EARTH FLATTENING APPROXIMATION AND FOCK'S THEORY FOR LOW ALTITUDE EM PROPAGATION OVER THE OCEAN

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Introduction

Low altitude propagation of electromagnetic (EM) waves over the ocean is strongly affected by the presence of water vapor, spray and the ocean waves. One way to describe the marine boundary environment for EM propagation is by specifying the measurable environmental parameters which determine the statistics of the index of refraction in the boundary layer. Specifically, there is a turbulent layer extending continuously from above the tip of ocean waves to somewhere below the ocean surface. At microwave frequencies, the index of refraction in this region varies continuously but often fluctuates strongly from a mean of about unity at the top to about nine in the sea water. As a random variable, the index of refraction governs the low altitude characteristics of EM wave propagation. Its variation with range and its fluctuation from the mean value give rise to sea clutter. To determine the statistics of the index of refraction in the marine boundary layer is a multi-disciplinary effort requiring the expertise of meteorologists, oceanographers, electrical engineers and physicists. In anticipation of such a joint effort in the US to obtain the mean and the fluctuation, or even the complete statistics of the index of refraction, a framework for accurately computing low altitude propagation factor and sea clutter once such statistics are given is under development.

Earth Flattening Approximation

The major problem in low altitude propagation pertains to the grazing incidence of these waves into the air-ocean interface. When an incident wave makes a very small angle with the tangent plane to the earth at the point of incidence, the falloff of the earth surface below the tangent plane away from the point of incidence becomes important. Based on ray-optics argument, the earth flattening approximation and the associated notion of the modified index of refraction were introduced to take care of the effect of the earth curvature [1]. By assuming that the ground range of interest was small compared to the radius of the earth and that fields varied much slower along the vertical than along the horizontal direction, M.H.L. Pryce was able to deduce a set of approximate Maxwell equations which, when the ground range was allowed to run to infinite, took the form of the Maxwell equations above a flat earth, with the index of refraction replaced by the modified refractive index [1]. Using this set of equations, a FORTRAN program, M-Layer [2], was developed for the computation of the propagation factor of a wave in a piecewise linear modified refractive index profile. The outputs of the M-Layer program were utilized in the construction of interpolating formulae for the over-the-horizon portion of the computer programs "Integrated refractive effects prediction system" (IREPS) [3] and "Engineer's refractive effects prediction system" (EREPS) [4].

Fock's Theory of Diffraction

From 1945 through 1962, Fock [5] and his colleagues published a series of papers on diffraction theory which was later recognized as pioneering efforts to seek out the transverse behavior of fields near a caustic [6]. For the problem of the radiation of an electric dipole above a spherical earth, Fock began with the exact expression for the Hertz vector in terms of the Mie series sum over the Legendre polynomials. After a series of transformation of the Legendre polynomials, he showed that the contribution to the series sum mainly came from terms having an order close to the earth circumference-to-radio wavelength ratio. Making use of the fact that this ratio is extremely large at radio frequencies, he replaced the Legendre polynomials with their asymptotic forms and converted the series sum for the Hertz vector into an integral which takes on the order of the original series as the complex variable of integration. Fock went on to suggest that the contour of this integral be deformed as van der Pol and Bremmer [7] did with the Watson transform approach to form a residue series. The Airy functions were introduced as the asymptotic representations for the Bessel functions encountered. Fock then found that the method of parabolic equation proposed by Leontovič together with the simplifying Leontovič boundary condition (also known as the impedance boundary condition) led to the Airy functions and the residue series directly. In fact, the method of parabolic equation can be used to deduce the fields near many types of caustics [Babič and Buldyrev, 1991]. It applies to the problem of the diffraction of waves by the earth because the surface of the earth is a caustic of the whispering gallery type. On the other hand, the method of parabolic equation applies only to the neighborhood of a caustic. Away from a caustic, it fails to be a good approximation to the wave equation. Nor can its solution fulfill the radiation condition at a great height from the earth. Direct simplification of the wave equation to the parabolic equation thus restricts the applicable region of the solution to the neighborhood of the caustics whose locations in many cases are not known in *a priori*.

A New Approach to Earth Flattening

The earth flattening approximation as prescribed by Freehafer [1] takes into account the earth curvature while allowing the use of the cylindrical coordinate system over a flat earth. The adoption of a cylindrical coordinate system automatically converts the series sum over the spherical harmonic functions for the fields above a spherical earth into an integral. Effecting such a conversion to evaluate the series more expediently had been the underlying theme of Watson transformation and of Fock's endeavor. The earth flattening approximation, if properly formulated, should greatly simplify EM wave propagation problems over a small portion of the surface of the earth. This approximation keeps the equations governing the fields in the form of Maxwell equations; hence, unlike the method of parabolic equation, it is applicable away from a caustic. But oversimplification in its original form led to results [8] which disagreed with Fock's diffraction theory near the earth surface. Recent investigations of the earth flattening approximation by Lee [9] revealed that, only the assumption that the ground range of interest is small compared to the radius of earth is adequate for deducing a set of approximate Maxwell equations above a flattened earth. The use of the modified index of refraction is not recommended under this new formulation.

When applied to the problem of radiation of a dipole in a homogeneous atmosphere above the earth, this new set of Maxwell equations leads to a solution which is equivalent to Fock's integral expression for the Hertz vector. By adopting Langer's uniform asymptotic expansions for the Bessel functions [10] instead of using the Airy functions directly, this solution is applicable everywhere, including infinity where the radiation condition has to be enforced, and is no longer restricted to the vicinity of a caustic. The extension to a profile of layers of constant refractive indices is immediate. Numerical procedures to search for the creeping wave modes and the waveguide modes of propagation have been developed and is being tested.

Discussions

Modeling the atmosphere and the air-ocean interface with a continuous, piecewise linear refractive index profile introduces complications which did not occur in the original earth flattening approximation. The height dependence due to the effect of earth curvature becomes quadratic and there is no difference, as far as mathematical complexity is concerned, to use a piecewise linear or a piecewise quadratic refractive index profile. The possibility of the coalescing of two nearby modes under a quadratic profile has been discussed by Langer [11] and by Pekeris [12]. The problem with the new formulation is more severe: the solution involves the Whittaker function of which uniform asymptotic expansion over only limited regions of its variable and parameters on the complex plan are known.

Problems involving range inhomogeneities can also be considered. Since the approximate Maxwell equations are separable in other flat coordinates such as the elliptic-cylindrical system, canonical problems such as those having a hyperbolic weather front or a coastline can be investigated using this new set of approximate equations.

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