Small reflection source impedance design for Maximum power Feed Efficiency of MIMO antenna

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Abstract

Maximum power feed efficiency of MIMO antenna with intensive mutual coupling on handset is achieved not by match or conjugate match source impedance configuration, but by the source impedance that introduces small reflection between the sources and the antenna network to reduce the power transmitted from the one source and absorbed by other sources. A novel model for calculating the absorbed power by the sources is developed for multiple antenna network to overcome the defect of Thévenin and Norton equivalent circuits model. The algorithm to calculate the source impedance for maximum feed efficiency is proposed.

1. INTRODUCTION

The recent efforts aimed at improving available capacity of wireless system have generated interest to use multi transmit and receive antennas. Through utilizing the additional spatial dimension of rich scattering wireless channel, the Multi Input Multi Output (MIMO) system surpasses the conventional systems utilizing frequency and time alone in achievable data rates [1-3]. Assume a MIMO system with N transmitters. Each transmitter transmits an independent data stream via a port of the MIMO antenna with N local ports. The MIMO antenna for transmitting is represented schematically in Fig.1. The power really fed into the MIMO antenna equals to the total source power minus the power absorbed by the sources. And the feed efficiency ($\boldsymbol{\eta}$) is defined as the power fed into the MIMO antenna over the total source power, as the equation (1). Since Thévenin and Norton equivalent circuits model can not be relied upon for calculating the internal power dissipation in the sources, especially in the unmatched cases [4], it is necessary to develop a novel model to calculate the absorbed power and the feed efficiency. Extended from the circuit model of two antennas [5] to that of multi-antennas, each equivalent source impedance is separated by the real part (**Rs$_i$**) and the imagine part (**jx$_i$**), between which a transmission line is arranged matched to the source impedance (**Rs$_i$ = Zc$_i$**). So the power absorbed by each own source equals the square of the normalized reflected wave ($\boldsymbol{b_i}$) of the incident source wave ($\boldsymbol{a_i}$), on the interface between the transmission line and the imagine part. And hence, the feed efficiency can be calculated correctly. In MIMO handset, high feed efficiency of MIMO antenna is expected for the limited cell power, but the intensive mutual coupling reduces radiated power [6], so the optimum source impedance (**Z$_{Si}$ = Rs$_i$ +jx$_i$**) is solved by means of (9) and ***MFE Algorithm*** for Maximum Feed Efficiency (***MFE***) to against the efficiency decrease caused by mutual coupling.

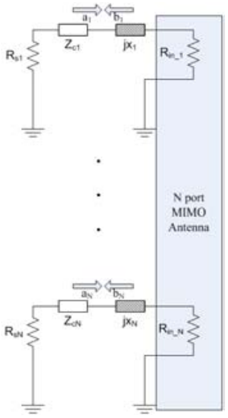

Fig. 1, Source end impedance separated model of MIMO antenna.

2. THEORETICAL ANALYSIS

The feed efficiency of a MIMO antenna is calculated as follows,

$$\begin{aligned} &\max \quad \boldsymbol{\eta} = \boldsymbol{1} - \frac{\boldsymbol{P_{abs}}}{\boldsymbol{P_{src}}} \\ &s.t. \quad \boldsymbol{Z_S} = \boldsymbol{Z_S} \odot \boldsymbol{I} \end{aligned} \qquad (1)$$

where $\boldsymbol{P_{src}}$, $\boldsymbol{P_{abs}}$ denote, respectively, the average power provided by the sources and the average total power absorbed by all the sources. The constraint in (1) determines that $\boldsymbol{Z_S}$ is a diagonal matrix, whose diagonal elements ($\boldsymbol{Z_{S_i}}$) denote the source impedances. $\boldsymbol{P_{src}}$ and $\boldsymbol{P_{abs}}$ are determined by means of (2) and (3). $tr(\square)$ and $(\square)^H$ denote, respectively, trace operator and hermitian operator of a matrix. $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{R_{in}}$ are $N\times N$ dimensional diagonal matrix, which determined by (4)-(6). The $\boldsymbol{i^{th}}$ diagonal element of $\boldsymbol{a}$ and $\boldsymbol{b}$ denote, respectively, the incident wave impressed by the $\boldsymbol{i^{th}}$ source and it's reflected wave. The $\boldsymbol{i^{th}}$ diagonal element ($\boldsymbol{R_{in\_i}}$) of $\boldsymbol{R_{in}}$ is equivalent input impedance looking into the $\boldsymbol{i^{th}}$ port of the MIMO antenna. $\boldsymbol{U}$ is a $N\times N$ dimensional matrix determined by (7), whose $\boldsymbol{i^{th}}$ column denotes the voltages on the N local ports stimulated by the unit source voltage (1V) impressed on the $\boldsymbol{i^{th}}$ port. $\boldsymbol{Z}$ is the normalized impedance matrix that can be measured or computed from measured scattering parameters. $\boldsymbol{I}$ is identity matrix. $1_{N\times N}$ is all-one matrix.

$$\boldsymbol{P_{src}=\frac{1}{2N}tr\left(a^H\cdot a\right)} \tag{2}$$

$$\boldsymbol{P_{abs}=\frac{1}{2N}\left[tr\left(b^H\cdot b\right)+tr\left(\left(U\square\left(1_{N\times N}-I\right)\right)^H\cdot\right.\right.}$$
$$\boldsymbol{\left.\left.\cdot\frac{Z_S{}^{-1}+Z_S{}^{-H}}{2}\cdot\left(U\square\left(1_{N\times N}-I\right)\right)\right)\right]} \tag{3}$$

$$\boldsymbol{a=\left(U\square\ I\right)\cdot\left(I+\left(R_{in}-Z_S{}^H\right)\cdot\left(R_{in}+Z_S\right)^{-1}\right)^{-1}\cdot}$$
$$\boldsymbol{\cdot\left(I+\frac{Z_S-Z_S{}^H}{2}R_{in}{}^{-1}\right)\cdot\left(\frac{Z_S+Z_S{}^H}{2}\right)^{-\frac{1}{2}}} \tag{4}$$

$$\boldsymbol{b=a\cdot\left(R_{in}-Z_S{}^H\right)\cdot\left(R_{in}+Z_S\right)^{-1}} \tag{5}$$

$$\boldsymbol{R_{in}=Z_S\cdot\left(U\square\ I\right)\cdot\left(I-\left(U\square\ I\right)\right)^{-1}} \tag{6}$$

$$\boldsymbol{U=Z\cdot\left(Z+Z_S\right)^{-1}} \tag{7}$$

3. MAXIMUM FEED EFFICIENCY IMPEDANCE ALGORITHM

Given the scattering parameter matrix ($\boldsymbol{S}$) of a MIMO antenna, the ***MFE*** of the antenna is fixed, because the ***MFE*** equals 1 minus the minimum Raleigh entropy of $\boldsymbol{\left(S^H\cdot S\right)}$. The ratio ($\boldsymbol{R}$) of absorbed power to source power is

$$\boldsymbol{R=\frac{P_{abs}}{P_{src}}=\frac{x^H\cdot\left(S^H\cdot S\right)\cdot x}{x^H\cdot x}} \tag{8}$$

where $\boldsymbol{R}$ is Raleigh entropy of $\boldsymbol{\left(S^H\cdot S\right)}$ [7], $\boldsymbol{x}$ is the N dimensional incident wave vector on the local ports. We name $\boldsymbol{R}$ the Raleigh entropy of the MIMO antenna. Since $\boldsymbol{\left(S^H\cdot S\right)}$ is a hermitian matrix, $\boldsymbol{\lambda_{min}\le R\le\lambda_{max}}$ [7]. $\boldsymbol{\lambda_{min}}$, $\boldsymbol{\lambda_{max}}$ are, respectively, the minimum and the maximum eigenvalue of $\boldsymbol{\left(S^H\cdot S\right)}$. And only when $\boldsymbol{x}$ is the minimum or maximum eigen vector of $\boldsymbol{\left(S^H\cdot S\right)}$, $\boldsymbol{R}$ equals $\boldsymbol{\lambda_{min}}$ or $\boldsymbol{\lambda_{max}}$, respectively. So mathematically ***MFE*** equals $\boldsymbol{1-\lambda_{min}}$.

However, this simple ***MFE*** is not what we want, because the simple maximum feed efficiency requires unbalance source impedance so that the antenna element with low feed efficiency does not work. The measured scattering parameter matrix of a MIMO antenna with four PIFA elements is as follows,

$$S=\begin{bmatrix}\left(-11.2dB,5.69^o\right) & \left(-10.6dB,-41^o\right) & \left(-8.56dB,-28.5^o\right) & \left(-25.2dB,-98.7^o\right)\\ \left(-10.6dB,-41^o\right) & \left(-14.51dB,-1.06^o\right) & \left(-26.4dB,-66.4^o\right) & \left(-12dB,-36.7^o\right)\\ \left(-8.56dB,-28.5^o\right) & \left(-26.4dB,-66.4^o\right) & \left(-15.7dB,94.8^o\right) & \left(-14.7dB,3.16^o\right)\\ \left(-25dB,-98.7^o\right) & \left(-12dB,-36.7^o\right) & \left(-14.7dB,3.16^o\right) & \left(-9.97dB,118^o\right)\end{bmatrix}.$$

The $\boldsymbol{\lambda_{min}}$ of $\boldsymbol{\left(S^H\cdot S\right)}$ is 0.0270, so theoretically the ***MFE*** can reach ***97.3%***. And given the source impedance [158.4+j5.6 , 1612.3-j155.2 , 3130+j400.4 , 38.2+j17.3] ohm, the feed efficiency of the MIMO antenna is ***91.64%***, while the input impedance and the reflect coefficient on the local ports are, respectively, [150.75-j18.82，83.49-j20.73，53.93-j19.05，42.08-j15.94] ohm and [-0.0214-j0.0798，-0.9001-j0.0141 , -0.9680-j0.0158 , 0.0417-j0.4142]. Although on this condition the feed efficiency is high, the impedances on the port 2 and port 3 are almost totally mismatch, so the element 2 and 3 of the MIMO antenna doesn't work at all.

In order to avoid this unbalance impedance configuration, the penalty factor $\left[\sqrt[N]{\det\left(\overset{\square}{P}_{src}-\overset{\square}{P}_{abs}\right)}\right]^{-1}$ is added into (1). When some branch of MIMO antenna doesn't work, the source power and the absorbed power on the branch is almost equal so that the penalty factor is a large number. As the result, the unbalance impedance output is avoid.

$$\boldsymbol{\eta'=1-\frac{tr\left(P_{abs}\right)}{tr\left(P_{src}\right)}-\frac{w}{\sqrt[N]{det\left(\overset{\square}{P}_{src}-\overset{\square}{P}_{abs}\right)}}}$$
$$\boldsymbol{\overset{\square}{P}_{abs}=\frac{1}{2}\left(b^H\cdot b\right)} \tag{9}$$
$$\boldsymbol{\overset{\square}{P}_{src}=\frac{1}{2}\left(a^H\cdot a\right)}$$

where $\boldsymbol{\eta'}$ is feed efficiency, $\boldsymbol{\overset{\square}{P}_{src}}$ and $\boldsymbol{\overset{\square}{P}_{src}}$ are, respectively, the source power matrix and the absorbed power matrix, the weight $\boldsymbol{w}$ is a small number set as 10E-6 to control the effect of the penalty factor.

After the penalty factor is added into the goal function as (9), the numeric algorithm to search the source impedance in a 2N dimensional space for ***MFE*** is indispensable. We assume the

MFE source impedance is close to the input impedance of MIMO antenna. So a fast iterative algorithm is proposed as follows.

MFE Algorithm

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- STEP1
    Set $Z_c$ as the character impedance of the system measuring MIMO antenna scattering parameter.
    Set $k = 1$, $Z_{si} = Z_c$.
    Set the maximum conjugate gradient times $M_g$ and the maximum random select times $M_s$.

- STEP2
    Get the local optimal value of $\eta = f(Z_s)$ by conjugate gradient method within the iterative times $M_g$.
    Get the local optimum solution $Z_{s(opt\_L)}$ and the input impedance $R_{in}$ of the $Z_{s(opt\_L)}$.

- STEP3
    If the searched $Z_{s(opt\_L)}$ never changes, set $Z_P(k) = Z_{s(opt\_L)}$ as the local solution, goto STEP4.
    Else, set $Z_s = R_{in\_i}$, repeat STEP2.

- STEP4
    If $k \geq M_s$, stop.
    Else, select a new $Z_s$ randomly in the physically feasible range, $k = k + 1$, repeat STEP2.
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The above algorithm is not sure to get the global optimum solution, but usually it is very fast to get the balance source impedance $Z_P(M_s)$ to improve the feed efficiency.

The algorithm is valid to improve the feed efficiency of real MIMO antenna. For the above mentioned four PIFA antenna, when 50 ohm source impedance is used, the feed efficiency on local ports are, respectively, **[*70.8%, 78.03%, 79.6%, 84.8%*]**, whose average feed efficiency is ***77.6%***. However, the algorithm gives out the source impedance for ***MFE*** as $[\text{74.05-j5.78} \quad \text{60.67-j3.15} \quad \text{43.94-j16.20} \quad \text{35.19-j24.41}]$ ohm, and the feed efficiencies on the ports reach **[*88%, 83%, 81%, 75%*]**, whose average efficiency is ***82.8%***, ***5.2%*** higher than the former ***77.6%***. In wireless handset, the intensive mutual coupling reduces feed efficiency, ***5.2%*** higher feed efficiency saves the cell power, improving working time of the handset.

4. SIMULATION VALIDATION

In order to indicate the difference between source impedance configuration for ***MFE*** and the configuration for match and conjugate match, the three dimensional figure and the contour of the feed efficiency versus the source impedances are, respectively, shown in Fig. 2 and Fig. 3, for an example MIMO antenna with 4 identical elements. The scattering parameter matrix is written as

$$S = \begin{bmatrix} \text{-20dB, }45^\circ & \text{-10dB, }45^\circ & \text{-10dB, }45^\circ & \text{-10dB, }45^\circ \\ \text{-10dB, }45^\circ & \text{-20dB, }45^\circ & \text{-10dB, }45^\circ & \text{-10dB, }45^\circ \\ \text{-10dB, }45^\circ & \text{-10dB, }45^\circ & \text{-20dB, }45^\circ & \text{-10dB, }45^\circ \\ \text{-10dB, }45^\circ & \text{-10dB, }45^\circ & \text{-10dB, }45^\circ & \text{-20dB, }45^\circ \end{bmatrix}$$

The scattering parameters of the example antenna elements are identical so that the there are only 2 degrees of freedom to optimize the source impedance, and hence the figure and the contour can be shown on paper. The source impedance configuration getting ***MFE*** is not the match impedance or conjugate match impedance of the MIMO antenna, nor the input impedance (R_{in_i}) of the network. The ***MFE*** source impedance ($\mathbf{Z}_{\mathbf{ME}}$) is 34+j12 ohm, while the match impedance ($\mathbf{Z}_{\mathbf{mi}}$) is 54+j8 ohm, the conjugate match impedance ($\mathbf{Z}_{\mathbf{mi}}^*$) is 60-j14 ohm, and the input impedance for ***MFE*** is 52.8-j2.98 ohm. The ***MFE*** is achieved at ***73.5%*** by ***MFE*** impedance, while the feed efficiencies of match impedance and conjugate impedance are, respectively, ***69.2%***, ***61.9%***. When the mutual coupling can not be ignored, small reflection introduced by slight mismatch at the source reduces the absorbed power coming from other sources. But when the mutual coupling is small, the ***MFE*** configuration approximates the conjugated match configuration, as shown in Fig. 4. The mutual coupling between the elements of the four MIMO antennas A, B, C, D, decreases step by step, while MFE is close to feed efficiency of the conjugated match configuration along the decrease of mutual coupling.

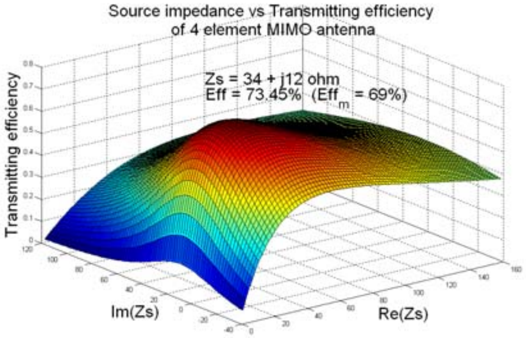

Fig. 2 Three dimensional figure of the feed efficiency (transmitting efficiency) for different source impedance configurations.

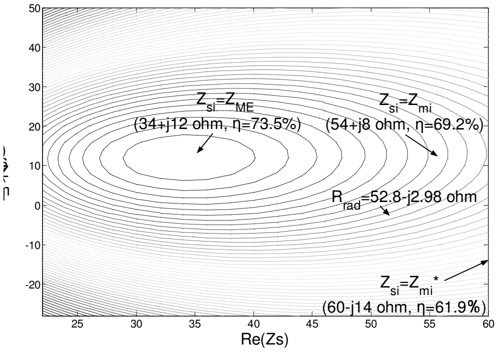

Fig. 3 Contour of the feed efficiency for different source impedance configurations.

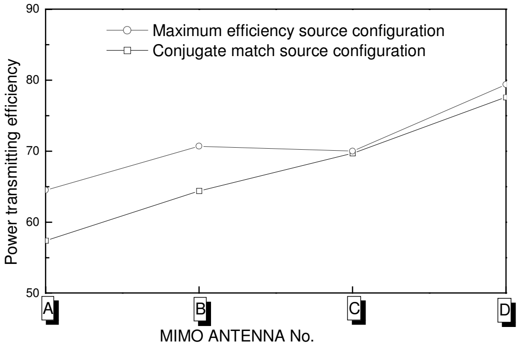

Fig. 4 Power feed efficiency of different source impedance configurations for MIMO antennas with different mutual coupling intensity.

5. CONCLUSION

A source impedance separated model for multiple antennas is developed to analyze power absorbed by sources of multiple-antenna-network. By this model, it is proved that the source impedance configuration for Maximum power Feed Efficiency is not the same as the conventional match or conjugate match configuration, especially when mutual coupling of MIMO antenna can not be ignored. So the ***MFE algorithm*** is proposed for searching the Maximum Feed Efficiency impedance, which gives out about 5% of power feed efficiency higher than that of match and conjugate match configuration when the efficiency is lower than 70%. When the efficiency is higher than 70%, the feed efficiency by maximum efficiency configuration is close to that by the conjugate match configuration. In MIMO handset, the feed efficiency decreases for mutual coupling, so source impedance optimized by the ***MFE algorithm*** can reduce the power absorbed by the sources, and hence, improves the feed efficiency and the performance of wireless handset.

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REFERENCE

[1] E. Telatar, “Capacity of multi-antenna Gaussian channels”, European Trans. Telecomm., Vol.10, pp.585-595, 1999.

[2] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas”, Bell Labs Technical Journal, Vol.1, pp. 41-59, 1996.

[3] A. Lozano, C. Papadias, “Layered space-time receivers for frequency-selective wireless channels”, IEEE Trans. Comm., Vol.50, pp.65-73, 2002.

[4] R.E.Collin, “Limitations of the Thévenin and Norton equivalent circuits for a receiving antenna”, IEEE antennas and propagation magazine, Vol. 45, April, 2003.

[5] Lindmark B. “Capacity of a 2/spl times/2 MIMO antenna system with mutual coupling losses”, IEEE Antennas Propagation Symposium, Vol.2, pp.1720-1723, 2004.

[6] Inder J. Gupta, Aharon A. Ksienski, “Effect of mutual coupling on the performance of Adaptive Arrays”, IEEE Trans. Antennas Propagation, Vol.31, pp. 785-791, 1983.

[7] Xianda Zhang, Matrix analysis and applications, 1st Ed., Springer, Beijing, 2004, pp.528.