

EXTRACTION OF THE TROPOSPHERIC REFRACTIVE INDEX PROFILE  
FROM THE RANGE DATA

ATEF M. GHUNIEM

Department of E.M. Fields, M.T.C.  
1 El-Ebour Bldg. Salah Salim St., Cairo, Egypt

ABSTRACT- The tropospheric refractive index is important for calculating the refracting effects on the wave propagation. This profile has to be determined either by direct measurement or by inference using other related data. In this paper, a simple and fast method is used to obtain this profile by inversion from the measured range data. Geometrical optics is used to solve the direct problem and a functional gradient method is used for inversion. Computations are made using linear, bilinear, trilinear and exponential profiles for both noise free and noisy data.

INTRODUCTION

Radio refraction is an important factor that affects the wave propagation through the troposphere. An example of these effects is the variation of the angle of arrival at the receiving antenna of radio links which leads to an additional transmission loss [1]. Another example is the variation of the time delay and deviation from the direct path which lead to additional errors in range measurement using radio waves.

The refractive index profile for the troposphere can be determined either directly or indirectly. The direct way is to determine it using the measured profiles for temperature, pressure and humidity or to measure it using a refractometer [1,2-4]. The indirect methods are basically inversion techniques that use related measured data to obtain the refractivity profile [5,6].

In this work, a simple and fast indirect method will be used. It is based on an inversion technique which utilizes the range data measured by radar for an aeroplane flying at a constant height. First, the geometrical optics is used to obtain the range as a function of the refractive index profile. Then the range data are spoiled by adding noise representing the random errors of measurement. Finally, an inversion technique based on a functional gradient method is used to recover the original profile.

FORMULATION OF THE PROBLEM

THE DIRECT PROBLEM

For an aeroplane flying at a constant height  $H$ , as shown in fig.1, the geometrical optics is used to calculate the radar range  $R$  by integrating the optical distance along the ray trajectory, i.e.,

$$R = \int_0^P m(z) ds \quad (1)$$

where  $m(z)$  is the modified refractive index profile which is related to the excess refractive index  $N(z)$  by [7] :

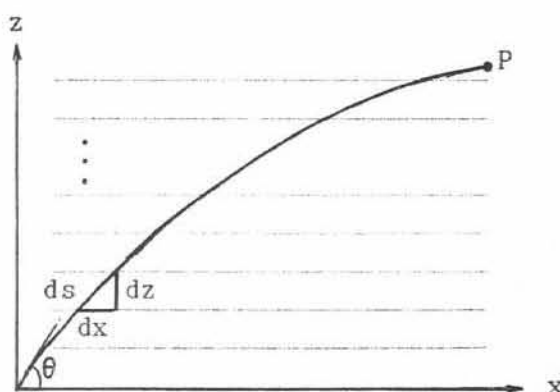


Fig.1. Geometry of the problem.

$$m(z) = 1 + N(z) \times 10^{-6} + z/r \quad (2)$$

where  $r$  is the radius of the earth.

The region of the troposphere from the ground to the height  $H$  is stratified into thin layers. For the horizontally stratified medium, the slope of the ray is [7]:

$$\frac{dx}{dz} = a[m^2(z) - a^2]^{-1/2} \quad (3)$$

where  $a = m(0) \sin \theta_0$ .

Putting  $ds = (dx^2 + dz^2)^{1/2}$  in (1) and using (3), we get:

$$R = \int_0^H m^2(z) [m^2(z) - a^2]^{-1/2} dz \quad (4)$$

Assuming that  $m(z)$  is constant for each layer, and  $R$  is calculated for different initial angles  $\theta_i$ , we can transform (4) to the discrete form:

$$R_i = h \sum_{j=1}^M u_j (u_j - b_i)^{-1/2}, \quad i=1,2,\dots,K \quad (5)$$

where  $M$  is the number of layers,  $K$  is the number of data points,  $h=H/M$ ,  $u_j = m^2(z_j)$  and  $b_i = a^2(\theta_i)$ .

#### THE INVERSE PROBLEM

The problem as given by (5) is nonlinear. As a result, an inversion technique using a functional gradient method [8] can be used. The method consists of defining a positive functional as:

$$F = \sum_{i=1}^K (R_i - R_{mi})^2 \quad (6)$$

where  $R_{mi}$  is the measured range ( $i$ th data point), and  $R_i$  is the calculated range using an assumed refractive index profile as an initial guess. Then we obtain the gradient of (6), in the form:

$$\Delta F = \int_0^H g(z) \Delta u(z) dz \quad (7)$$

where:

$$g(z) = \sum_{i=1}^K (R_i - R_{mi}) [u(z) - 2b_i] [u(z) - b_i]^{-3/2} \quad (8)$$

To decrease  $F$  in each iteration, the required step correction  $\Delta u(z)$  is chosen to ensure that  $\Delta F$  is negative, i.e.

$$\Delta u(z) = -t g(z) \quad (9)$$

where  $g(z)$  is the correcting function given by (8) and  $t$  is a constant which may be determined using a search algorithm [8] for minimization of the functional  $F$  in each step.

The value of the refractive index at the ground level  $N_s$  is either known or can be easily measured. It can be used as a constraint on the solution and on the assumed profile as well. In this case equation (7) may be modified to take the form :

$$\Delta F = \int_0^H z g(z) \Delta u(z) dz \quad (10)$$

#### NUMERICAL COMPUTATIONS AND RESULTS

The numerical computations are done using linear, bilinear, trilinear and exponential profiles as examples. For a given profile, the radar ranges (data points) are calculated using the direct problem. A noise vector from a random number generator is added to the data to simulate the real range measurements. The inverse problem is then solved to obtain the original profile. The solution starts with an arbitrary profile as an initial guess, provided that the value of  $N$  at the ground level is taken as the measured one.

The best results were obtained for the linear profile as given in fig.2, in both cases of noise free and noisy data. For the bilinear profile agreement was also good in spite of the little deviation at the discontinuity as shown in fig.3. The worst results were obtained for the trilinear profile shown in fig.4. It is clear that the method is unable to cope with the sharp discontinuities. This example is intended to examine the efficiency of the method in dealing with severe cases such as those having a super-refractive layer. However, this is an extreme case because sharp changes in the refractive index gradient are unlikely to occur in nature. For the exponential profile, the noise free case gives excellent results, but in case of the noisy data, there is some deviation at large heights as it is clear from fig.5.

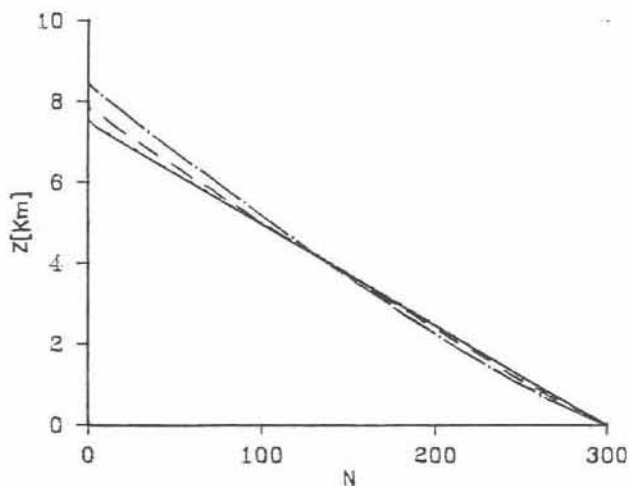


Fig.2. Linear profile.

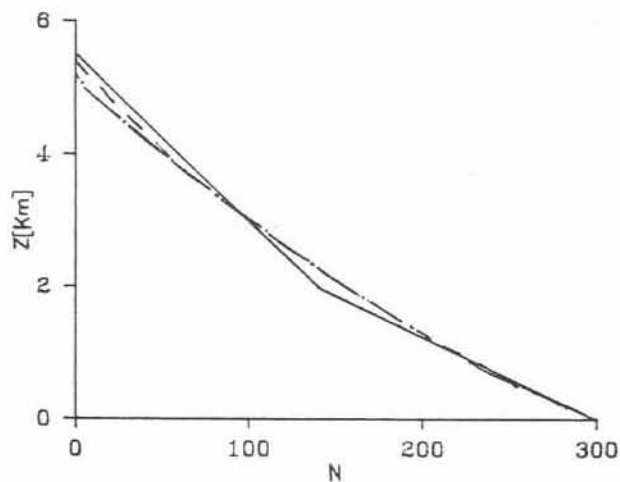


Fig.3. Bilinear profile.

- Original profile
- - - Calculated from noise free data
- . - Calculated from noisy data

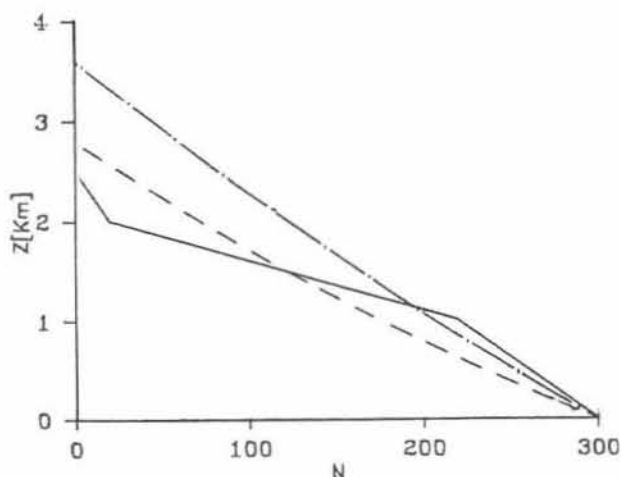


Fig.4. Trilinear profile.

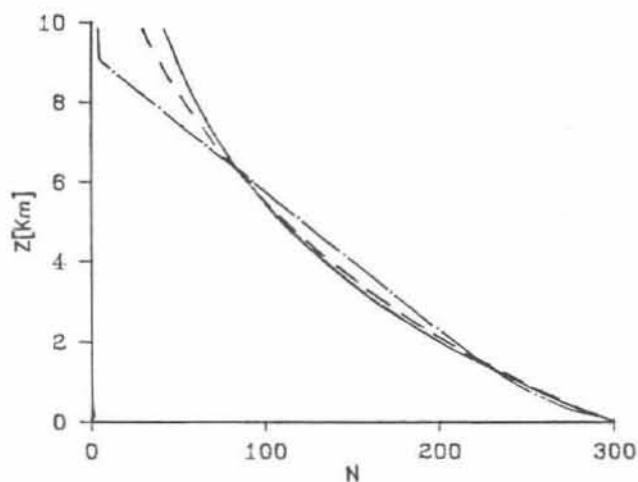


Fig.5. Exponential profile.

— Original profile  
 - - - Calculated from noise free data  
 - · - Calculated from noisy data

#### CONCLUSION

An inverse technique using a gradient method is presented. In this technique the radar range data are inverted to get the tropospheric refractive index profile. This method is found good except at sharp discontinuities of the refractive index gradient.

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