ANGULAR SUPERRESOLUTION FOR PHASED ARRAY ANTENNA BY RECONSTRUCTING APERTURE COMPLEX DISTRIBUTION—THEORY AND EXPERIMENTS*

Da-Gang FANG *, Wei-Xing SHENG *, Dao-Jing LI**, Pei-Kang LIANG**
*Department of Elec. Eng., Nanjing University of Science and Technology, 210094, China
**Xi'an Electronic Engineering Research Institute, 710100, Xi'an, China

1. Introduction

In order to realize the angular superresolution, both the amplitude and the phase aperture distribution related to the incident waves should be obtained[1]. The digital beamforming antenna array may be applied for this purpose. Unfortunately it is quite complicated and costly. Alternatively, in this paper, the time sequence element phase weighting (TSEPW) technique is proposed for the conventional phased array antenna[2]. After measuring the outputs corresponding to the different weighting of this antenna, both the amplitude and the phase distributions may be reconstructed. Consequently, the angular superresolution may be achieved through the non-linear spectral estimation. The outputs and the aperture distribution are related through matrix transformation. For easy implementation, the Walsh-Hadamard transform is specifically chosen. With the aperture distribution, the angular superresolution is realized through the RELAX algorithm[3] which possesses good performance under the low SNR and is robust. The equiphase surface of the array is shifted to face the targets. In so doing, the phase distribution is smooth, consequently the spatial signal can be sampled sparsely to reduce the number of phase weighting and increase the speed for real time processing. Because of the time sequence of the phase weighting, the movement of the radar targets will affect the outputs. To solve this problem, a motion compensation method is presented. This method requires the measurement of the number of the targets and their corresponding Doppler frequencies. The phase shifts of the outputs of the phased array antenna may be obtained from the Doppler frequencies. With these phase shifts, the complex distribution may be found out which involves the motion compensation. For an X band one dimension phased array antenna with 139 phased elements, by using the TSEPW technique, the angular resolution is improved by a factor of 2 under the case of 15 dB SNR. The TSEPW technique may be applied in the design of new phased array radar systems or in the reform of the existing conventional phased array radar systems. It is also possible to be applied in the communication systems.

2. Theory

We consider a one-dimensional phased linear array antenna with N phased elements as shown in Fig.1. The total output of the phased array antenna is the vector summation of the individual element outputs and is received by the coherent receiver. The arrival angle of echo from a target located in the far-field region is denoted by θ. With the received signal x_i by the ith element, the complex amplitude distribution on the array forms a column vector $X=[x_1,x_2,\ldots,x_N]^T$. N different phase weighting are used, and we assume that in the ith phase weighting, corresponding to each element, the shifted phase is $\varphi_{i1},\varphi_{i2},\cdots,\varphi_{iN}$ respectively, and the weighting coefficient is $w_{ik}=e^{j\varphi_{ik}},\ k=1,2,\cdots,N$. The output of the antenna array is y_i. Therefore, the output of the phased array antenna with N phase weighting is given by the

*Supported by the National Nature Science Foundation of China (69771004)

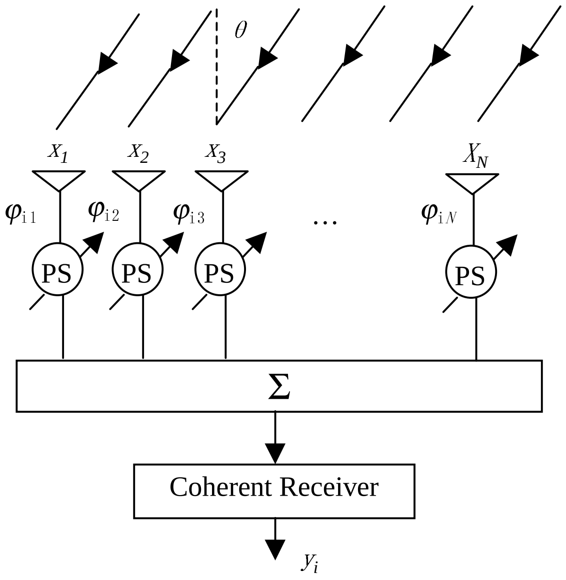


Fig.1 Scheme of phased array antenna

following relationship.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \tag{1}$$

That is, $\boldsymbol{Y}=\boldsymbol{W}\cdot\boldsymbol{X}$. The reconstruction of the amplitude and phase distribution on the antenna array aperture is calculated by

$$\boldsymbol{X}=\boldsymbol{W}^{-1}\cdot\boldsymbol{Y} \tag{2}$$

For easy implementation, Hadamard matrix $\boldsymbol{H_N}$ is chosen as $\boldsymbol{W}$, provided N satisfies the following relationship

$$\begin{cases} N \bmod 4 = 0 \\ N = 2^k \text{ or } \dfrac{N}{12} = 2^k \text{ or } \dfrac{N}{20} = 2^k \end{cases} \tag{3}$$

where k is a positive integer.

It is well known that the inverse matrix of $\boldsymbol{H_N}$ is $\boldsymbol{H_N}^{-1}=\boldsymbol{H_N}/N$. Thus, Eq.2 can be rewritten as

$$\boldsymbol{X} = \boldsymbol{H}_N^{-1}\boldsymbol{Y} = \frac{1}{N}\boldsymbol{H}_N\boldsymbol{Y} \tag{4}$$

Since the elements of matrix $\boldsymbol{H_N}$ are +1 or –1, therefore, the phase weighting may be realized by the two-state 0 or π phase shifter.

If N cannot satisfy Eq.3, we choose the maximum integer M which is less than N and satisfies Eq.3 to form an M by M matrix $\boldsymbol{H_M}$, only M elements of the array antenna are phase weighted according to the N coefficients of a row in $\boldsymbol{H_M}$ to get y_1, y_2, …, y_M .

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{M} \\ \vdots & \vdots & \vdots & \vdots \\ h_{M1} & h_{M2} & \cdots & h_{MM} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \sum_{i=M+1}^{N} x_i \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{5}$$

To solve for Eq.5, we need the following additional phase weighting:

$$y_{M+1} = \begin{bmatrix} -1 & -1 & \cdots & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \sum_{i=M+1}^{N} x_i \tag{6}$$

Because $h_{11}=h_{12}=\cdots=h_{1M}=1$, we have

$$\sum_{i=M+1}^{N} x_i = \frac{1}{2}(y_1 + y_{M+1}) \tag{7}$$

By substituting Eq.7 into Eq.5, we get

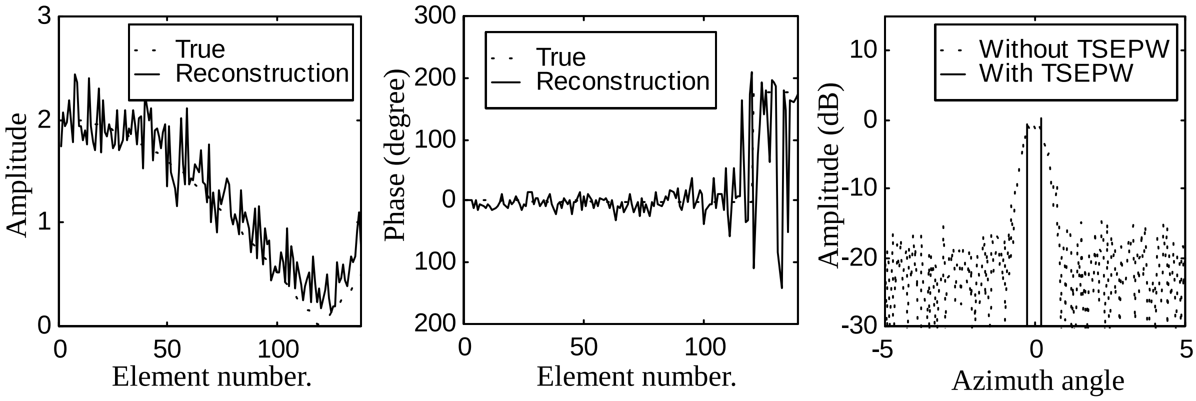
$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \frac{1}{M} \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ h_{M1} & h_{M2} & \cdots & h_{MM} \end{bmatrix} \begin{bmatrix} y_1 - \frac{1}{2}(y_1 + y_{M+1}) \\ y_2 - \frac{1}{2}(y_1 + y_{M+1}) \\ \vdots \\ y_M - \frac{1}{2}(y_1 + y_{M+1}) \end{bmatrix} \tag{8}$$

The equiphase surface of the array is always shifted to nearly face the targets. In so doing, the phase distribution on the aperture is smooth. Consequently the spatial signal can be sampled sparsely to reduce the number of phase weighting and increase the speed for real time processing.

Because of the time sequence of the phase weighting, the movement of the radar targets will affect the outputs *Y*. As a result, the aperture complex field distribution will be contaminated. Therefore the motion compensation method is introduced. This method requires the measurement of the targets and their corresponding Doppler frequencies. The phase shifts of the outputs of the antenna may be obtained from the corresponding Doppler frequencies. With these phase shifts, the outputs *Y* may be corrected and the clean aperture complex field distribution is finally found. The higher accuracy of measuring the Doppler frequencies, the less error of the aperture distribution. It is seen that the accuracy of 1Hz is enough for the requirement of the aperture distribution reconstruction. This distribution is extrapolated through the non-linear spectral estimation algorithm to get the angular superresolution. After careful comparison, we choose the RELAX algorithm to do this job. The reason is that this algorithm possesses good performance under the lower SNR and is quite robust. These advantages are important in our application.

3.Numerical and experimental results

We first present a numerical example for superresoluton angle estimation by the TSEPW method. Consider an X band one-dimensional phased linear array antenna radar with 139 phased elements. The carrier frequency is 10 GHz. The half-power beam width of the antenna is 0.84°. The pulse repetition frequency (PRF) is 1KHz. Consider the imperfect of radar components and the received noise, we assume that the root-mean-squared error (RMSE) of the shifting phase of the element phase shifter (including the phase quantization error) is 5°. The RMSE of insertion loss of the phase shifter in different phase is 0.5dB. The output SNR of the receiver is 15dB and the 10 bits analogue-to-digital (A/D) converter is used.



(a) (b) (c)

Fig.2 Results of numerical data. (a)True and reconstructed amplitude distribution of aperture (b) True and reconstructed phase distribution of aperture (c) Detected radar targets with and without TSEPW

We assume that there are two targets in front of the radar at same distance of 10Km and their azimuth angles are –0.2° and 0.2° respectively. One of these two targets moves toward radar in 5m/s, and the other moves backward radar in 3m/s. Let *M*=128. Two Doppler frequencies are detected as 344.7Hz and –207.0Hz. Fig.2 (a) and Fig.2 (b) show the true and the reconstructed amplitude and phase distribution on the antenna aperture. Fig.2 (c) shows the angle information of the targets with and without the TSEPW method.

We also apply the TSEPW method to experimental data. An X band phased array radar with same parameters as those in numerical example is used as shown in Fig.3. Fig.4 shows the angle information of the targets with and without the TSEPW under 15dB SNR. The superresolution performance is observed in different SNR and different number of phase weighting. The results are shown in Table 1.

Table 1 Angular resolution (degree) in different SNR and different *M*

SNR \ *M*	32	64	128
30 dB	0.54	0.42	0.42
25 dB	0.54	0.42	0.42
20 dB	-	0.42	0.42
15 dB	-	0.50	0.42

Fig.3 Phased array radar used in experiments

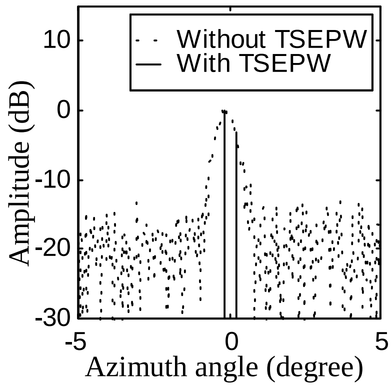


Fig.4 Results of experimental data

4. Conclusions

Azimuth superresolution for phased array radar can be achieved by reconstructing the complex aperture field distribution and using the superresolution algorithm. The reconstruction is obtained through time sequence phase weighting. The imperfection of the components may affect the performance of the reconstruction. This imperfection includes insertion loss and phase error of phase shifter, additional noise and nonlinearity in receiver, non-uniformity between inphase and quadrature channels, quantization error in analog-to-digital converter and so on. It can be shown that Walsh-Hadamard transform can reduce the influence of error and noise, and the requirements are quite reasonable to the conventional radar and existing techniques. The experimental results confirm the applicability and the validity of the proposed TSEPW method.

References

[1] U.Nickel, Dipl-Math, Dr rer nat, “Angular superresolution with phased array radar: a review of algorithms and operational constraints”, *IEE Proceedings Pt.F*, Vol.134, February 1987, 53-59.

[2] D.G.Fang, J.T.Sun, Y.Wang and W.X.Sheng, ”Realization of radar cross-range resolution by array phase weighting”, *Chinese Journal of Radio Science*, Vol.10, June 1995, 1-3.

[3] J.Li, D.M.Zheng and P.Stocia, “Angle and waveform estimation via RELAX”, *IEEE Trans. on AES*, Vol.33, July 1997, 1077-1087