A NOVEL TECHNIQUE TO THE SOLUTION OF TRANSIENT ELECTROMAGNETIC SCATTERING FROM CONDUCTING BODIES

Sadasiva M. Rao,Tapan K. Sarkar,and Soheil A. Dianat
Department of Electrical Engineering,
Rochester Institute of Technology,
Rochester,NY 14623 U.S.A.

1.INTRODUCTION

Previous approaches to the problem of transient scattering by conducting bodies have utilized the well-known marching-on-in-time solution procedures. In this solution procedure the space and time are discretized into number of subintervals.A recurrence relation for the value of the current at the present time and space interval is obtained as a function of currents at previous instants.The Marching-on-in-time solution procedure is simple to derive and easily applicable to any geometrical shape.An important advantage of this method,which has been frequently stressed by Auckenthaler and Bennet[1],Mitzner[2],and Herman[3,4] is the fact that no matrix inversion is required if one carefully chooses the time and space discretizations.Unfortuantely this method suffers from a serious disadvantage which result in a rapidly growing spurious oscillations at later instants of time.The exact theoretical cause for this behaviour is not known but it is speculated that the accumulation of errors during the calculations such as round-off and truncation errors triggers these instabilities.Although it may be possible to reduce these instabilities by employing certain smoothing procedures,these procedures are not simple. Moreover,there is no straight-forward way of selecting the best possible smoothing procedure for a given geometry.As a result the accumulation of round-off errors puts a serious limitation on the applicability of this procedure to arbitrary geometries. Another disadvantage of this procedure is that the accuracy of solution cannot be easily verified and usually there is no error estimation.

Recently an alternate procedure based on the solution of operator equations using iterative techniques is proposed by Rao et al.[5].In this method again the space and time are divided into number of subintervals but these discretizations are independent of one another.A suitably defined integrated squred error over a rectangular grid of time and space is sucessively minimized at the end of each iteration by employing either the method of steepest descent or the method of conjugate gradient.The solution is obtained after the error falls below a certain pre-selected value. The solution is usually obtained in a finite number of steps and being a solution procedure based on the minimization of integrated squared error,the round-off and the truncation errors are limited only to the last stage of iteration.The major advantage of this method is that one can obtain accuracy estimate at each iteration and may terminate the algorithm at a desired value of the error.However the method described in [5] is computationally very slow and also is not easily extendable to other geometrical structures.

In this paper,we describe a new solution procedure which retains all the advantages of marching-on-in-time method and also of the iterative methods.This procedure is stable with round-off and truncation errors and simple to apply.

2.METHOD OF CONJUGATE GRADIENT

Let S denote the surface of a perfectly conducting scatterer for which we wish to formulate the time domain scattering problem.The time domain electric field integral equation is given by [6]

$$\left[\frac{\partial E^{inc}}{\partial t}\right]_{tan} = \left[\frac{\partial^2 \bar{A}}{\partial t^2} - \frac{1}{\mu\varepsilon} \nabla(\nabla.\bar{A})\right]_{tan}, \qquad (1)$$

and

$$\bar{A} = \frac{\mu}{4\pi} \int_S \frac{\bar{J}(\bar{r}', t-R/c)}{R} ds' \qquad (2)$$

where μ and ε represent the permeability and permittivity of the surrounding medium,R is the distance between the observation point $\bar{r}$ and the source point $\bar{r}'$.The velocity of propagation in the surrounding medium is $c=(\mu\varepsilon)^{-\frac{1}{2}}$ .In the operator notation,eq.(1) can be written as

$$AI=Y \qquad (3)$$

For the application of conjugate gradient method, we define the integrated squared error as

$$E = \int_S |AI-Y|^2 \, ds \tag{4}$$

at each instant of time.It should be mentioned here that in the present way of defining the integrated squared error the adjoint operator turns out to be same as the original operator. The current $\bar{J}(\bar{r})$ at the time instant $t = i\Delta t$ is computed by starting with an initial guess and generates

$$I_{k+1} = I_k + \alpha_k P_k \tag{5}$$

$$R_{k+1} = R_k + \alpha_k AP_k \tag{6}$$

$$\alpha_k = \frac{1}{\|AP_k\|^2} \tag{7}$$

$$P_{k+1} = P_k - b_{k+1}A\, R_{k+1} \tag{8}$$

$$b_k = \frac{1}{\|A\, R_k\|^2} \tag{9}$$

to minimize the error at this instant.The iterations are continued until the error falls below a pre-selected value. The conjugate gradient method described by eqs.(5)-(9) converges for any initial guess and the simplest guess would be to assume the current to be zero at the beginning of the iterative procedure for each instant of time.

The main difference in the present way of applying the conjugate gradient method to the earlier work described in [5] is that the error in the solution vector is minimized at each instant separately rather than trying to minimize on the whole rectangular grid of space and time.The new approach is not only much faster with respect to computational speed but also requires much less core storage than the one described in [5].In the present case we only have to store seven column vectors of spatial descretization plus a rectangular matrix containing the values of current at each instant and at the center of each spatial subdomain.In the earlier procedure described in [5] one has to store five rectangular matrices representing the time and spatial grid.Since the error is minimized at each instant separately there is no error accumulation as the time progresses and as a result the solution is guaranteed to be stable in the earlier as well as late times.Moreover it is also possible to have a control over the error in the solution vector because basically the method is an iterative method. Details of the numerical procedure along with numerical results will be presented.

3.REFERENCES

1. A.M.Auckenthaler and C.L.Bennet,"Computer Solution of Transient Time Domain Thin-Wire Antenna Problems.",IEEE Trans.Microwave Theory and Tech., No.19, pp 892-893,1971.

2. K.M.Mitzner,"Numerical Solution for Transient Scattering from a Hard Surface by Arbitrary Shape-Retarded Potential Technique.",Journal of Acoust.Soc.Am., Vol.42,pp 391-397,1967

3. G.C.Herman,"Scattering of Acoustic Waves by an Inhomogeneous Obstacle.", International URSI Symp. Munich 1980.

4. G.C.Herman,"Scattering of Transient Acoustic Waves in Fluids and Solids." Ph.D.Thesis,Delft Inst.of Tech. The Netherlands. 1981.

5. S.M.Rao,T.K.Sarkar,and S.A.Dianat,"The Application of the Conjugate Gradient Method to the Solution of Transient Electromagnetic Scattering from Thin Wires." Journal of Radio Science,Vol.19,No.5,pp 1319-1326,Sept-Oct.1984

6. S.M.Rao,Electromagnetic Scattering and Radiation of Arbitrarily-shaped surfaces by Triangular Patch Modeling.,Ph.D. Thesis,University of Mississippi,Aug. 1980