldsymbol{\Lambda}\mathcal{F}^{-1}\left\{\mathcal{F}\left\{\boldsymbol{g}\right\}\cdot\mathcal{F}\left\{\boldsymbol{\Lambda}^T\boldsymbol{I}\right\}\right\} \tag{12}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ stand for FFT and inverse FFT, respectively.

The (11) and (12) are used to accelerate the solution of the problems contain only same type of current densities. However in the CFVIE, the electric and magnetic current densities are not directly expanded and used as the unknowns. As shown in (4), the electric and magnetic current densities are coupled with the κ_e and κ_h, respectively and hence they need to be projected separately. In this context, two set of projection matrix Λ_e and Λ_h need to be set up for the respective $\kappa_e\boldsymbol{J}_V$ and $\kappa_h\boldsymbol{K}_V$.

The modified AIM procedures for solving the CFVIE can be summarized as the following:

1) Project the current densities ($i.e.$ $\boldsymbol{J}_V$ and $\boldsymbol{K}_V$) to surrounding grid points
2) Compute the grid potentials due to these two set of current densities with the aid of fast Fourier Transform
3) Translate the resultant grid potentials back to the element
4) Replace the inaccurate contribution from near-zone grid sources with the correct interactions among the elements

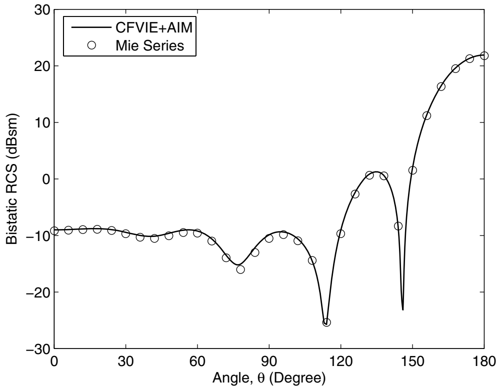

Fig. 1: Bistatic RCS of a complex material sphere of radius $r = 1.0$ ($\epsilon_r = 1.6 - j0.8$ and $\mu_r = 0.8 - j0.2$).

3. NUMERICAL RESULTS

In this section, some numerical examples are presented to demonstrate the applicability of the proposed method for analyzing large scale electromagnetic scattering of structure of mixed permittivity and permeability. Throughout the simulation, the grid spacing and near-zone threshold distance used in AIM are set to $0.1\lambda_0$ and $0.15\lambda_0$, respectively.

The first example is a sphere of radius 1.0m. The relative permittivity and permeability of the sphere are $\epsilon_r = 1.6 - j0.8$ and $\mu_r = 0.8 - j0.2$, respectively. The complex material sphere is modeled by 25 290 tetrahedrons and lead to a total of 104 016 unknowns. The bistatic RCSs of the sphere are computed and shown in Fig. 1. The results obtained by using Mie series are plotted for comparison. It is observed that a very good agreement between the results obtained by using our method and Mie series.

The second example we consider is a dielectric sphere coated with two complex materials as shown in Fig. 2. The radius of the dielectric core is 0.6m and has a relative permittivity $\epsilon_{r1} = 1.44$. The complex relative permittivity and permeability of the inner coating material are $\epsilon_{r2} = 1.5 - j0.2$ and $\mu_{r2} = 1.6 - j0.4$. While for the outer coating material, the material property is $\epsilon_{r3} = 1.6 - j0.4$ and $\mu_{r3} = 1.5 - j0.2$. The thickness of the each coating material is 0.2m. The discretization of the sphere results in a total of 55 864 tetrahedrons where 5 791 are the dielectric core, 8 268 and 13 164 are for the respective inner and outer coating layer. The total number of unknowns of this example is $N = 100\,738$. The bistatic RCSs of the coated sphere are computed and shown in Fig. 3. The results obtained by using FEM are also plotted for comparison.

The last example considered is a five-period mixed dielectric/magnetic slab shown in Fig. 4. The parameters of the slab are $k_0h = 2.0$, $h/d = 1.713$, $W = 6d$, $d_1 = d_2 = d/2$. In computing this example, the wavelength of the incident wave is equal to 4.5m. Each period of the slab comprises one dielectric and magnetic bars. The relative permittivity of the

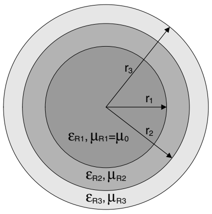

Fig. 2: Geometry of a coated dielectric sphere with $\epsilon_{r1} = 1.44$,$\epsilon_{r2} = 1.5 - j0.2$ and $\mu_{r2} = 1.6 - j0.4$, $\epsilon_{r3} = 1.6 - j0.4$ and $\mu_{r3} = 1.5 - j0.2$

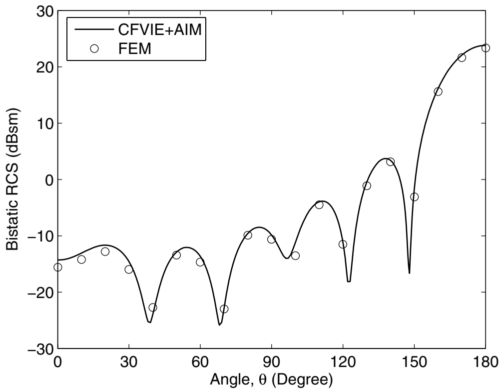

Fig. 3: Bistatic RCS of a coated dielectric sphere with two coating layers.

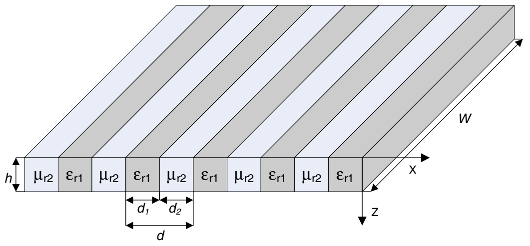

Fig. 4: Geometry of a five-period complex material slab

dielectric bar is $\epsilon_r = 1.44$ and the relative permeability of the magnetic bar is $\mu_r = 2.56$. The periodic slab is modeled by using 9 056 tetrahedrons which results in 20 250 unknowns. The monostatic RCSs in XZ-plane are shown in Fig. 5. We also compute the RCSs of a piecewise homogeneous dielectric slab and a piecewise homogeneous magnetic slab of the same size as the periodic slab and shown in the Fig. 5.

4. Conclusion

In this paper, an efficient method based on the combined field volume integral method is presented for solving the scattering problems of object with arbitrary permittivity and permeability. This method includes the interaction between dielectric and magnetic properties in the object, and is applicable to model arbitrarily shaped object. The resultant integral equation is subsequently converted into a matrix equation by using method of moments. Adaptive integral method (AIM) has been implemented in the method of moments to reduce the

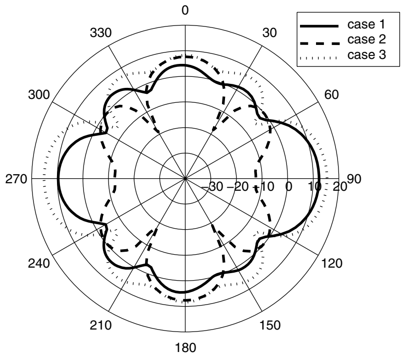

Fig. 5: Monostatic RCS of a five periodic slab with $k_0 h = 2.0$ and different material properties. Case 1: $\epsilon_{r1} = 1.44$, $\mu_{r2} = 2.56$; Case 2: $\epsilon_{r1} = 1.44$, $\epsilon_{r1} = 1.44$; Case 3: $\mu_{r1} = 2.56$, $\mu_{r2} = 2.56$.

memory for matrix storage and CPU time for matrix-vector multiplication in iterative solver. Three examples have been presented in this paper to demonstrate the applicability and accuracy of the method for scattering problems of electrically large object with complex material properties.

References

[1] D. H. Schaubert, D. R. Wilton, and A. W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies", *IEEE Trans. Antennas Propagat.*, vol. 32, no. 1, pp. 77–85, Jan 1984.

[2] D. H. Schaubert and P. M. Meaney, "Efficient computation of scattering by inhomogeneous dielectric bodies", *IEEE Trans. Antennas Propagat.*, vol. 34, no. 4, pp. 587–592, 1986.

[3] J. L. Volkakis, "Alternative field representations and integral equations for modeling inhomogeneous dielectrics", *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 3, pp. 604–608, 1992.

[4] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: Adaptive Integral Method for solving large-scale electromagnetic scattering and radiation problems", *Radio Sci.*, vol. 31, no. 5, pp. 1225–1251, Sept.-Oct. 1996.

[5] Z. Q. Zhong and Q. H. Liu, "A volume adaptive integral method (VAIM) for 3-D inhomogeneous objects", *IEEE Antennas and Wireless Propagat. Letters*, vol. 1, no. 5, pp. 102–105, 2002.

[6] W. B. Ewe, L. W. Li, and M. S. Leong, "Fast solution of mixed dielectric/conducting scattering problem using volume-surface Adaptive Integral Method", *IEEE Trans. Antennas Propagat.*, vol. 52, no. 11, pp. 3071–3077, Nov. 2004.

[7] W. B. Ewe, L. W. Li, Q. Wu, and M. S. Leong, "Analysis of reflector and horn antennas using Adaptive Integral Method", *IEICE Trans. on Commun.*, vol. E88-B, no. 6, pp. 2327–2333, June 2005.