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Short-term prediction of rain attenuation using financial time series models

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1. Introduction

The constant demand for increased capacity of communication channels has led the SATCOM industry to develop new satellite systems operating at frequencies above 20 GHz where large bandwidths are available (EHF band – Extremely High Frequencies). Nevertheless, attenuation effects of atmospheric gases, clouds and rain can reach significant levels at these frequencies and it is no longer cost-effective to use a fixed power margin.

Rain attenuation, due to scattering and absorption by water droplets, is the major limitation for satellites links in the EHF band. As rain events have a very restricted extension in time and space, various adaptive Fade Mitigation Techniques (FMT) have been developed to make communications at these frequencies possible. Whatever the FMT used, short-term prediction of rain attenuation is necessary to track the propagation channel variations and trigger power compensation only when needed. Typically, the forecast interval is 10 seconds, corresponding to the control loop reaction time. Unfortunately, rain attenuation behaves almost like a random walk (Brownian motion) and existing prediction models do hardly better than considering that the process remains constant over the forecast interval. Nevertheless, as rain attenuation time series show periods with high 'volatility' or strong variations, its statistics are similar to some stocks and currency exchange rates, therefore suggesting that the use of models originally developed for financial applications might be appropriate [1].

2. Measurements

A database was created from OLYMPUS 20 GHz satellite beacon data measured at Gometz-la-Ville, France, during 15 months at an elevation angle of 30°. The data were recorded at a sampling frequency of 100 Hz and averaged over 1 second intervals. 67 attenuation events were selected, which represent a total of 550 hours, including at least 57 hours of rainfall (the criterion being that the attenuation level exceeds a threshold of 1.5 dB).

3. TARIMA-GARCH prediction Model

Following the theory of time series analysis, the study of attenuation and its autocorrelation has led us to describe it as a TARIMA process (Threshold Auto Regressive Integrated Moving Average). First, a threshold of 1.5 dB separates situations with and without rain. Then the attenuation time series is differentiated to obtain a first-order stationary process that can be approximated by a classical ARMA model. The identification procedure yielded to the following equations:

$$Y_t = X_t - X_{t-1}$$

$$Y_t = \sum_{i=1}^2 a_i Y_{t-i} + \varepsilon_t + \sum_{i=1}^2 b_i \varepsilon_{t-i}$$

Where X_t is the attenuation at time t (Figure 1.a), Y_t , the differentiated process and ε_t the prediction error made by the ARMA predictor (Figure 1.b). The four parameters are estimated by minimizing a robustified quadratic prediction error criterion [2].

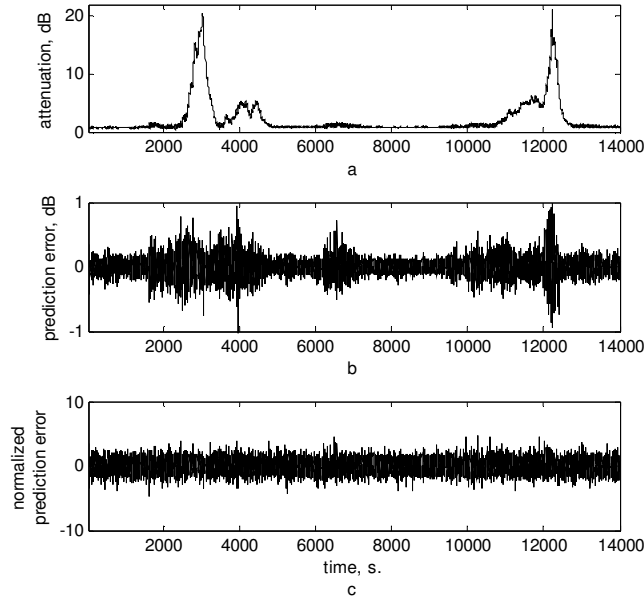


Figure 1: Rain event occurring on 30th June 1992
a OLYMPUS 20 GHz beacon attenuation X_t
b TARIMA prediction error ε_t
c GARCH normalized prediction error η_t

The periods of large variations of ε_t (Figure 1.b) indicates that the prediction error variance σ_t^2 is not constant and could be optimized by an appropriate time-varying model. The fact that the prediction error distribution has a high Kurtosis (equal to 6) compared to the normal distribution (Figure 2.a) confirms that the linear ARMA predictor is not able to model the second-order non-stationarity of the process. Since it was found that the autocorrelation of ε_t^2 is significant over 10 seconds time-lags, a non-linear GARCH model (Generalized Auto Regressive Conditional Heteroskedasticity) was selected to describe σ_t^2 [1] :

$$\varepsilon_t = \eta_t \cdot \sigma_t$$

$$\sigma_t^2 = K + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$$

where η_t is the normalized prediction error and σ_t the conditional standard deviation of ε_t . The GARCH parameters K , α and β are estimated by maximum likelihood. Analyzing the normalized prediction error η_t shows that it is a second-order stationary process (Figure 1.c) with a Gaussian distribution (Figure 2.b). Moreover it was found that η_t is an uncorrelated process, which allows us to conclude that η_t is a second-order Gaussian white noise, and consequently that the prediction error variance has been successfully modelled. Thus, the GARCH model can be used to predict in real time σ_t^2 and compute the adaptive error margin that has to be added to the TARIMA prediction to reach a required link availability.

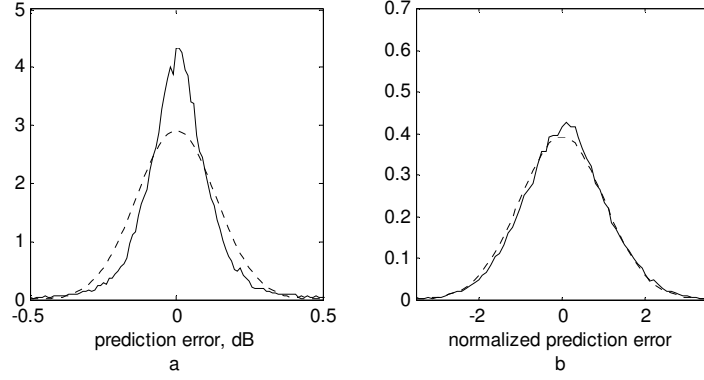


Figure 2: Comparison with the Gaussian distribution
a pdf of TARIMA prediction error ε_t
b pdf of GARCH normalized prediction error η_t

4. Comparison and results:

A comparison has been performed with three other models and a forecast interval set at 10 seconds. The simplest model, called ‘persistence’, relies on the assumption that the attenuation level remains constant over the forecast interval and adds a fixed error margin corresponding to the required availability.

We also implemented the Van de Kamp model based on a ‘two-samples’ Markov chain with an error margin depending on the attenuation level [3]. This model is presented with a 0.1 Hz cutoff frequency filtering to remove scintillations. However, this preprocessing is not suitable in real time because the causal filter’s phase shifting effect lowers the model’s performance. For this reason, filtering is used to determine the parameters, given in [3], but not in the real time comparison.

Finally, the TARIMA-GARCH model is compared with the ARMA(3,3) model, as proposed by Grémont [4], whose parameters are updated in real time by a RLS algorithm. This model assumes that the prediction error corresponds to a scintillation process, whose variance is known to be constant over short periods of time (up to 1 min). Thus, the error margin is adapted according to a real time estimation of the error standard deviation over the past 60 seconds.

The link availability is conventionally defined as the percentage of time during which the measured attenuation level does not exceed the power margin. In order to compare the models for a given availability, a ‘mean cost’, C , is defined in dB, as the mean overestimation of the measured attenuation level by the power margin:

$$C = \frac{1}{T} \int_{PM_t - X_t > 0} [PM_t - X_t] dt$$

The power margin PM_t is the predicted attenuation level \hat{X}_t plus the corresponding error margin \hat{M}_t :

$$PM_t = \hat{X}_t + \hat{M}_t$$

The ‘mean cost’, C , corresponds to the unnecessary part of the power margin that does not compensate for any channel attenuation.

The results can be influenced by the clear sky time percentage in the database because the presented models behave differently according to the meteorological situation. Therefore, to provide a comparison that is independent from the database composition, the cost / availability relationships given in Figure 3 are computed only over the rainy part of the database. The comparison shows that

the TARIMA-GARCH model reduces the ‘mean cost’ by 20% for a required availability of 99% during rain events.

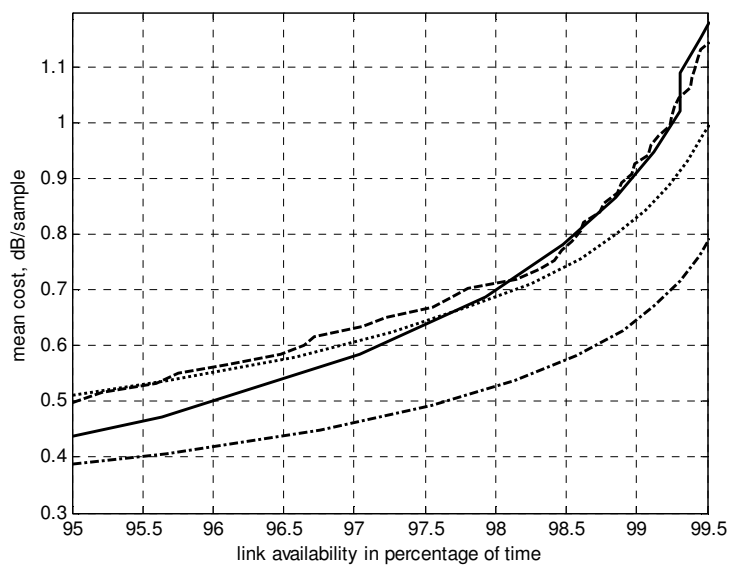


Fig. 3 ‘Cost / availability’ performances of the models

— persistence
 adaptative ARMA(3,3)
 - - - - 'two samples' model
 - . . . - TARIMA-GARCH

5. Conclusion

The short-term modeling of the prediction error variance allows the TARIMA-GARCH model to outperform the existing models with respect to the optimization of the power margin used in FMTs. This result can be explained by the fact that the prediction error contains not only scintillations, but also a fast fluctuating rain attenuation component that can not be modeled by linear prediction methods. Therefore the prediction error variance can not be supposed constant over 60 seconds intervals as proposed in the Grémont model. The error margin used in the ‘two samples’ model is also limited because it depends only on the attenuation level and does not take into account the autocorrelation of the prediction error variance. Future work will focus on the estimation of the frequency scaling factor needed in an Uplink Power Control FMT in which the prediction has to be made from a given frequency to a higher one.

References

- [1] WEI W.W.S.: ‘Time Series Analysis : Univariate and Multivariate Methods’, 2nd edn., Addison Wesley, chap. 15, 2005.
- [2] LJUNG, L. : ‘System identification - theory for the user’, 2nd edn., Prentice-Hall, chap. 10.2, 1999.
- [3] BOLEA-ALAMAÑAC A., BOUSQUET M., CASTANET L., VAN DE KAMP M.M.J.L. ‘Implementation of short-term prediction models in fade mitigation techniques control loops’, COST 272 / 280 Workshop., Noordwijk, The Netherlands, ESA/ESTEC, ESTEC, 26-28 May 2003.
- [4] GREMONT B., FILIP M., GALLOIS P. and BATE S. : ‘Comparative analysis and performance of two predictive fade detection schemes for Ka-band fade countermeasures’, *IEEE Sel. Areas Commun.*, 17, (2), pp. 180–192, 1999.