Reduction of AOA estimation error due to perturbation in array response by spatial smoothing preprocessing

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Abstract

In this paper, reduction of angle-of-arrival (AOA) estimation error owing to amplitude and phase perturbation in the array response using spatial smoothing preprocessing (SSP) is proposed. The performance improvement of the proposed method is validated by Monte-Carlo simulation. To show applicability of the proposed method in practice, it was applied to estimate AOAs of measured data obtained in an anechoic chamber and in an open site. According to the results, SSP can reduce the random error in the array response, thus reducing the error of estimated AOAs in the measured data.

1. Introduction

Very precise AOA estimation is required for mobile localization and illegal radio surveillance. In the last few decades, a variety of AOA estimation algorithms have been extensively discussed in the literature [1], especially the subspace based methods because of their high resolution. The well-known subspace based algorithms are the multiple signal classification (MUSIC) algorithm [2] and the estimation of signal parameters via rotational invariance techniques (ESPRIT) [3]. However, subspace-based approaches give high resolution only when the true array response is obtained. Therefore, to achieve the true array response, array calibration is one of the necessary issues. There are some research works proposing the calibration techniques by applying the data in the real scenarios, e.g. [4]. However, it is often found that array calibration cannot perfectly remove the error in the array response, and thus leading to an error of estimated AOAs. This issue is also discussed in [5]. There are some researches that analyze the perfomance of subspace-based methods in the presence of array response errors [6]-[7] in which complicated theoretical expressions were derived and compared with simulation. Nevertheless, the applicability of these researches was very limited in practice. In this paper, reduction of the random error in the array response of measured data using the spatial smoothing preprocessing (SSP) is proposed. Although SSP is well-known for using in the highly correlated or coherent-sources scene to de-correlate the signals [8]-[10], we will show here that SSP can reduce the random error perturbation in the array response. In [7], the effect of spatial smoothing on the performance of subspace methods in the presence of the perturbation in the nominal array response is already shown; however, its objective is to deal with the correlated/coherent sources in the presence of the array model error. Moreover, though the expressions in [7] are well-derived, it is validated only by the simulation. In this paper, the simple expressions are illustrated and validated not only by simulation, but also with the measured-data estimation.

This paper is organized as follows. Section 2 describes the problem formulation starting with the description of the ideal array model, error in the array model and explanation of the reduction of the random error by SSP. Section 3 shows simulation results, followed in section 4 by demonstrating the applicability in real scenarios and finally, the conclusion is in section 5.

2. Problem Formulation

A. Ideal Array Model

We first briefly describe the signal model for an ideal array with no perturbation. For simplification, assume that we consider only a single source impinging on an M-element uniform linear array (ULA), an array output vector $\mathbf{x}(t)$ can be written as

$$\mathbf{x}(t) = \mathbf{a}s(t) + \mathbf{n}(t), \tag{1}$$

where $s(t)$ is an arriving signal at time t, $\mathbf{n}(t)$ is an $M \times 1$ additive noise vector and $\mathbf{a}$ is a so-called steering vector describing the array response for a source with an arrival angle θ and assumed to be known from some calibration procedures. The steering vector can be defined as

$$\mathbf{a}(\theta) = [1, e^{j\omega}, e^{j2\omega}, \cdots, e^{j(M-1)\omega}]^T, \tag{2}$$

where $\omega = 2\pi d \sin\theta/\lambda$ is the phase difference between adjacent elements, d is the inter-element spacing, λ is the signal carrier wavelength, and the superscript T denotes the transpose. The output covariance matrix of the single source can be written as

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \sigma_s^2 \mathbf{a}\mathbf{a}^H + \sigma_n^2 \mathbf{I}, \tag{3}$$

where $E[\cdot]$ represents the statistical expectation, the superscript H denotes the complex conjugate transpose operator, σ_s^2 is the arriving-signal power and $\sigma_n^2\mathbf{I}$ is the noise covariance matrix that reflects the uncorrelated noise among all the elements and having a common variance at all elements.

B. Error in Array Model

Let $a_m(\theta) = e^{j\phi_m}$ be the nominal array response of the m^{th} element and $\phi_m = (m-1)\omega$. When amplitude and phase perturbations of the antenna array occur, the perturbed array response of the m^{th} element can be modeled as [6]

$$\begin{aligned} \tilde{a}_m(\theta) &= (1+\tilde{g}_m)e^{j(\phi_m+\tilde{\phi}_m)} \\ &= \gamma_m a_m(\theta), \end{aligned} \tag{4}$$

where $\gamma_m = (1+\tilde{g}_m)e^{j\tilde{\phi}_m}$, $\tilde{g}_m$ and $\tilde{\phi}_m$ are gain and phase errors of the m^{th} element, respectively and assumed to be independent of the AOA and very small. Then, to first order of the Taylor expansion, we have

$$\gamma_m \approx 1 + \tilde{g}_m + j\tilde{\phi}_m = 1 + \xi_m, \tag{5}$$

where $\xi_m = \tilde{g}_m + j\tilde{\phi}_m$ denotes a zero-mean complex Gaussian error of the m^{th} element with known covariance. We assume that the gain and phase perturbations of each element are independent identically distributed (i.i.d) Gaussian random variables with variances σ_g^2 and σ_ϕ^2, respectively, then the covariance of the random error in each element is obtained by

$$E[\xi_m {\xi_m}^*] = \sigma_g^2 + \sigma_\phi^2. \tag{6}$$

The variance σ_g^2 and σ_ϕ^2 determine the deviation of the gain and phase response from their nominal values, respectively. In general, the gain deviation is defined in decibels as $20\log_{10}(\sigma_g)$ and for phase deviation, it can be defined as $180(\sigma_\phi/\pi)$ degrees.

From the above description of the array imperfection, the perturbed array output vector $\tilde{\mathbf{x}}(t)$ can be modeled as

$$\begin{aligned} \tilde{\mathbf{x}}(t) &= \Gamma \mathbf{a} s(t) + \mathbf{n}(t) \\ &= \tilde{\mathbf{a}} s(t) + \mathbf{n}(t), \end{aligned} \tag{7}$$

where $\Gamma = \text{diag}[\gamma_1, \gamma_2, \ldots, \gamma_M]$, then the perturbed output covariance matrix can be expressed as

$$\tilde{\mathbf{R}} = E[\tilde{\mathbf{x}}(t)\tilde{\mathbf{x}}^H(t)] = \sigma_s^2 \tilde{\mathbf{a}}\tilde{\mathbf{a}}^H + \sigma_n^2 \mathbf{I}. \tag{8}$$

C. Reduction of the random error in the array response by SSP

In this subsection, we shall prove that SSP can reduce the random error. In the forward-only spatial smoothing (FSS) scheme [8], the array is divided forward into L smaller subarrays, each of which contains M_0 elements (see Fig. 1),

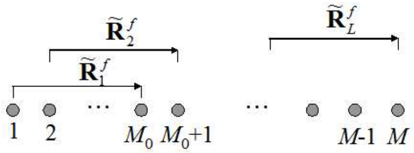

Fig. 1: The forward spatial smoothing scheme

where $L = M - M_0 + 1$. Let $\tilde{\mathbf{x}}_l^f(t)$ stand for the perturbed output vector of the l^{th} subarray, we have

$$\begin{aligned} \tilde{\mathbf{x}}_l^f(t) &= [\tilde{x}_l(t), \tilde{x}_{l+1}(t), \cdots, \tilde{x}_{l+M_0-1}(t)]^T \\ &= \tilde{\mathbf{a}}_l s(t) + \mathbf{n}_l(t), \quad l = 1, 2, \ldots, L \end{aligned} \tag{9}$$

where $\tilde{\mathbf{a}}_l = [\tilde{a}_l, \tilde{a}_{l+1}, \ldots, \tilde{a}_{l+M_0-1}]^T$. Then, the perturbed covariance matrix of the l^{th} forward subarray is obtained by

$$\tilde{\mathbf{R}}_l^f = E[\tilde{\mathbf{x}}_l^f(t)(\tilde{\mathbf{x}}_l^f(t))^H] = \sigma_s^2 \tilde{\mathbf{a}}_l \tilde{\mathbf{a}}_l^H + \sigma_n^2 \mathbf{I}. \tag{10}$$

Following [8], the perturbed forward spatially smoothed covariance matrix $\tilde{\mathbf{R}}_F$ is defined as the average value of the perturbed covariance matrices of all forward subarrays:

$$\begin{aligned} \tilde{\mathbf{R}}_F &= \frac{1}{L}\sum_{l=1}^{L} \tilde{\mathbf{R}}_l^f \\ &= \sigma_s^2 \frac{1}{L}\sum_{l=1}^{L} \tilde{\mathbf{a}}_l \tilde{\mathbf{a}}_l^H + \sigma_n^2 \mathbf{I}. \end{aligned} \tag{11}$$

The analysis of gain errors and phase errors can be considered separately since gain and phase errors are independent with each other. However, the analysis of phase errors is only discussed in this paper because of the page limitation.

By assuming that only the phase response of the array elements deviates from the nominal response, the perturbed steering vector of the l^{th} forward subarray $\tilde{\mathbf{a}}_l$ can be written as

$$\tilde{\mathbf{a}}_l = \begin{bmatrix} (1+j\tilde{\phi}_l)e^{j\phi_l} \\ (1+j\tilde{\phi}_{l+1})e^{j\phi_{l+1}} \\ \vdots \\ (1+j\tilde{\phi}_{l+M_0-1})e^{j\phi_{l+M_0-1}} \end{bmatrix}. \tag{12}$$

Focusing only on the part of the array response of (11) which is expanded in (13) shown at the top of next page. Given $\mathbf{B}$ is the matrix obtained by averaging all of the matrix $\mathbf{B}_l$ over L, where $\mathbf{B}_l = \tilde{\mathbf{a}}_l\tilde{\mathbf{a}}_l^H$. Each element of the matrix $\mathbf{B}_l$, denoted by $b_{pq,l}$, consists of two parts: i) the part of the phase shift of the nominal array response in terms of $e^{j(p-q)\omega}$ which is constant for each matrix $\mathbf{B}_l$ and ii) the part of the random error in terms of $(1 + j(\tilde{\phi}_{l+p-1} - \tilde{\phi}_{l+q-1}))$ where subscripts p and q denote the row and column of the matrix, respectively.

Given b_{pq} be the p^{th}-row, q^{th}-column element of the matrix $\mathbf{B}$. Each element b_{pq} is also composed of the part of the phase shift of the nominal array response and the part of the phase error which is a new random phase error produced by those before averaged over L and denoted by ζ_{pq}.

$$
\begin{aligned}
\mathbf{B} &= \frac{1}{L}\sum_{l=1}^{L}\mathbf{B}_l \\
&= \frac{1}{L}\sum_{l=1}^{L}\begin{bmatrix}
1 & (1+j(\tilde{\phi}_l-\tilde{\phi}_{l+1}))e^{-j\omega} & \cdots & (1+j(\tilde{\phi}_l-\tilde{\phi}_{l+M_0-1}))e^{-j(M_0-1)\omega} \\
(1+j(\tilde{\phi}_{l+1}-\tilde{\phi}_l))e^{j\omega} & 1 & \cdots & (1+j(\tilde{\phi}_{l+1}-\tilde{\phi}_{l+M_0-1}))e^{-j(M_0-2)\omega} \\
\vdots & \vdots & \ddots & \vdots \\
(1+j(\tilde{\phi}_{l+M_0-1}-\tilde{\phi}_l))e^{j(M_0-1)\omega} & (1+j(\tilde{\phi}_{l+M_0-1}-\tilde{\phi}_{l+1}))e^{j(M_0-2)\omega} & \cdots & 1
\end{bmatrix}
\end{aligned} \tag{13}
$$

As previously assumed, the phase errors of each array element are i.i.d. normal random variables with zero mean, i.e.,

$$\tilde{\phi} \sim N\left(0, \sigma_\phi^2\right), \tag{14}$$

then it can be proved that the new random phase error ζ_{pq} is distributed with

$$\zeta_{pq} \sim N\left(0, 2\mid p-q\mid \frac{\sigma_\phi^2}{L^2}\right) \quad \text{for} \quad p \neq q. \tag{15}$$

To analyze the gain error case in the same manner, the part of a new random gain error produced by those before averaged over L, denoted by η_{pq}, is distributed with

$$\eta_{pq} \sim N\left(0, \frac{2\sigma_g^2}{L}\right). \tag{16}$$

We can see that the deviation of the gain and phase errors decrease according to the number of subarray L. Therefore, by applying SSP, more precise AOA estimation can be obtained.

3. Simulation Results and Discussion

To illustrate the performance of the proposed method, computer simulations were performed. The simulation condition is described in Table 1 and 100 Monte-Carlo trials were run for each simulation. The MUSIC algorithm and ESPRIT method are used to estimate AOA. We consider the gain and phase error separately. Fig. 2 shows the standard deviation (STD) of AOA estimation versus the number of subarrays (L) compared between FSS-root MUSIC and FSS-ESPRIT when the gain error only occurs in the array response with a standard deviation of the gain error (σ_g) of -10 dB. This figure shows that the results of AOA estimation by FSS-ESPRIT are improved for all number of subarrays, while those by FSS-root MUSIC are improved in some number of subarrays. Fig. 3 shows the standard deviation of AOA estimation versus the number of subarrays (L) compared between FSS-root MUSIC and FSS-ESPRIT when the phase error only occurs in the array response with a standard deviation of the phase error (σ_ϕ) of $\approx$ 3 degrees. This figure shows that the results of AOA estimation by FSS-ESPRIT are improved for all number of subarrays, while those by FSS-root MUSIC are improved in some number of subarrays. From both the gain and phase error, SSP improves the performance of ESPRIT, but it is not so for MUSIC. Therefore, for further simulations, we use only ESPRIT. All condition parameters of this simulation are same as those of the simulation in Fig. 2.

Table 1: Simulation condition

Number of element (M)	16
Element spacing (d)	half a wavelength
AOA	6 [deg]
Signal SNR	10 dB
Number of snapshots	100

In Fig. 4, AOA is estimated by the ESPRIT algorithm with and without SSP and the standard deviation of the estimated AOA versus the varied random gain errors are shown. In this result, we use FSS with different number of subarrays: L = 3, 6, 9, and 12. It is shown that the standard deviation of the estimated AOA by applying FSS with the number of subarrays shown is smaller than that without applying FSS.

Fig. 5 illustrates the standard deviation of the estimated AOA versus the varied random phase errors. We also use FSS with the different number of subarrays: L = 3, 7, 10, and 14. It is also shown that the standard deviation of the estimated AOA by applying FSS with the number of subarrays shown is smaller than that without applying FSS.

4. Applicability in Real Situation

A. Measurement System

To show that this method is applicable in practice, it is applied to estimate AOAs from measured data. The experiments were done in two scenarios: i) in the anechoic chamber and ii) in the open site. For the experiment in the anechoic chamber, the measurement setup is illustrated in Fig. 6 and the specification of the measurement system is shown in Table 2. The receiving antenna was two parallel sets of uniform linear arrays (ULAs) as shown in Fig 6. We use terms “Array 1” and “Array 2” for easily refering to the upper ULA and lower ULA, respectively. The transmiting frequency was 1.74 GHz. The distance between transmitter and receiver was 10.42 m where a wavefront of the signal was perceptively curved because of

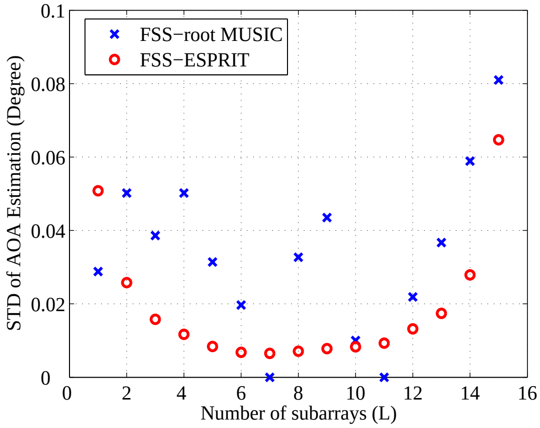

Fig. 2: Comparison between FSS-root MUSIC and FSS-ESPRIT when only gain error occurs ($\sigma_g = -10$ dB).

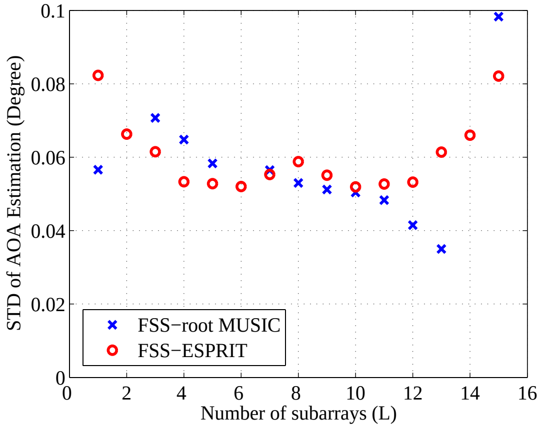

Fig. 3: Comparison between FSS-MUSIC and FSS-ESPRIT when only phase error occurs ($\sigma_\phi \approx 3$ deg).

a near-field measurement. Therefore, a phase curvature of a near-field wavefront was compensated to a far-field wavefront before data were processed [11]. The receiving antenna was rotated in an azimuth direction controlled by an antenna position controller. The experiments were repeated to obtain data whose AOAs were in range of -6 to 6 [deg]. The received data from the antenna array with the received SNR of 36 dB were downconverted to an intermediate frequency (IF) of 1.9648 MHz, then digitized with a sampling rate of 4.375 MHz, and further downconverted to baseband. Then, this baseband data were used for AOA estimation.

B. Results from Indoor Measured Data

Since the receiving array antenna is composed of two ULAs as shown in Fig 6, data received at each ULA is processed separately. MUSIC and ESPRIT were used to estimate AOAs. The calibration was applied to the measured data before AOA estimation was undertaken. The measured data obtained in

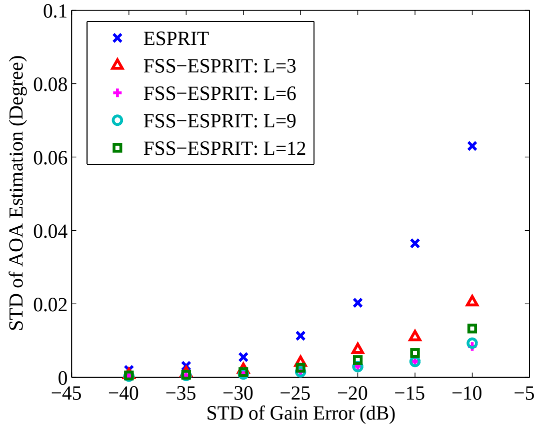

Fig. 4: STD of AOA estimation versus STD of gain error when AOA is estimated by ESPRIT and FSS-ESPRIT.

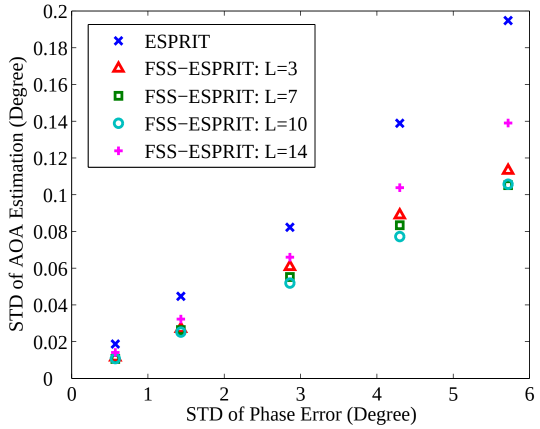

Fig. 5: STD of AOA estimation versus STD of phase error when AOA is estimated by ESPRIT and FSS-ESPRIT.

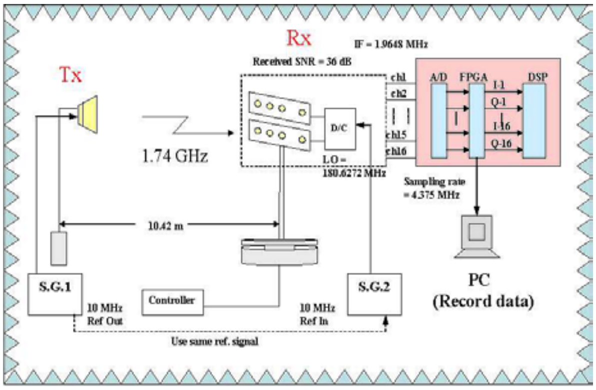

Fig. 6: The measurement setup in the anechoic chamber

the anechoic chamber at 0 degrees was used for calibration in all AOAs by using the amplitude and phase compensation technique discussed in [4]-[5]. Since the results obtained from

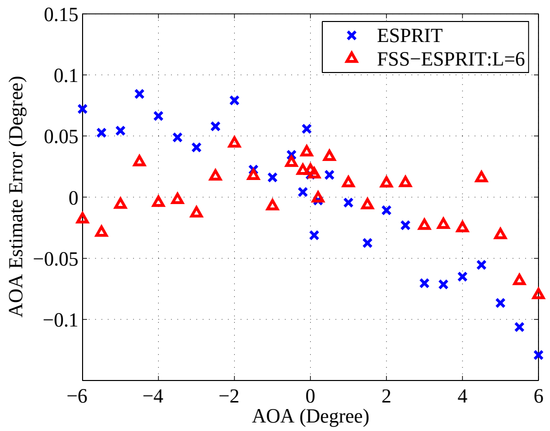

Fig. 7: AOA estimation error versus true AOAs when AOA is estimated from the anechoic chamber data by ESPRIT and FSS-ESPRIT.

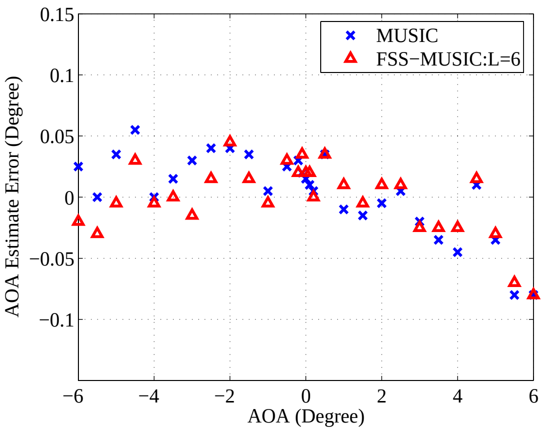

Fig. 8: AOA estimation error versus true AOAs when AOA is estimated from the anechoic chamber data by MUSIC and FSS-MUSIC.

Array1 and Array 2 have the same trend, the result from Array 1 is only shown in this paper. The AOA estimation error versus the true angle are demonstrated in Figs. 7 and 8 where the results were obtained by ESPRIT and MUSIC, respectively. For both Figs. 7 and 8, the results with and without FSS were compared. In the case of FSS, the number of subarrays was 6. It is illustrated that SSP can improve the AOA estimation both by MUSIC and ESPRIT; however, the performance of ESPRIT was better by applying FSS than MUSIC.

C. *Outdoor Experiment Condition*

In the previous subsection, the measured data in the anechoic chamber were used to observe the effect of SSP on the performance of the AOA estimation. Obviously, it was not realistic scenario. In this subsection, a field test was undertaken in an open space to find the issues in a real deployment. The data were obtained from a field experiment at Hokkaido, Japan as shown in Fig. 9. The antenna arrays were mounted at a height of 10 m above ground on top of a building. The transmit antenna was a single patch antenna placed at a known position in an open area with Line-Of-Sight (LOS) condition. The distance between the transmitter and receiver was approximately 800 m in the LOS path where the AOA of 0 degrees was considered. The experiment condition is shown in Fig 10. Other measurement equipments were the same as those used for the experiment in the anechoic chamber.

TABLE 2: SPECIFICATION OF MEASUREMENT SYSTEM

Tx	Antenna	Horn antenna
	Antenna gain	14.67 dBi
	Tx power	30 dBm
	Signal	GMSK
Rx	Antenna array	2 parallel ULAs
	Number of element (M)	10 (of each array)
	Element spacing (d)	0.8λ
	Antenna type	Patch antenna
	Antenna gain	7 dBi
	size of array	135×30 cm

Fig. 9: Field of Experiment

The GMSK modulated signal with the carrier frequency of 1.74 GHz was transmitted and arrived at the antenna array with SNR of 36 dB. The experiments were repeated 15 times by changing the position of the transmitter whose known angles of arrival relative to the receive array anntenna were $-6, -5, -2, -0.5, -0.2, -0.1, 0, 0.1, 0.2, 0.5, 1, 2, 5$ and 6 degrees.

D. *Results from Outdoor Measured Data*

The same calibration data applied in the anechoic chamber data was also applied in the open-site data before AOA estimation was undertaken. Figs. 11 and 12 show the AOA estimation error versus the true angle where the results were obtained by ESPRIT and MUSIC, respectively. For both Figs. 11 and 12, the results with and without FSS were also compared. The

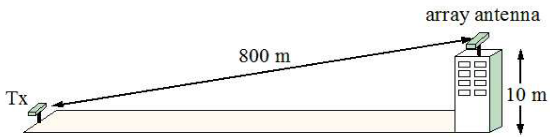

Fig. 10: Experiment condition

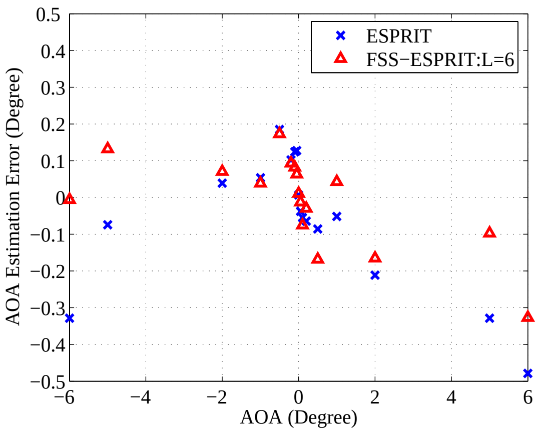

Fig. 11: AOA estimation error versus true AOAs when AOA is estimated from the open-site data by ESPRIT and FSS-ESPRIT.

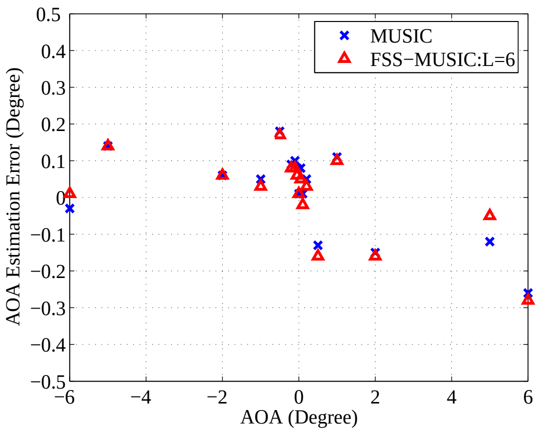

Fig. 12: AOA estimation error versus true AOAs when AOA is estimated from the open-site data by MUSIC and FSS-MUSIC.

number of subarrays was 6, in the case of FSS. Though, it is illustrated that SSP can improve the AOA estimation both by MUSIC and ESPRIT, the performance of ESPRIT was better by applying FSS than MUSIC.

TABLE 3: STANDARD DEVIATION OF AOA ESTIMATION ERROR (DEGREES) BY ESPRIT AND FSS-ESPRIT ($L = 6$) FOR BOTH CASES OF THE ANECHOIC CHAMBER DATA AND THE OPEN-SITE DATA.

Method	Chamber data	Open-site data
MUSIC	0.0331	0.1142
FSS-MUSIC	0.0293	0.1122
ESPRIT	0.0581	0.1760
FSS-ESPRIT	0.0291	0.1218

The standard deviation of AOA estimation error by ESPRIT and MUSIC with and without FSS are tabulated in Table 3 for both cases of the anechoic chamber data and the open-site data. It is verified that SSP can reduce the random error in the array response, leading to the more accuracy of AOA estimation and also observed that the performance of ESPRIT had more improvement than MUSIC by applying FSS.

5. Conclusion

This paper shows that the spatial smoothing preprocessing can reduce the random error in the array response, leading to lessening of the error of estimated AOAs. The performance improvement is verified by the Monte-Carlo simulation as well as with the measured-data estimation. The results of the measured-data estimation are in good agreement with the simulation results. Therefore, this approach is applicable in real scenarios. Moreover, it was found that the SSP can improve the performance of ESPRIT more than that of MUSIC.

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