Quantization Analysis for Reconfigurable Phased Array Beamforming

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Abstract

In reconfigurable phased array radars beamforming is performed in the digital domain. Digital implementation can be carried out using finite precision and infinite precision representation of the phased array signal. We consider finite precision representation, since it takes considerably less implementation resources compared with the infinite case. In this paper we analyze quantization effects when quantization is introduced in the phasing network. The quantization problem is extensively addressed in the literature for phased array radars. But it mainly focuses on steering weights and delay quantization in a long phased array. For reconfigurable digital beamforming (DBF), it is possible to store all the phasing samples and process them in real time. Therefore the quantization issue is different than digitizing a few phasing samples.

Digital samples of phased signals are sensitive to finite precision. Finite precision reduces mainlobe level as well as sidelobe gain. For multiple beam generation, it degrades not only the transmitted beam but adjacent beams as well. The quantization effects depend on position of the quantizer. They are different when the quantizer is placed before and after beamforming network. Simulations have been performed to demonstrate the results.

1 Introduction

Quantization is a representation of data samples with certain number of bits per sample after rounding to a suitable level of precision. Quantization errors in a DBF system can be introduced from three sources; one source is input quantization, a second is coefficient quantization and the third is the finite precision in arithmetic operations [1]. The quantization error in the arithmetic operations can be controlled by carefully selecting the size of buffer registers according to the input word length [2]. We address quantization issue resulting from fixed word length coefficients. Quantization errors from phasing network of an ionospheric radar are topic of this article. Tasman International Geospace Environment Radar (TIGER) is an HF radar that investigates ionospheric irregularities in the Southern Hemisphere. Recently a second component of the TIGER is commissioned in New Zealand to acquire higher resolution.

This article is divided into two main sections; quantization effects for DBF before and after phasing network. The HF radar comprises phasing array of sixteen antenna elements with modulating frequency range of 8 to 20MHz. The radar antenna geometry is described in section 2. Fixed length samples cause periodic phase quantization error. We discuss finite precision effects on phasing weights in section 3. Quantization problem in phasing networks is significantly addressed in the literature, for example [3]-[7]. Most of the analysis is limited to arrays where steering weights are quantized. In other words beam patterns are calculated using infinite precision and quantization error is modeled after the beamforming network. We discuss effects of quantization in such situation in section 4. In section 5, quantization effects are discussed when quantization is introduced before beamforming. This is recent issue in digital beamforming where beam width, directivity and accuracy can be changed on the fly. Examples of such DBF systems are reconfigurable transmitters and receivers in wireless environment. In the last section conclusions are drawn.

2 HF Radar Antenna Geometry

The phasing of signals extends the transmission or detection range of a transmitter or receiver by forming a narrower beam than the individual antennas. Phased array antennas are used to steer a narrow beam over an arc from a fixed antenna array. For each transmitter or receiver the direction is adjusted with systematic phase delays to each of the antennas in the array. In the linear phased array antenna the antenna elements are arranged with uniform spacing, as shown in Figure 1, where d is inter-element spacing, and θ is angle normal to modulated wave. It is obvious from the figure that in the general case, the modulated wave at element $N-1$ will be delayed by a differential distance of $d\sin\theta$ compared with a wave at element $N-2$. If we consider the phase of the transmitted signal is zero at the origin, then the phase lead at element N to that element at origin is $Nkd\sin\theta$, where $k = 2\pi/\lambda$ is a constant. The operational frequency of the signal varies with the wavelength using the relationship $c = f\lambda$, where c is the velocity of light in the vacuum. The number of array elements and space between them determine the beamwidth and size of sidelobes [3]

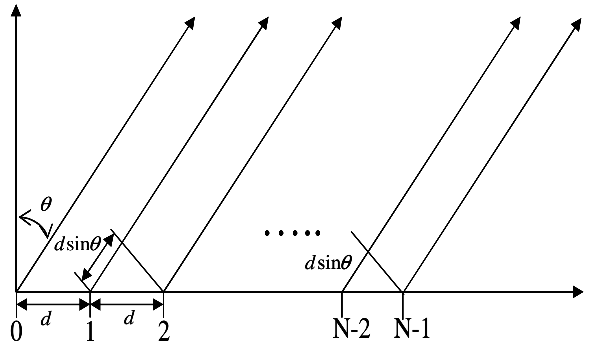

Figure 1 TIGER phased array antenna geometry.

3 Periodic Phase Quantization

One source of periodic phase quantization is uniform array structure. Uniform distances between the array elements produce periodic phase quantization when they are steered to certain directions. The periodic quantization error is also caused by factors other than the regular geometry. For example continuous wave transmission and far field operation. In TIGER phased array system, pulsed transmission is employed therefore the quantization error would be reduced because overlapping pulses are reduced compared with continuous transmission [3].

In digital beamforming, weighting coefficients are quantized using a suitable quantizer. For a q bit quantizer, minimum step size of phase shifter is $\pi/2^q$. In other words the quantization error is distributed between $-\pi/2^q$ to $\pi/2^q$. Variance of phase quantization error can be written as assuming $C = \pi/2^q$ for interval of [-C C] [6]

$$\sigma_q^2 = \int_{-c}^{c} \frac{x^2}{2c} dx = \frac{c^2}{3} \qquad (1)$$

Substituting the values of C again we get

$$\sigma_q^2 = \frac{\pi^2}{3.2^{2q}} \qquad (2)$$

The above expression shows that variance decreases with increase in precision level of quantizer.

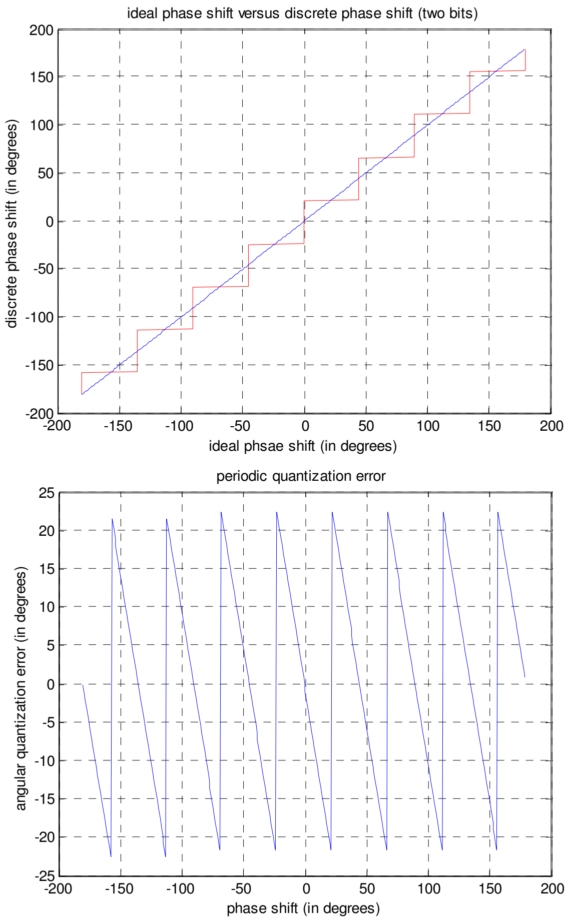

Figure 2 Comparison of ideal and digital phase shifter for two bits (b) periodic quantization error when the phase shift in each array element is quantized by the two bit quantizer.

Digital phase shifters are employed in digital implementation of phased array beamforming. The size of the phase shifter plays important role in beam accuracy and is dependent on the number of precision levels used in the phase increments [5]. For reduced number of bits, the digital phase shifter coarsely approximate the analog phase shifter as shown in the Figure 2. For higher number of bits, the approximation is close to the ideal phase shifting.

The periodic quantization error depends on step size of the quantizer. For two bit quantizer, the step size is 45 degrees. It is clear from Figure 2 that the quantization error is plus or minus half of the quantization step.

Periodic phase quantization error is addressed in the literature for different applications. In [4] Gray has reported quantization error in the beamforming for continuous waveform receivers. Gray also suggested interpolation to minimize the quantization effects since higher sampling rate causes less delay errors.

4 Effects of Quantization After Beamforming Network

The quantization of infinite precision samples into fixed word length degrades the phased signals. As described in the previous section, the use of more levels for higher precision decreases the quantization error at the expense of larger hardware resources. For a reduced precision level, quantization error is spread to the main beams and to the grating lobes as well. In this section we study effects of quantization on beam resolution and associated grating lobes.

Quantization causes different beam patterns when it is introduced before beamforming network and after beamforming network. We first look at most common form, when quantizer is placed after beamforming network, as shown in Figure 3. All the phasing calculation is performed in the analog domain and reconfigurable computing is applied after analog to digital conversion.

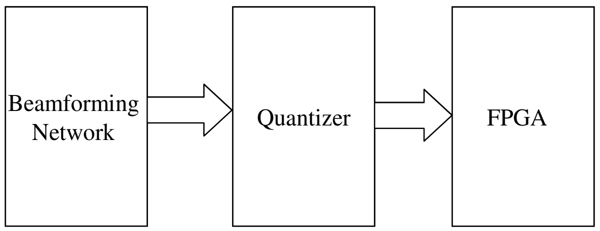

Figure 3 Quantization effects after beamforming network

In this case beamforming is based on infinite precision samples and quantized into suitable quantization levels. Beam steering coefficients are processed using FPGA to realize reconfigurable beam direction and resolution.

In order to get deep understanding we develop mathematical models for array patterns and grating lobes. We can define array factor of a uniform array pointing to a direction θ_0 [7]

$$Array\ Factor = \frac{\sin\left(N\left(\frac{2\pi}{\lambda}d(\sin\theta - \sin\theta_0)\right)\right)}{N\sin\left(\left(\frac{2\pi}{\lambda}d(\sin\theta - \sin\theta_0)\right)\right)} \qquad (3)$$

where N is number of array elements, d is spacing between two adjacent elements, λ is wavelength, θ is phase change in the array and θ_0 is pointing angle. If the nulls of the array occur at directions

$$\text{nulls} = \frac{n\lambda}{2Nd} \qquad (4)$$

where n is an integer. Putting equation (4) into (3)

$$Array\ Factor = \frac{\sin\left(N\left(\frac{2\pi}{\lambda}d(\sin\theta - \sin\theta_0)\right)\right)}{N\sin\left(\left(\frac{2\pi}{\lambda}d\left(\sin\theta - \sin\theta_0 + \frac{n\lambda}{2Nd}\right)\right)\right)} \qquad (5)$$

Equation (5) can be used to represent beam pattern of a uniform array including grating lobes. The largest grating lobe occurs when $n = 1$.

Now we discuss array factor in case of quantization. If the phase progression in the phased array is equal to the least quantized bit of the quantizer then

$$\text{phase progression} = \frac{\pi d N}{\lambda}(\sin\theta - \sin\theta_0) = \frac{2\pi}{2^q} \qquad (6)$$

putting into expression (5) we have

$$Array\ Factor = \frac{\sin\left(\frac{\pi}{2^q}\right)}{N\sin\left(\frac{\pi}{N2^q} + \frac{n\pi}{N}\right)} \qquad (7)$$

Which can be written as

$$Array\ Factor = \frac{\sin\left(\frac{\pi}{2^q}\right)}{N\sin\left(\frac{\pi}{N}\left(\frac{1}{2^q} + n\right)\right)} = \frac{\sin\left(\frac{\pi}{2^q}\right)}{N\sin\left(\frac{n'\pi}{N}\right)} \qquad (8)$$

We calculate power by approximating numerator angle

$$Power = \left[\frac{\left(\frac{\pi}{2^q}\right)}{N\sin\left(\frac{n'\pi}{N}\right)}\right]^2 \qquad (9)$$

Equation (9) represents approximate power of quantized phased array. This can also be used to approximate grating lobe levels resulting from quantization.

In order to investigate the quantization effects, an example is presented with fixed word length phasing samples. Amplitude of the phasing vector is quantized into six bits; the increased number of bits will reduce the quantization effect. For an actual design the fixed bit width depends on available hardware resources. The quantized beam in Figure 4(a) shows that the quantizer does not adequately represent the beam pattern and thus introduces quantization noise. As can be seen from this simple example, six bit compromises the first and second sidelobes at the -20dB level. For a DBF system of more than ten bits, the sidelobe level will be essentially unaffected by the quantization at the -20dB level.

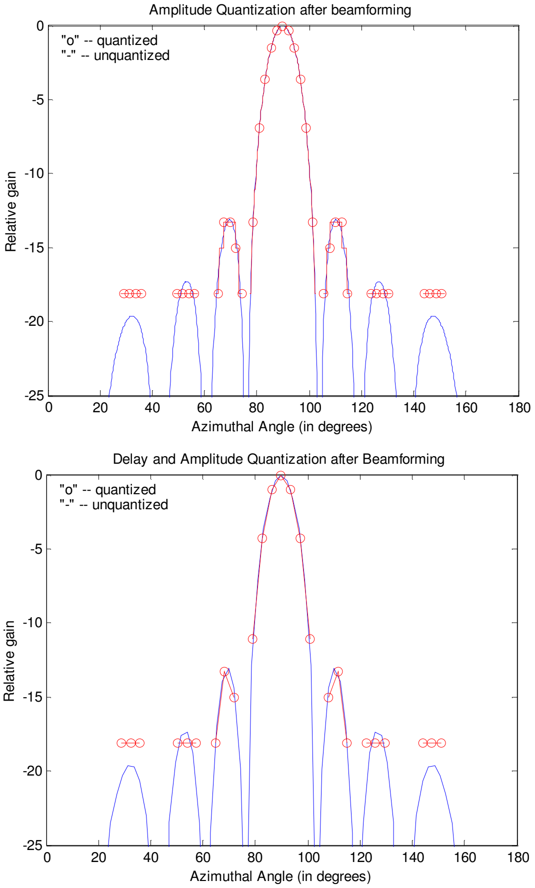

Figure 4 Quantization effects on beam pattern when samples are rounded using (a) amplitude quantization and (b) delay and amplitude quantization.

The quantization causes gain errors in sidelobe levels. Higher resolution in quantization introduces lower quantization error. The quantization error changes the dynamic range of the grating lobes and degrades the adjacent beam resolution for multiple beam systems.

We can also study quantization effects of a slow speed analog to digital conversion. An example is shown in Figure 4 (b). The quantization effects degrade beam pattern even more when both delay and amplitude quantization are introduced. The amplitude is quantized to six bits and the sampling delay is sixteenth times less than the ideal one. The two dimensional scenario causes severe reduction in main lobe gain and produces distorted energy in the unwanted region of sidelobes.

5 Effects of Quantization Before Beamforming

For FPGA implementation of DBF, all weights of the DBF are stored in registers. Therefore we look at the issue when DBF is performed using such weights. Difference with the earlier issue is that now both beam generation and steering are based on fixed numbers. This problem has not been addressed in the literature partly because technology was not advanced enough to handle large amount of calculation in the digital domain.

A block diagram is shown in Figure 5 to realize the concept of digitizing the phase samples. In this case beamforming calculation is based on rounded integers. As mentioned in the section on periodic quantization, the periodic quantization is now included in the phase calculation. Therefore the digitization error causes higher degradation in the beam resolution compared to the previous case.

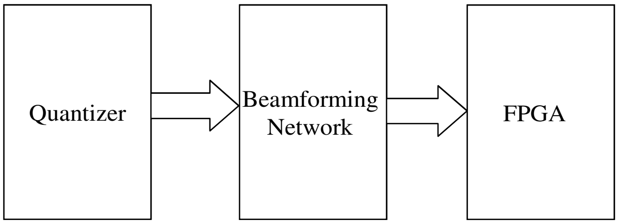

Figure 5 Quantization effects before beamforming network

Simulations have been performed to demonstrate the amplitude and delay quantization effects on the beam pattern. An example is shown in Figure 6 where pashing weights are rounded to six bits and sampling is sixteen times less the ideal one. A comparison of Figure 6 with Figure 4 shows that for identical precision of the quantizer, beam pattern exhibit higher periodic error in the latter case. The delay quantization components add constructively and destructively that causes low fidelity beam pattern. The degradation is visible in the dynamic range of the main lobe and sidelobes as well.

The output beam resolution is adversely affected when the phasing coefficients are quantized before beamforming network. In order to circumvent these components, precision level of the amplitude and timing quantizer should be at adequate level.

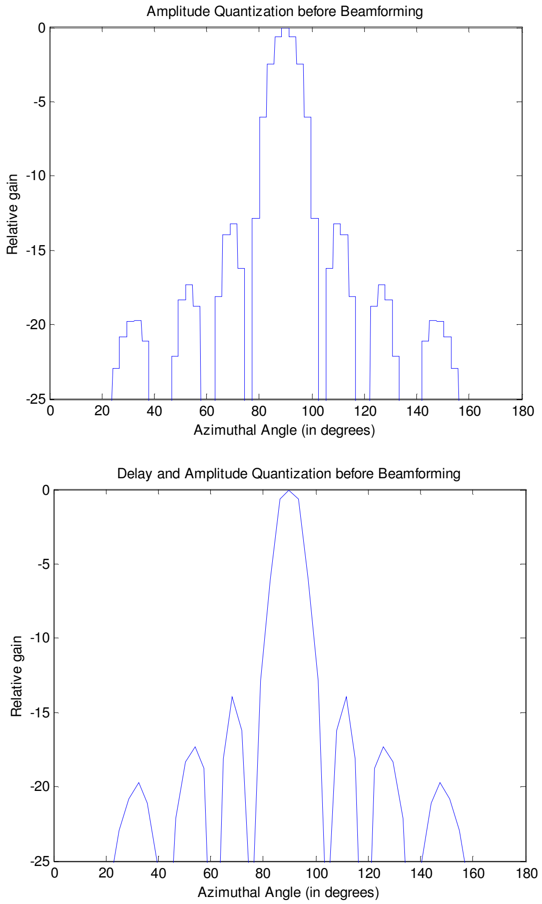

Figure 6 Quantization effects before beamforming when (a) amplitude quantization and (b) both amplitude and delay quantization are introduced.

The potential benefits in reconfigurable beamforming are variable beam direction, beam width and accuracy with a software flick. On the other hand there are least minimum conditions to control phasing components; for example amplitude and timing precision should be at adequate levels.

6 Conclusions

In this paper, effects of fixed word length have been discussed on phasing of the digital TIGER system. Finite precision is described in two scenarios. In first case, conventional approach of quantized weights is addressed after beamforming. Secondly a new approach of quantization before beamforming is adopted. The second scenario exhibit sub optimum results since quantization error accumulates in beamforming calculations. The quantization error is higher for lower precision level. In order to overcome these effects, a higher precision level is required.

7 References

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