

ELECTROMAGNETIC SCATTERING BY A TWO DIMENSIONAL ELECTROMAGNETIC CRYSTALS EMBEDDED IN A DIELECTRIC SLAB WITH PERIODIC DEFECTS

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1. Introduction

Photonic bandgap structures [1] in discrete periodic systems have received growing attention because of their potential applications to narrow-band filters, high-quality resonant cavities, strongly guiding devices and substrates for antennas.

In this paper, we present a very accurate and efficient method to investigate a two-dimensional electromagnetic scattering from layered periodic arrays of cylindrical objects with the periodic defects and the arrays embedded in the dielectric slab. The formulation is based on the Lattice Sums technique [2], the T-matrix approach and the generalized reflection and transmission matrices for a layered system. The method is quite general and applies to various configurations of two-dimensional periodic arrays. The proposed method is used to analyze the influence of the periodic defects on the frequency response in reflectance from the multilayered periodic arrays of circular cylinders. Power reflection coefficient of different configuration of multilayered periodic arrays of the circular cylinders with periodic defects is numerically studied. It could be seen that periodic defects make substantial influence on the frequency response in reflectance. They lead to the appearance of a series of stop-band regions and effect on the location of the stop-bands with perfect reflection.

2. Formulation of the problem

A cross section of N-layered periodic arrays of cylindrical objects with the periodic defects situated in a background medium with a wave number k_0 is shown in Fig.1. The cylinders are infinitely long and parallel to each other.

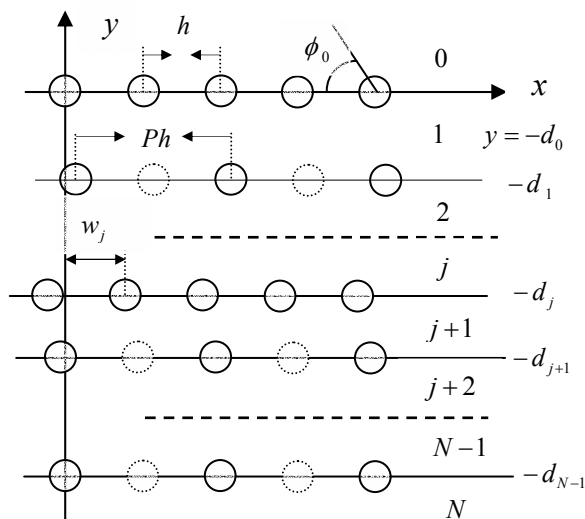


Fig.1. Cross section of N-layered periodic arrays of cylindrical objects with periodic defects in free space

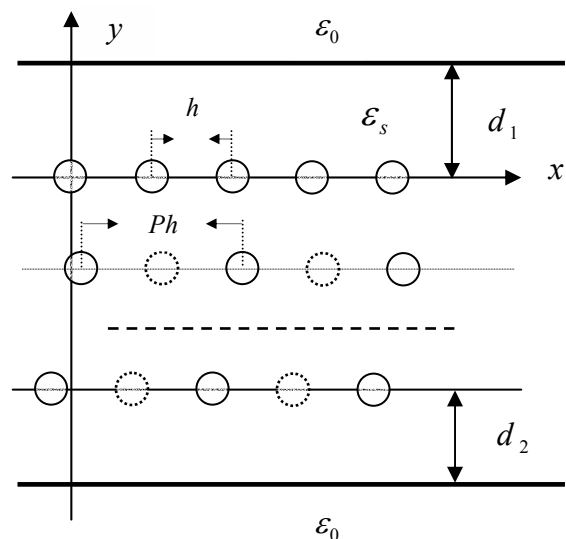


Fig.2. Cross section of N-layered periodic arrays of cylindrical objects with periodic defects embedded in dielectric slab

The period of the first (third, fifth... $N-1$) layers is h , while the period of the second (forth, sixth... N) layers is assumed to be Ph , where P is a natural number. The common period of the multilayered array of the circular cylinders is Ph . The local origin attached to the j -th layer is shifted by w_j along the x -axis from the global origin. The radius and dielectric permittivity of the cylinders are a and ε respectively. Generally, the cylinders in different layers need not be identical in material properties, dimensions or shape. We consider two-dimensional scattering problem for a plane wave, whose direction of incidence is normal to the cylinder axis.

A plane wave with unit amplitude is incident from the upper half-space $y > 0$. The wavevector forms an angle ϕ_0 with respect to the x axis and its x component is $k_{x0} = k_0 \cos \phi_0$. The incident plane wave is scattered into a set of Floquet space-harmonics with the x -dependence as $e^{ik_{x\ell}x}$ where $k_{x\ell} = k_{x0} + 2\ell\pi/Ph$ ($\ell = 0, \pm 1, \pm 2, \dots$). When a single layer of the array is isolated, scattering process is described in terms of the reflection and transmission coefficients, which relate the amplitudes of reflected and transmitted space harmonics to the amplitude of the incident wave. When the array is multilayered or embedded in the dielectric slab, the scattering space harmonics impinging on the neighbor arrays as new incident waves and are scattered into another set of space harmonics, which impinge back on the original array. A series of this process explains a multiple scattering of wave fields in the layered array. To describe the multiple scattering processes, we introduce the reflection and transmission matrices for each of the array planes, which relate a set of incident space harmonics to a set of reflected and transmitted ones. The results are then used to derive the generalized reflection and transmission matrices for the layered structure.

Following the same analytical treatment [3] which has been developed for the scattering problem of a single layer, the reflection matrix $\mathbf{R}_{j,j+1}$ and the transmission matrix $\mathbf{F}_{j,j+1}$ for the incident space harmonics downgoing from the regions j to $j+1$ are deduced as follows:

$$\mathbf{R}_{j,j+1} = \mathbf{U} \cdot \mathbf{P}_j^{sc}, \quad \mathbf{F}_{j,j+1} = \mathbf{I} + \mathbf{V} \cdot \mathbf{P}_j^{sc} \quad (1)$$

with

$$\mathbf{U} = [u_{\ell m}] = \left[\frac{2(-i)^m}{k_0 Ph \sin \phi_\ell} e^{im\phi_\ell} \right], \quad \mathbf{V} = [v_{\ell m}] = \left[\frac{2(-i)^m}{k_0 Ph \sin \phi_\ell} e^{-im\phi_\ell} \right] \quad (2)$$

$$\cos \phi_\ell = \cos \phi_0 + \frac{2\ell\pi}{k_0 Ph}, \quad \mathbf{P}_j^{sc} = (\mathbf{I} - \mathbf{T}_j \cdot \mathbf{L})^{-1} \cdot \mathbf{T}_j \cdot \mathbf{P}^{in}, \quad \mathbf{P}^{in} = [p_{mn}] = [i^m e^{im\phi_n}], \quad (m, n = 0, \pm 1, \pm 2, \dots) \quad (3)$$

where \mathbf{U} and \mathbf{V} are matrices, which transform the m -th order cylindrical wave into the upgoing and downgoing plane waves of the ℓ -th space harmonic, respectively, \mathbf{P}^{in} is a matrix, which transforms the downgoing n -th space harmonic into the m -th order cylindrical wave and \mathbf{I} is the unit matrix. \mathbf{T}_j is the T-matrix of the isolated single cylinder located at the local origin $(w_j, -d_j)$ on the j -th layer. \mathbf{L} is Lattice Sums, which is expressed by a semi-infinite series of Hankel functions. To overcome the difficulty of a very slow convergence of the series, we use an integral form of the Lattice Sums, which can be accurately and efficiently evaluated using a simple scheme of numerical integration [2].

The Lattice Sums and the T-matrix of cylindrical objects play an important role in our formulation. The Lattice Sums and related matrix \mathbf{L} characterize uniquely the periodic arrangement of scatterers and are independent of the polarization of the incident field and the individual configuration of scatterers. The details of scattering from each array element within a unit cell are described by the T-matrix. The generalized reflection and transmission matrices or the generalized scattering matrix, which characterize the reflection and transmission of the N -layered arrays, are obtained by linking the reflection matrices $\mathbf{R}_{j,j+1}, \mathbf{R}_{j+1,j}$ and transmission matrices $\mathbf{F}_{j,j+1}, \mathbf{F}_{j+1,j}$ over the N layers. We have omitted to describe the details of the formulation of generalized reflection and transmission matrices due to the limited space, however, please see the reference [3].

As the periods of the layers presented in Fig.1 and Fig.2 are h or Ph , the common period of the multilayered array is Ph . For the layers with Ph period, the whole set of space harmonics $\{e^{ik_{x\ell}x}\}$

could be divided into P subsets using the following expression:

$$\{e^{ik_x x}\} = \bigcup_{\tau=0}^{P-1} \left\{ e^{i \left[k_x \tau x + \frac{2\pi n}{h} \right] x} \right\} \quad (4)$$

Each subset corresponds to the array with a period h and there is no coupling between plane waves, which belong to the different subsets. We have used this formulation in order to study the effect of periodic defects on the frequency response in reflectance and transmittance.

When the layered periodic arrays of cylindrical objects with the periodic defects are embedded in a dielectric slab (Fig.2), a slight modification is required in the previous equations. If the permittivity and permeability of the slab are ε_s and μ_s respectively, the equations (1)-(3) are changed into the following expressions:

$$\mathbf{U} = [u_{\ell m}] = \left[\frac{2(-i)^m}{k_s Ph \sin \theta_\ell} e^{im\theta_\ell} \right], \quad \mathbf{V} = [v_{\ell m}] = \left[\frac{2(-i)^m}{k_s Ph \sin \theta_\ell} e^{-im\theta_\ell} \right], \quad \cos \theta_\ell = \frac{k_0}{k_s} \cos \phi_0 + \frac{2\ell\pi}{k_s Ph} \quad (5)$$

where $k_s = \omega \sqrt{\varepsilon_s \mu_s}$. The matrices \mathbf{L} and \mathbf{T} are calculated for the background medium with ε_s and μ_s ; θ_ℓ is defined taking into account Snell's law at the slab boundaries. Using those substitutions, we can apply the same procedure as that for the layered periodic structures. The generalized reflection and transmission matrices for the embedded periodic arrays are obtained by applying the Fresnel reflection and transmission matrices to the reflection and transmission of space-harmonic waves at the upper and lower boundaries of the dielectric slab.

3. Numerical results

The proposed approach has been used to analyze the reflection and transmission characteristics of layered periodic arrays of circular cylinders with the periodic defects. Although a substantial number of numerical results could be generated, we discuss here the power reflection coefficient of the fundamental space harmonic R_0 in the wavelength range $0 < h/\lambda < 1$ under the normal incidence with $\phi_0 = 90^\circ$. The numerical results in what follows were obtained with the errors in the energy conservation less than 10^{-8} by taking into account the lowest thirteen space harmonics and by truncating the cylindrical harmonic expansion at $m = \pm 6$ to calculate the T-matrix of the isolated circular cylinders. Distance between the layers of periodic grating is h . In order to study the effect of the periodic defects on the frequency response in reflectance, we have calculated the power reflection coefficient R_0 for the multilayered identical arrays of the circular cylinders at $P = 2$, i.e. the period of the second (forth, sixth...) layers is $2h$. The numerical results have been calculated using the following non-dimensional parameters: a/h , $\varepsilon/\varepsilon_0$, $\varepsilon_s/\varepsilon_0$.

Figure 3 illustrates the dependence of the power reflection coefficient R_0 for the one hundred identical layers of the periodic arrays of the dielectric circular cylinders with periodic defects in free space as a function of the non-dimensional wavelength h/λ for TM wave at $a/h = 0.2$, $\varepsilon/\varepsilon_0 = 5.0$, $\varepsilon_s/\varepsilon_0 = 1.0$ and $w_j = 0$. Figure 4 shows the similar plot for the same multilayered periodic grating without periodic defects. It could be seen that periodic defects lead to the appearance of a series of stop-band regions and make a substantial influence of the location of the stop-bands with perfect reflection. From the figures it follows that the bandwidths of the stop-band become considerably narrower than that of the same array without periodic defects, where the stop-band with a wide bandwidth at $0.34 < h/\lambda < 0.47$ could be observed.

Figure 5 illustrates the dependence of the power reflection coefficient R_0 for the three identical layers of the dielectric circular cylinders with periodic defects embedded in the dielectric slab versus non-dimensional parameter w/h at the normalized frequency $h/\lambda = 0.8$. The periods of the first and third layers are h and the cylinders in the second layer with a period $2h$ are shifted by w along the x -axis from the global origin. From the figure it follows that the shift from the global origin makes the

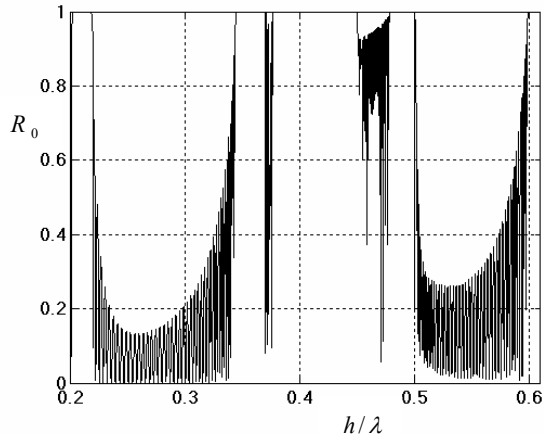


Figure 3. Dependence of R_0 for the one hundred identical layers of periodic arrays of the dielectric circular cylinders with periodic defects in free space versus h/λ .

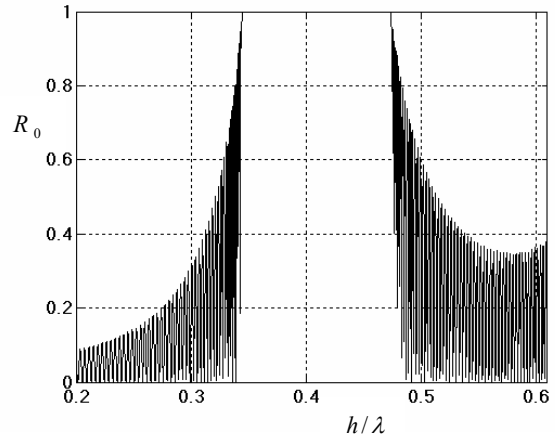


Figure 4. Same as is Fig.3 for the multilayered periodic array without periodic defects.

substantial effect on the power reflection coefficient. Two resonance peaks with a broad bandwidth appeared at $w/h=0.14$ and $w/h=0.3$ become to be well separated and a broad-band resonance profile is observed, when the parameters of multilayered array are properly chosen. In addition, as an example, the power reflection coefficient R_0 for the 12 layered array of the circular air-holes embedded in the dielectric slab ($a/h=0.2$, $\varepsilon/\varepsilon_0=1.0$, $\varepsilon_s/\varepsilon_0=2.0$, TE wave) with periodic defects (solid line) and without periodic defects (dashed line), when the second (forth, sixth...twelfth) layers are shifted by $w=h/2$ along the x -axis from the global origin is presented in figure 6.

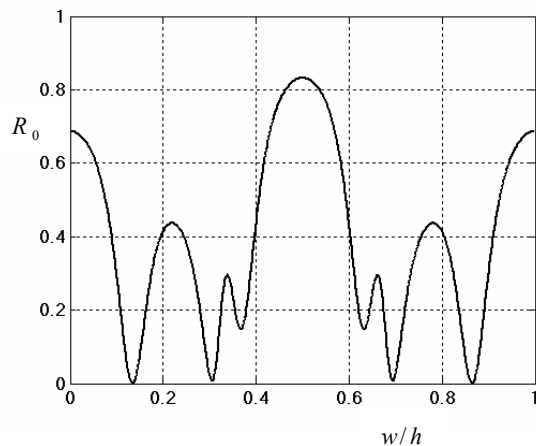


Figure 5. Dependence of R_0 for three identical layers of periodic arrays of the dielectric circular cylinders with periodic defects in dielectric slab versus w/h at $a/h=0.2$, $\varepsilon/\varepsilon_0=12.0$, $\varepsilon_s/\varepsilon_0=2.0$ and $h/\lambda=0.8$, TM wave.

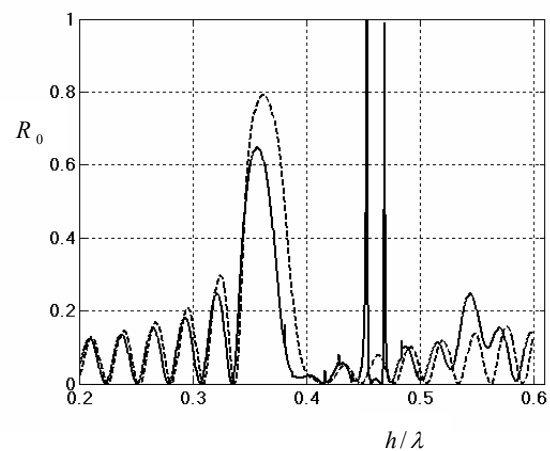


Figure 6. Power reflection coefficient R_0 for 12 identical layers of periodic arrays of the circular air holes embedded in the dielectric slab with periodic defects (solid line) and without periodic defects (dashed line) at $w=h/2$, TE wave

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