ON THE ADDITIONAL DISPERSION OF A WHISTLER IN THE EARTH-IONOSPHERE WAVEGUIDE

A.I.Sukhorukov[1], S.Shimakura[2] and M.Hayakawa[3]

[1]Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, 142092 Troitsk, Moscow Region, USSR

[2]Department of Electrical and Electronic Engineering, Chiba University, Chiba 260, Japan

[3]Sugadaira Space Radio Observatory, The University of Electro-communications, Chofu, Tokyo 182, Japan

Astract. - An analysis of the excitation of a night-time Earth-ionosphere waveguide near the cut-off frequencies of the waveguide modes is presented as applicable to very unusual whistlers with additional dispersion.

1. Introduction.

Shimakura et al.(1987) investigated whistlers under strong effect of propagation in the Earth-ionosphere waveguide. These kinds of whistlers, named very unusual whistlers are observed only under night-time conditions and are characterized by a clear-cut additional dispersion near the cut-off frequencies of the 1st and the 2nd waveguide modes. By simultaneously localizing the sources of very unusual whistlers and of their causative atmospherics received at two stations, Shimakura et al.(1991) and Hayakawa et al.(1991) showed that lightning discharges causing very unusual whistlers occur exactly at the duct entrance,and waves are trapped in a magnetospheric duct in the local sunrise meridian. Hence the additional traces of the Earth-ionosphere waveguide propagation (in the case in question ~ 3000 km westwards) are concluded to be due to the subionospheric propagation after leaving the ionosphere. The exit of a low-latitude whistler into the Earth-ionosphere waveguide is secured either by fairly large trapping cone of the duct propagation overlapping, at least partly, a transmission cone, or by horizontal gradients or density inhomogeneities in the ionosphere resulting in a wave scattering into the transmission cone (Helliwell, 1965; Walker, 1976; Hayakawa and Tanaka, 1978; Shimakura et al., 1991). While analyzing the additional waveguide dispersion one used earlier a simplest model of an ideal Earth-ionosphere waveguide what permitted, however, with sufficient precision, to determine distances to the source and to estimate an effective waveguide height (Shimakura et al., 1987). The purpose of the present paper is to obtain analytical estimations of the effect of the subionospheric whistler propagation in the framework of a nighttime waveguide model with a sharply bounded anisotropic ionosphere and with due regard to the inclination of a geomagnetic field.

2. Statement of the problem and method of solution.

A whistler which has entered into the Earth-ionosphere wave-guide is, at large distances from the entrance point, a super-position of normal waveguide modes. Near cut-off frequencies f_{cm},normal waves of the m-th order have relatively small angles of incidence onto ionosphere. In case of a horizontally homogeneous waveguide model, that we confined ourselves to,this means that for the formation of a very unusual whistler it is necessary that plane waves with almost vertical waveguide vectors be presented, to a sufficient extent, in the spatial spectrum of a downward whistler at the exit from ionosphere ($|k_\perp|^2 \ll k_0^2 \ll |k_z|^2$, k_0 is wave number in vacuum). Let us set spatial (x,y) distribution of a component $E_x(\vec{\rho},z,\omega) \equiv E_0$ of a downgoing whistler at a certain height z_0 ($z_0 > h$, h is a waveguide height) that can be presented in the form of expansion over plane waves

$$\tilde{E}_0(\vec{k}_\perp,z_0)=\int d\vec{\rho}\; e^{-i\vec{k}_\perp\vec{\rho}} E_0(\vec{\rho},z_0), \quad E_0(\vec{\rho},z_0)=\int \frac{d\vec{k}_\perp}{(2\pi)^2}\, e^{i\vec{k}_\perp\vec{\rho}} \tilde{E}_0(\vec{k}_\perp,z_0) \qquad (1)$$

where $\vec{\rho} = (x,y)$, $\vec{k}_\perp = (k_x,k_y)$. The spectrum $\tilde{E}_0(\vec{k}_\perp,z_0)$ is assumed to be smooth inside the transmission cone (i.e. in the domain of wave numbers $|k_\perp| \leqslant k_0$).

Let us ionosphere $(z > h)$ homogeneous being characterized by a dielectric permeability tensor ε_{ij} of a cold magnetoactive plasma with a geomagnetic field $\vec{B}_0$ in the plane yOz (Fig.1). For the nighttime conditions of the given model, an inequality $\omega_{Be} \gg \nu$ is valid, where ω_{Be} and ν are a gyrofrequency and an effective collision frequency of electrons at height $z \sim h$. The model with a sharply bounded ionosphere is quite acceptable when examining the processes in fairly narrow spectral bands under conditions of the nighttime undisturbed ionosphere. The Earth is assumed ideally conductive.

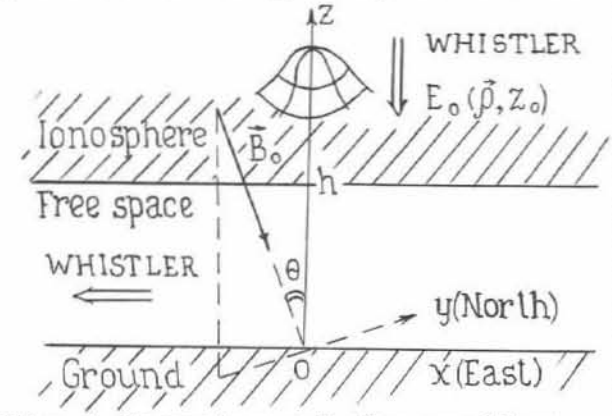

Fig.1 Geometry of the problem

The method of solving the problem goes to finding tangential spatial Fourier field components $\vec{E}_\tau(\vec{k}_\perp,z)$ and $\vec{H}_\tau(\vec{k}_\perp,z)$ in the ionosphere and in the waveguide, their futher stitching at the boundary $z = h$, and reverse Fourier synthesis of the form (1). In this case, the field in ionosphere is a superposition of a downgoing right-hand polarized wave (a incident whistler) and of right- and left-hand polarized waves reflected upward off the boundaries. Since the inequality $|k_{zs}| \gg k_0$ is valid for VLF waves inside transmission cone in ionosphere, in a quasilongitudinal aproximation ($tg\,\theta \ll 2\omega_{pe}^2/\omega\,\omega_{Be}$, ω_{pe} is a plasma frequency) we can use the expansion

$$k_{zs} = k_0 n_s + \alpha_s k_y + \beta_s k_x^2 + \gamma_s k_y^2 \qquad (2)$$

where s=1,2,3, n_s is a refractive index of a wave of the s-type propagating along the Oz axis, the linear over $k_\perp$ term determines the shift of a wave beam, the real part of quadratic over $k_\perp$ terms determines divergence of the beam (Bellyustin and Polyakov, 1977).

For large distances from the entrance point of a whistler into the waveguide ($k_0\rho \gg 1$), a reverse Fourier trasform (integration via the method of a stationary phase over an angular variable and contour integration in a complex plane $k_\perp$) leads to a series over normal quasi-TM and quasi-TE waveguide modes (Wait, 1962) (right-hand and left-hand polarized near corresponding cut-off frequencies).

3.Results and discussion.

The analysis of the pole equation yields the values of the cut-off frequencies of the right-hand and left-hand polarized modes

$$f^{R}_{cm} = \frac{cm}{2h}\left[1 + O\left(\frac{\nu}{\mu\omega_{Be}},\ \mu^{-2}\right)\right], \qquad f^{L}_{cm} = \frac{cm}{2h}\left[1 - \frac{1}{\pi m\mu}\right] \qquad (3)$$

$m=1,2,\ldots$, $\mu = \omega_{pe}/(\omega\ \omega_{Be}\cos\theta)^{1/2}$; structurally similar to those for $\theta=0$ (Edemskii et al.,1988); their dependence on the propagation direction of a wave is neglegibly small. Making use of expansion over small parameters μ and ν/ω_{Be}, for L_m-mode at $k_0\rho \gg 1$ and near f^{L}_{cm} $(\delta \equiv 1 - f^{L}_{cm}/f)$ we have

$$E^{L}_{zm}(\rho,z=0) = \frac{(-1)^{m+1}\tilde{E}_0(S_m,\varphi,z_0)k_0^{1/2}S_m^{3/2}}{(\pi\rho)^{1/2}\ h}\left[S_m e^{i\varphi} + \left(1 - S_m^2\right)\frac{tg\ \theta}{\mu}\right]\times$$
$$\times\ exp\left[ik_0\mu(z_0 - h) + ik_0S_m(\rho + \frac{z_0 - h}{2}\ tg\ \theta\ sin\ \varphi)\right] \qquad (4)$$

$H^{L}_{\varphi m} \simeq -E^{L}_{zm}/S_m$, $H^{L}_{\rho m}/H^{L}_{\varphi m} \simeq -i$

(polarization is circular with precision to terms $\sim \delta \ll 1$). Here $S_m = k_{\perp m}/k_0$ is a complex sine of the incidence angle of a wave onto ionosphere, the imaginary part of which determines attenuation coefficient $\alpha^{L}_{m} = 8.7\ k_0 Im\ S_m$ (in *dB/Mm*)

$$Re\ S_m \simeq \sqrt{2\delta}$$
$$Im\ S_m \simeq \frac{1}{2k_0h\mu\sqrt{2\delta}}\left[\frac{\nu}{\omega_{Be}\cos^2\theta} + \delta^2 + \frac{\sqrt{2}\delta^{3/2}}{\mu}tg\ \theta\ \cos\ \varphi + \frac{\delta}{2\mu^2}tg^2\theta\right] \qquad (5)$$

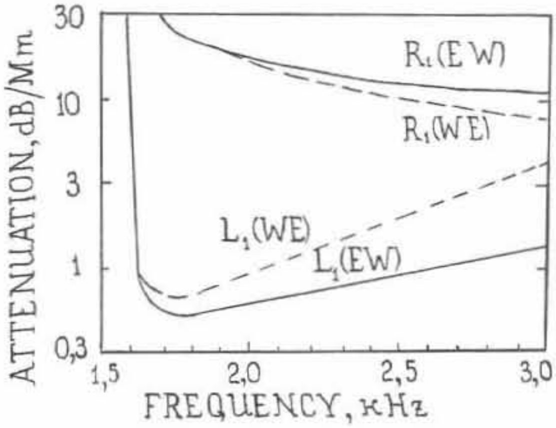

Fig.2 Attenuation coefficient of the first R- and L-modes as a function of frequency for WE and EW propagation for $\theta=50°$, $h=90$ km, $\nu=10^5 c^{-1}$, $N_e=10^3 cm^{-3}$.

The expressions (5) are valid for $\sqrt{2\delta} \gg (\pi m/\mu^2)tg\ \theta$, $(\pi m/2\mu)(\nu/\omega_{Be}\cos^2\theta) \ll \delta \ll 1$. From them it follows that under the night-time undisturbed ionospheric conditions the attenuation coefficients of the first two L modes near the corresponding cut-off frequencies remain small even at appreciable incidence angles of a geomagnetic field (Fig.2), (except near-equtor latitudes). As noted in the work of Edemskii et al. (1988) who analysed the behavior of the waveguide modes near cut-off frequencies in the frame work of a model with a vertical geomagnetic field, the deep minima of the attenuation coefficients of the L_m - modes are due to the cessation of their luminescence into a whistler (R) mode for $f \to f_{cm}$. It is also clear that a drastic decrease of the luminescence into the whistler mode for $f \to f_{cm}$, which depends appreciably upon azimuth, causes a relatively weak azimuthal dependence of the attenuation coefficients of the L_m - modes near cut-off frequencies. However, far from cut-off frequencies, for frequencies such that $\sqrt{2} - 1 < p_m \equiv f^L_{cm}/f < 1$ we have

$$\alpha^L_m(WE) - \alpha^L_m(EW) = \frac{8.7}{h\mu^2} \frac{(1 - p^2_m)\ tg\ \theta}{\sqrt{8p^2_m - (1 + p^2_m)^2}} \quad (6)$$

and, for instance, for $f = 3$ *kHz*, $\theta = 50^\circ$, the difference in the attenuation coefficients of the first L - mode for the directions *WE* and *EW* already reaches ~ 3 *dB/Mm*.

As to the R_m - waveguide modes, their attenuation coefficients near the cut-off frequencies are very large (tens of *dB/Mm*). Therefore, already at distances $\rho \sim 2 - 3$ *Mm* from the entrance point of a whistler into the waveguide their contribution into the intensity of the signal for $f \sim f_{cm}$ is neglegibly small, compared to the contributions of strongly disperse L_m - modes and of a weakly disperse quasi-TM_0 mode (of a nearly linear polarization). [The contributions of the latter into the signal's intensity at distances $\rho \sim 2 - 8$ *Mm*, owing to reverse relations between the excitation factors and the attenuation coefficients turn to be of the same order]. As a result, the additional dispersion of a whistler in the waveguide (the time of the group delay τ as a function of frequency) has the form (Outsu, 1960; Shimakura et al., 1987) of

$$\tau(f) = \frac{\rho}{c} \left(\frac{f}{\sqrt{f^2 - f^{L\ 2}_{cm}}} - 1 \right) \quad (7)$$

where f^L_{cm} is given by the expression (3).

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