TWO DIMENSIONAL ANALYSIS OF ELECTROMAGNETIC CRYSTAL WAVEGUIDES CONSISTING OF PERFECTLY CONDUCTIVE RECTANGULAR CYLINDERS

Hongting Jia and Kiyotoshi Yasumoto
Department of Computer Science and Communication Engineering
Kyushu University 36, Fukuoka 812-8581, Japan
jia@csce.kyushu-u.ac.jp, yasumoto@csce.kyushu-u.ac.jp

1. Introduction

Electromagnetic crystals are periodic arrangements of discrete dielectric or metallic objects in which any electromagnetic wave propagation is forbidden within a fairly large frequency range. An electromagnetic crystal waveguide may be composed by either introducing a line defect or chains of defects in an electromagnetic crystal or bounding a space by two pieces of electromagnetic crystals. The mode propagation in two-dimensional electromagnetic waveguide has been extensively investigated using various approaches such as the plane wave expansion method[1], the finite-difference time-domain technique[2], the beam-propagation method[3], the Lattice sums and T-matrix method[4, 5], and the improved Fourier series method[6]. Most of the studies concerning guide modes of electromagnetic crystals are focused at dielectric electromagnetic crystals. Very few studies can be found in the literature to discuss those of metallic electromagnetic crystal waveguides.

In this paper, we shall propose a rigorous and simple approach to guide modes of a metallic electromagnetic crystal waveguide. This method is an extension of an S-matrix and mode-matching method which has been recently developed[7]to analyze the electromagnetic scattering characteristics of two-dimensional metallic electromagnetic crystals with arbitrary cross section. Numerical examples show both of fast convergence and shot computer time. The dispersion characteristics and field distributions are presented for the lowest TM and TE modes.

2. Formulation of Dispersion Equations

Fig.1 shows the side view of a two-dimensional electromagnetic crystal waveguide composed by metallic rectangular cylinders. The cylindrical elements should be same along each layer of the arrays but those in difference layers do not need to be same. The background medium is a homogeneous dielectric with relative permittivity ϵ_{rb} and relative permeability μ_{rb}. The guided waves are assumed to be uniform in the z-direction and vary in the form $e^{i\alpha x}$ in the x-direction, where α is a propagation constant of guided modes. Since this structure has the same period in x-direction, all the electromagnetic fields may be expressed by the superposition of space harmonics $\{e^{ik_{xm}x}\}$, $k_{xm} = \alpha + \frac{2\pi m}{h}$, which make an orthogonal and complete system. Therefore, the scattering characteristics are described by the reflection and transmission matrices based on these space harmonics. If we use the reflection matrices $\boldsymbol{R}_0^{\cup}$ and $\boldsymbol{R}_0^{\cap}$ to be the scattering properties at the upper and lower boundary plane as viewing from the 0-th region. Then the downgoing vector $\boldsymbol{a}_0^-$, which denote the z-component of the electrical fields and the magnetic fields for TM and TE modes, in the 0-th region as shown in Fig.1 must satisfy the following equation.

$$\boldsymbol{Y}_0\boldsymbol{R}_0^{\cup}\boldsymbol{Y}_0\boldsymbol{R}_0^{\cap}\boldsymbol{a}_0^- = \boldsymbol{a}_0^- \tag{1}$$

where $\boldsymbol{Y}_0$ is a diagonal matrix, whose diagonal elements are $e^{ik_{ym}d_0}$, $k_{ym} = \sqrt{k_b^2 - k_m^2}$, $\mathrm{Im}(k_{ym}) \geq 0$, $k_b = k_0\sqrt{\epsilon_{rb}\mu_{rb}}$. For nontrivial $\boldsymbol{a}_0^-$ the discrete value α_q of α must be a root of the following equation.

$$\det[\boldsymbol{Y}_0\boldsymbol{R}_0^{\cup}\boldsymbol{Y}_0\boldsymbol{R}_0^{\cap} - \boldsymbol{I}] = 0 \tag{2}$$

ISBN: 89-86522-78-0 94460©KEES

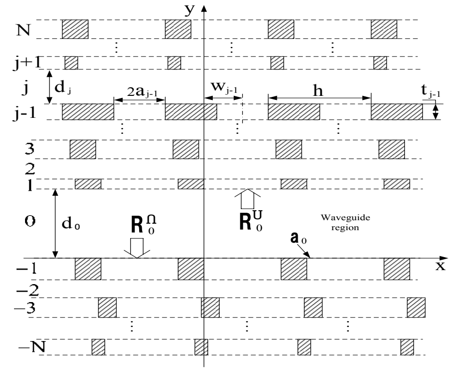
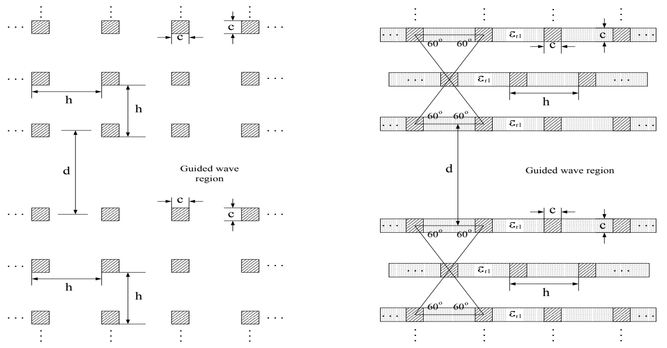


Fig.1 Schematic of a two-dimensional electromagnetic crystal waveguide consisting of rectangular metallic cylinders.



(a) Square lattice (b) Regular triangular lattice

Fig.2 Cross section of metallic electromagnetic crystal waveguides.

The refection matrices $\boldsymbol{R}_0^{\cup}$ and $\boldsymbol{R}_0^{\cap}$ may be obtained by setting $\boldsymbol{R}_0^{\cup} = \tilde{\boldsymbol{R}}_N^{-}$ and $\boldsymbol{R}_0^{\cap} = \tilde{\boldsymbol{R}}_{-N}^{+}$, where matrices $\tilde{\boldsymbol{R}}_N^{-}$ and $\tilde{\boldsymbol{R}}_{-N}^{+}$ are derived from the following formulas through the $N/2$ times recursion process starting with $\tilde{\boldsymbol{S}}_{\pm 1} = \boldsymbol{S}_{\pm 1}$. Let $(\tilde{\boldsymbol{R}}_j^{\pm},\ \tilde{\boldsymbol{T}}_j^{\pm}$ be the total reflection and transmission matrices for the entire system of j-layered arrays. When the $(j \pm 2)$-th array is stacked with the separation distance $d_{j\pm 1}$ upper and lower the j-th array, respectively, the total reflection and transmission matrices $(\tilde{\boldsymbol{R}}_{j\pm 2}^{\pm},\ \tilde{\boldsymbol{T}}_{j\pm 2}^{\pm}$ for the $(j \pm 2)$-layered system is calculated by the following relations:

$$\begin{bmatrix} \tilde{\boldsymbol{R}}_{j+2}^{+} & \tilde{\boldsymbol{T}}_{j+2}^{+} \\ \tilde{\boldsymbol{T}}_{j+2}^{-} & \tilde{\boldsymbol{R}}_{j+2}^{-} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{j+2}^{+} + \boldsymbol{F}_j^{+} \tilde{\boldsymbol{R}}_j^{+} \boldsymbol{V}_j \boldsymbol{T}_{j+2}^{-} & \boldsymbol{F}_j^{+} \tilde{\boldsymbol{T}}_j^{+} \\ \boldsymbol{F}_j^{-} \boldsymbol{T}_{j+2}^{-} & \boldsymbol{F}_j^{-} \boldsymbol{R}_{j+2}^{-} \boldsymbol{V}_j \tilde{\boldsymbol{T}}_j^{+} + \tilde{\boldsymbol{R}}_j^{-} \end{bmatrix}, \quad j \geq 1 \tag{3}$$

$$\begin{bmatrix} \tilde{\boldsymbol{R}}_{j-2}^{+} & \tilde{\boldsymbol{T}}_{j-2}^{+} \\ \tilde{\boldsymbol{T}}_{j-2}^{-} & \tilde{\boldsymbol{R}}_{j-2}^{-} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{R}}_j^{+} + \boldsymbol{Z}_j^{+} \boldsymbol{R}_{j-2}^{+} \boldsymbol{V}_j \tilde{\boldsymbol{T}}_j^{-} & \boldsymbol{Z}_j^{+} \boldsymbol{T}_{j-2}^{+} \\ \boldsymbol{Z}_j^{-} \tilde{\boldsymbol{T}}_j^{-} & \boldsymbol{Z}_j^{-} \tilde{\boldsymbol{R}}_j^{-} \boldsymbol{V}_j \boldsymbol{T}_{j-2}^{+} + \boldsymbol{R}_{j-2}^{-} \end{bmatrix}, \quad j \leq -1 \tag{4}$$

and

$$\boldsymbol{F}_j^{+} = \boldsymbol{T}_{j+2}^{+} \left(\boldsymbol{I} - \boldsymbol{V}_j \tilde{\boldsymbol{R}}_j^{+} \boldsymbol{V}_j \boldsymbol{R}_{j+2}^{-} \right)^{-1} \boldsymbol{V}_j, \qquad \boldsymbol{F}_j^{-} = \tilde{\boldsymbol{T}}_j^{-} \boldsymbol{V}_j \left(\boldsymbol{I} - \boldsymbol{R}_{j+2}^{-} \boldsymbol{V}_j \tilde{\boldsymbol{R}}_j^{+} \boldsymbol{V}_j \right)^{-1} \tag{5}$$

$$\boldsymbol{Z}_j^{+} = \tilde{\boldsymbol{T}}_j^{+} \left(\boldsymbol{I} - \boldsymbol{V}_j \boldsymbol{R}_{j-2}^{+} \boldsymbol{V}_j \tilde{\boldsymbol{R}}_j^{-} \right)^{-1} \boldsymbol{V}_j, \qquad \boldsymbol{Z}_j^{-} = \boldsymbol{T}_{j-2}^{-} \boldsymbol{V}_j \left(\boldsymbol{I} - \tilde{\boldsymbol{R}}_j^{-} \boldsymbol{V}_j \boldsymbol{R}_{j-2}^{+} \boldsymbol{V}_j \right)^{-1} \tag{6}$$

The matrices $(\boldsymbol{R}_j^{\pm},\ \boldsymbol{T}_j^{\pm})$ are the reflection and transmission matrices of j-th layer array where j is an odd number. These matrices can be obtained using mode-matching technique. For the detail discussion please refers to [7]

$$\begin{bmatrix} \boldsymbol{R}_j^{+} & \boldsymbol{T}_j^{+} \\ \boldsymbol{T}_j^{-} & \boldsymbol{R}_j^{-} \end{bmatrix} = \pm \begin{bmatrix} \boldsymbol{\Gamma}_j^{+} + \boldsymbol{I} & \boldsymbol{\Gamma}_j^{-} \\ \boldsymbol{\Gamma}_j^{-} & \boldsymbol{\Gamma}_j^{+} + \boldsymbol{I} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{\Gamma}_j^{+} - \boldsymbol{I} & \boldsymbol{\Gamma}_j^{-} \\ \boldsymbol{\Gamma}_j^{-} & \boldsymbol{\Gamma}_j^{+} - \boldsymbol{I} \end{bmatrix} \tag{7}$$

with

$$\boldsymbol{\Gamma}_j^{+} = \left[\Gamma_{j,mn}^{+} \right] = \begin{cases} \left[\dfrac{k_{yn} a_j \mu_{rj}}{h \mu_{rb}} \displaystyle\sum_{\nu=1}^{\infty} \dfrac{\left(1 + e^{2i\gamma_{j\nu} t_j}\right) \xi_{j\nu}^2 G_{\nu mn}}{\left(1 - e^{2i\gamma_{j\nu} t_j}\right) \gamma_{j\nu}} \right] & \text{for TM wave} \\ \left[\dfrac{\alpha_m k_{xn} \epsilon_{rb} a_j}{h k_{ym} \epsilon_{rj}} \displaystyle\sum_{\nu=0}^{\infty} \dfrac{\left(1 + e^{2i\gamma_{j\nu} t_j}\right) \gamma_{j\nu} G_{\nu mn}}{\left(1 - e^{2i\gamma_{j\nu} t_j}\right)\left(1 + \delta_{\nu 0}\right)} \right] & \text{for TE wave} \end{cases} \tag{8}$$

$$\boldsymbol{\Gamma}_j^{-} = \left[\Gamma_{j,mn}^{-} \right] = \begin{cases} \left[\dfrac{2 k_{yn} a_j \mu_{rj}}{h \mu_{rb}} \displaystyle\sum_{\nu=1}^{\infty} \dfrac{e^{i\gamma_{j\nu} t_j} \xi_{j\nu}^2 G_{j,\nu mn}}{\left(1 - e^{2i\gamma_{j\nu} t_j}\right) \gamma_{j\nu}} \right] & \text{for TM wave} \\ \left[\dfrac{2 \alpha_m k_{xn} \epsilon_{rb} a_j}{h k_{ym} \epsilon_{rj}} \displaystyle\sum_{\nu=0}^{\infty} \dfrac{e^{i\gamma_{j\nu} t_j} \gamma_{j\nu} G_{j,\nu mn}}{\left(1 - e^{2i\gamma_{j\nu} t_j}\right)\left(1 + \delta_{\nu 0}\right)} \right] & \text{for TE wave} \end{cases} \tag{9}$$

$$G_{j,\nu mn} = \frac{-(-1)^{\nu} e^{i(k_{xn}-k_{xm})w_j}}{a_j^2(k_{xm}^2-\xi_{j\nu}^2)(k_{xn}^2-\xi_{j\nu}^2)} \left[(-1)^{\nu} e^{ik_{xn}a_j} - e^{-ik_{xn}a_j}\right] \left[(-1)^{\nu} e^{ik_{xm}a_j} - e^{-ik_{xm}a_j}\right] \quad (10)$$

$$\xi_{j\nu} = \frac{\nu\pi}{2a_j}, \qquad \gamma_{j\nu} = \sqrt{k_j^2 - \xi_{j\nu}^2}, \qquad \mathrm{Im}(\gamma_{j\nu}) \geq 0, \qquad k_j = k_0\sqrt{\epsilon_{rj}\mu_{rj}} \quad (11)$$

The signs $\pm$ in Eq.(7) are corresponding to TM wave and TE wave, respectively.

3. Numerical Examples

We first consider an electromagnetic crystal waveguide consisting of square conductive cylinders located in free space with square arrangement as shown in Fig.2(a). Fig.3 shows the dispersion curves and electrical field distributions of the lowest even and odd TM modes in the metallic electromagnetic crystal waveguide, where $c = 0.2h$, $d = 1.5h$, and the truncated number M is chosen to be 15 including 31 space harmonics. In Fig.3(a) the solid lines denote the even modes, the dotted line denotes an odd mode, and the square mark means cutoff point. In these three modes, only mode 1 locates in the perfect bandgap range, both of mode 2 and mode 3 are in a part bandgap range. The distribution of electric fields are plotted in Fig.3(b)-(d) for three mode at $h/\lambda_0 = 0.4$ and 0.7. The corresponding normalized propagation constants are $\alpha h/2\pi = 0.288769$, 0.353135, and 0.427878, respectively. It is found that the electric fields concentrated around defect region for even modes, whereas, the electric fields are distributed in a larger region for odd mode. This means that the odd mode is weak mode. It notes that the present method can calculate all the guided modes, even they is not in perfect bandgap ranges. Fig.4 shows the dispersion curves and magnetic field distributions of the lowest even and odd TE modes in the same waveguide discussed in Fig.3. This waveguide has two modes. One is an even mode. The another one is an odd mode. Their magnetic fields distributions are plotted in Fig.4(b)-(c), at $h/\lambda_0 = 0.8$ point and $\alpha h/2\pi = 0.207316$ and $\alpha h/2\pi = 0.192617$. Since there is no perfect bandgap for TE polarization case, both two guided modes are in part bandgaps. Fig.5 shows the dispersion curves of another electromagnetic crystal waveguide consisting of metallic cylinders embedded in dielectric slabs with regular triangular arrangement whose cross section is shown in Fig.2(b), where $c = 0.2h$, $\epsilon_{r1} = 10$, and $d = 1.5h$. There are four even modes and no odd mode for TM polarized modes, whereas there is only one odd mode and no even mode.

4. Conclusion

We have proposed a rigorous and simple approach to guide modes of a metallic electromagnetic crystal waveguide. The method is based on a generalized scattering matrix and mode-matching technique. The validity of the proposed method has be confirmed by considerable numerical experiments. The dispersion characteristics and field distributions are presented for the lowest TM and TE modes of two types of metallic electromagnetic waveguides.

Acknowledgments

This research was supported in part by the 21st Century COE Program "Reconstruction of Social Infrastructure Related of Information Science and Electrical Engineering".

References

[1] H. Benisty, "Modal analysis of optical guides with two-dimensional photonic band-gap boundaries," J. Appl. Phys., vol.79, pp.7483-7492, 1996.

[2] Y. Naka and H. Ikuno, "Guided mode analysis of two-dimensional air-hole type photonic crystal optical waveguides, *IEICE Tech. Rep.,* EMT-00-78, pp.75-80, 2000.

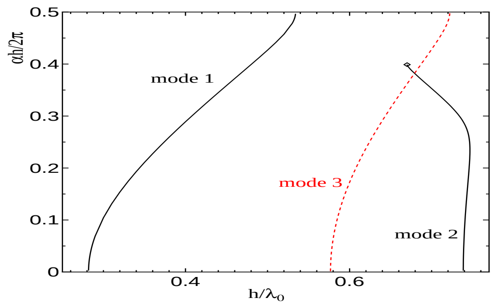
[3] M. Koshiba, Y. Tsuji, and M. Hikari, "Time-domain beam propagation method and its application to photonic crystal circuits," *J. Lightwave Technol.* vol.18, pp.102-110, 2000.

[4] K. Yasumoto and H. Jia, "Modal analysis of two-dimensional photonic crystal waveguides formed by rectangular cylinders," *Proc. URSI EMT-S*, pp.739-741, 2004.

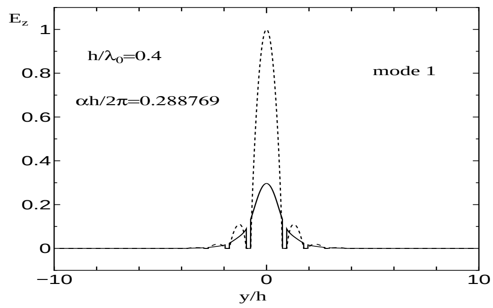
[5] H. Jia and K. Yasumoto, "Modal analysis of coupled cavity waveguides using lattice sums technique," *Proc. of ICMMT*, pp.746-749, Aug. 2004.

[6] H. Jia and K. Yasumoto, "Modal analysis of two-dimensional photonic crystal waveguides formed by rectangular cylinders," *Proc. of APMC*, No.393, 2004.

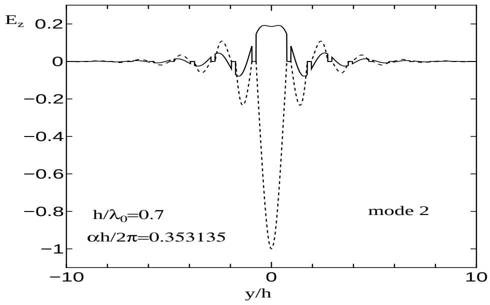
[7] H. Jia and K. Yasumoto, “S-matrix solution of electromagnetic scattering from periodic arrays of metallic cylinders with arbitrary cross section,” *IEEE Antennas and Wireless Propagation Lett.* vol.3, pp.41-44, 2004.



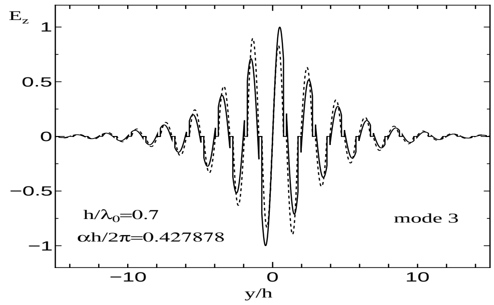
(a) Dispersion curves



(b) Field distributions of mode 1

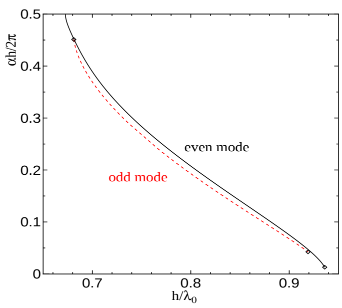


(c) Field distributions of mode 2

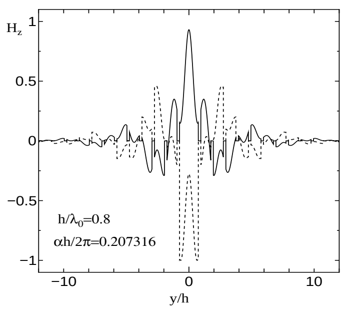


(d) Field distributions of mode 3

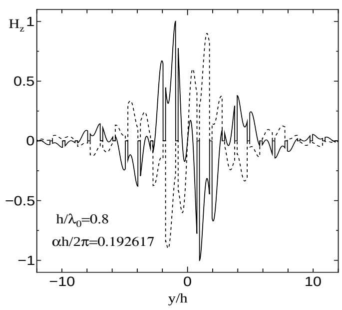
Fig.3 Dispersion curves and electrical field distributions of the lowest even and odd TM modes in the electromagnetic crystal waveguide.



(a) Dispersion curves

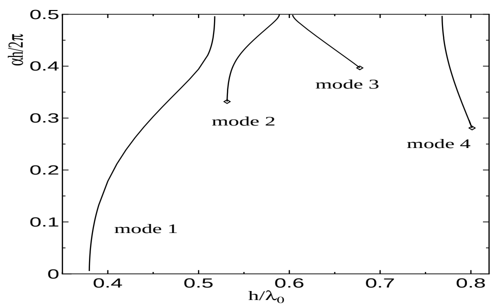


(b) Field distributions of even mode

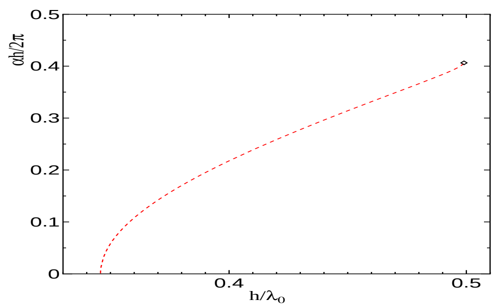


(c) Field distributions of odd mode

Fig.4 Dispersion curves and magnetic field distributions of the lowest even and odd TE modes in the electromagnetic crystal waveguide.



(a) TM modes



(b) TE mode

Fig.5 Dispersion curves of the lowest TM and TE modes in the electromagnetic crystal waveguide consisting of rectangular conductive cylinders embedded in dielectric slabs.