Integral field representations for the propagation of waves near a spherical Earth

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I. INTRODUCTION

Tropospheric propagation of radio waves above VHF over a distance small compared to the radius a of a spherical Earth is studied. The transmitter, modelled as a vertical dipole, is located at a height h_0 above the Earth's surface. This height can vary from a small fraction to many wavelengths. The receiver is located at a height h. Its range l is measured along the Earth's surface from the transmitter. The case when l is much greater than the wavelength, the transmitter and the receiver heights h_0 and h is of interest.

The fields from a dipole near a finitely conducting sphere can be represented exactly over the spherical harmonic functions. At a radius of about 6,350 km, such a series solution is not useful to the problem of current interest because, at a wavelength in the order of one meter or less, millions of terms of the series would have to be summed, and the lower order terms have to be cancelled out almost completely. It is well known that the Waston transformation can be utilized, as was done by van der Pol and Bremmer [Bremmer, 1949], to convert the series into sub-series of faster convergence. Under the approximate Leontovich (surface impedance) boundary conditions, Bremmer (1958) and Wait (1956) obtained an expansion of the fields in powers of the Earth's curvature. In the flat Earth limit, their results were reduced to the asymptotic form of the Sommerfeld integrals.

It is clear that very close to the transmitter, Sommerfeld's integral representations for the fields of a dipole above a flat surface [King and Smith, 1981] are good approximations to the present problem. To advance beyond the Leontovich boundary conditions and the asymptotic results, it is natural to look for extensions of those integrals to incorporate into them the radius of the Earth. Such extensions are desirable for several reasons: First, numerical evaluations of this type of integrals have been investigated extensively [Chang and Fisher, 1974; Kuo and Mei, 1978; Rahmat-Samii, et. al., 1981]. Secondly, with the parameters l, a, h_0, h and the wave numbers k_1 and k_2 of the air and of the Earth respectively appearing explicitly in the integrands of single integrals, approximate formulas can be found for different combinations of these parametric values similar to what had been done by Baños [1966]. Finally, such a formulation can be readily extended to treat problems involving stratified atmospheric or ground layers.

It should be noted that atmospheric profiles and surface roughness may have substantial effects on radio wave propagation at the frequencies under consideration [Hitney, et. al., 1984]. The effects of the surface roughness are a completely separate topic, though the results of this work may provide mean values of the field strengths near the surface. On the other hand, it is hoped that this approach can be further extended to include the diffraction effects due to continuous variations of the dielectric constant with height in the atmosphere.

In what follows, the time dependence $e^{-i\omega t}$ is assumed. In region 1 (air, $r > a$) the wave number is $k_1 = \omega\sqrt{\mu_1\epsilon_1}$, where μ_1 is the permeability, $\epsilon_1 = \epsilon_0\epsilon_{R1} + i\sigma_1/\omega$ is the generalized permittivity and ϵ_{R1} is the relative dielectric constant. Such quantities in region 2 ($r < a$, Earth) are similarly defined. Wherever ϵ, μ, k and $\zeta = \sqrt{\mu/\epsilon}$ are used without subscripts, it should be understood that they stand both for ϵ_1, μ_1, k_1, ζ_1 in region 1 and for ϵ_2, μ_2, k_2, ζ_2 in region 2.

II. FORMULATION

The radiation from an infinitesimal current element of unit strength $\vec{J} = \hat{z}\delta(\vec{r} - \vec{r}_0)$ is considered where $r_0 = a + h_0 > a$. The total fields can be expressed in terms of the Hertzian dipole $\hat{r}\Pi(r, \theta)$ in the spherical coordinates as:

$$E_r(r, \theta) = \frac{\zeta}{4\pi k r r_0 \sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} \Pi(r, \theta) \tag{1}$$

$$E_\theta(r, \theta) = -\frac{1}{4\pi r_0} \frac{\partial}{\partial\theta} \Lambda(r, \theta) \tag{2}$$

$$H_\varphi(r, \theta) = -\frac{i}{4\pi r_0} \frac{\partial}{\partial\theta} \Pi(r, \theta) \tag{3}$$

where $\Lambda(r, \theta) = \frac{\zeta}{kr} \frac{\partial}{\partial r} r\Pi(r, \theta)$; $\Pi(r, \theta)$ satisfies the equation:

$$\nabla^2 \Pi(r,\theta) + k^2 \Pi(r, \theta) = 4\pi i\delta(\vec{r} - \vec{r}_0) \tag{4}$$

together with the boundary conditions that $\Pi(r, \theta)$ and $\Lambda(r, \theta)$ are continuous. In terms of the field from an isolated dipole, $\Pi(r, \theta)$ can be splitted into a primary field $\Pi^{pr}(r, \theta)$ and a secondary field $\Pi^{sec}(r, \theta)$:

$$\Pi(r, \theta) = \begin{cases} \Pi^{pr}(r, \theta) + \Pi^{sec}(r, \theta) & r > a \\ \Pi^{sec}(r, \theta) & r < a \end{cases} \tag{5}$$

By the Kirchhoff-Huygens principle [Stratton, 1941], integral expressions for the secondary fields can be given in terms of $\Pi(a, \theta)$ and $\Lambda(a, \theta)$. These integrals run over the source points $\vec{r}'$ on the Earth's surface. The distance between two points $\vec{r}$ and $\vec{r}'$ in the spherical coordinates is:

$$\begin{aligned} |\vec{r} - \vec{r}'| &= \sqrt{(r - r')^2 + 4rr' \sin^2(\frac{\theta - \theta'}{2}) + 4rr' \sin\theta \sin\theta' \sin^2(\frac{\phi - \phi'}{2})} \\ &= \sqrt{(r - r')^2 + \frac{rr'}{a^2} [(\bar{l} - \bar{l}')^2 + 4\,\bar{l}\bar{l}' \sin^2(\frac{\phi - \phi'}{2})]} \end{aligned} \tag{6}$$

where $\bar{l} = 2a \sin\frac{\theta}{2} \cos\frac{\theta'}{2}$, $\bar{l}' = 2a \sin\frac{\theta'}{2} \cos\frac{\theta}{2}$.

For small θ and θ', $\bar{l}$ and $\bar{l}'$ can be approximated by the distances along the Earth surface, or more accurately, by the chords $l = 2a \sin\frac{\theta}{2}$ and $l' = 2a \sin\frac{\theta'}{2}$. Such an approximation leads to the following integral expressions for the fields in $r > a$:

$$\Pi(\mathrm{r}, \tfrac{l}{a}) - \Pi^{\mathrm{pr}}(\mathrm{r}, \tfrac{l}{a}) = \frac{-1}{\mathrm{k}_1}\int^{\infty} \lambda d\lambda J_0(\lambda l)[Q(\lambda)R(\mathrm{r}, a; \mathrm{k}, \lambda) + P(\lambda)S(\mathrm{r}, a; \mathrm{k}_1, \lambda)] \quad (7)$$

$$\Lambda(\mathrm{r}, \tfrac{l}{a}) - \Lambda^{\mathrm{pr}}(\mathrm{r}, \tfrac{l}{a}) = \frac{-\zeta_1}{\mathrm{k}_1}\int_0^{\infty} \lambda d\lambda J_0(\lambda l)[Q(\lambda)S(a, \mathrm{r}; \mathrm{k}_1, \lambda) + P(\lambda)T(\mathrm{r}, a; \mathrm{k}_1, \lambda)] \quad (8)$$

where:

$$R(\mathrm{r}, \mathrm{r}'; \mathrm{k}, \lambda) = \frac{a^2 \mathrm{k}}{\mathrm{rr}'\beta_{\mathrm{rr}'}} e^{i|\mathrm{r}-\mathrm{r}'|\beta_{\mathrm{rr}'}} \quad (9)$$

$$\beta_{\mathrm{rr}'} = \sqrt{\mathrm{k}^2 - \frac{a^2}{\mathrm{rr}'}\lambda^2}$$

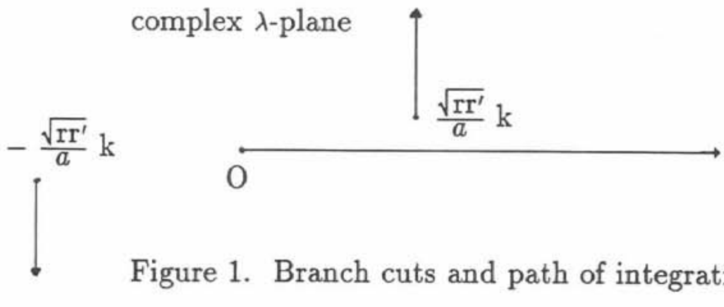

Figure 1. Branch cuts and path of integration

$$S(\mathrm{r}, \mathrm{r}'; \mathrm{k}, \lambda) = -\frac{a^2}{\mathrm{rr}'} e^{i|\mathrm{r}-\mathrm{r}'|\beta_{\mathrm{rr}'}} \{i[\mathrm{sgn}(\mathrm{r}-\mathrm{r}')] + \frac{1 - i|\mathrm{r}-\mathrm{r}'|\beta_{\mathrm{rr}'}}{2\mathrm{r}'\beta_{\mathrm{rr}'}^3}(\mathrm{k}^2 - \beta_{\mathrm{rr}'}^2)\} \quad (10)$$

$$T(\mathrm{r}, \mathrm{r}'; \mathrm{k}, \lambda) = -\frac{2ia^2}{\mathrm{r}^2}\delta(\mathrm{r}-\mathrm{r}') + \frac{a^2\beta_{\mathrm{rr}'}}{\mathrm{krr}'} e^{i|\mathrm{r}-\mathrm{r}'|\beta_{\mathrm{rr}'}}[1 - \frac{(\mathrm{r}-\mathrm{r}')^2}{4\mathrm{rr}'\beta_{\mathrm{rr}'}^4}(\mathrm{k}^4 - \beta_{\mathrm{rr}'}^4) + \frac{(1 - i|\mathrm{r}-\mathrm{r}'|\beta_{\mathrm{rr}'})}{4\mathrm{rr}'\beta_{\mathrm{rr}'}^6}(\mathrm{k}^2 - \beta_{\mathrm{rr}'}^2)(3\mathrm{k}^2 - \beta_{\mathrm{rr}'}^2)] \quad (11)$$

III. THE SOMMERFELD TYPE INTEGRALS

By requiring that $\Pi(\mathrm{r}, \frac{l}{a})$ and $\Lambda(\mathrm{r}, \frac{l}{a})$ be continuous, $P(\lambda)$ and $Q(\lambda)$ can be solved:

$$P(\lambda) = \frac{N_{PR}(\lambda)}{D(\lambda)} R(a, \mathrm{r}_0; \mathrm{k}_1, \lambda) + \frac{N_{PS}(\lambda)}{D(\lambda)} S(\mathrm{r}_0, a; \mathrm{k}_1, \lambda) \quad (12)$$

$$Q(\lambda) = \frac{N_{QR}(\lambda)}{D(\lambda)} R(a, \mathrm{r}_0; \mathrm{k}_1, \lambda) + \frac{N_{QS}(\lambda)}{D(\lambda)} S(\mathrm{r}_0, a; \mathrm{k}_1, \lambda) \quad (13)$$

where

$$N_{PR}(\lambda) = -[S(a, a^+; \mathrm{k}_1, \lambda) + S(a, a^-; \mathrm{k}_2, \lambda)] \quad (14)$$

$$N_{QS}(\lambda) = -[S(a^+, a; \mathrm{k}_1, \lambda) + S(a^-, a; \mathrm{k}_2, \lambda)] = N_{PR}(\lambda) \quad (15)$$

$$N_{PS}(\lambda) = R(a^+, a; \mathrm{k}_1, \lambda) + \frac{\zeta_1}{\zeta_2}R(a^-, a; \mathrm{k}_2, \lambda) \quad (16)$$

$$N_{QR}(\lambda) = \frac{\zeta_2}{\zeta_1} T(a^-, a; \mathrm{k}_2, \lambda) + T(a^+, a; \mathrm{k}_1, \lambda) \quad (17)$$

$$D(\lambda) = N_{QR}(\lambda) N_{PS}(\lambda) - N_{PR}(\lambda) N_{QS}(\lambda) \quad (18)$$

contain only algebraic terms; $\beta_1 = \sqrt{\mathrm{k}_1^2 - \lambda^2} = \beta_{1aa}$; $\beta_2 = \sqrt{\mathrm{k}_2^2 - \lambda^2} = \beta_{2aa}$.

The components of the $\vec{E}$ and $\vec{H}$ fields in $\mathrm{r} > a$ for small θ are:

$$E_{\mathrm{r}}(\mathrm{r}, \tfrac{l}{a}) = E_{\mathrm{r}}^{\mathrm{pr}}(\mathrm{r}, \tfrac{l}{a}) + \frac{\zeta_1 a^2}{4\pi \mathrm{k}_1^2 \mathrm{rr}_0}\int^{\infty} \lambda^3 d\lambda J_0(\lambda l)[Q(\lambda)R(\mathrm{r}, a; \mathrm{k}_1, \lambda) + P(\lambda)S(\mathrm{r}, a; \mathrm{k}_1, \lambda)] \quad (19)$$

$$E_{\theta}(\mathrm{r}, \tfrac{l}{a}) = E_{\theta}^{\mathrm{pr}}(\mathrm{r}, \tfrac{l}{a}) - \frac{\zeta_1 a}{4\pi \mathrm{k}_1 \mathrm{r}_0}\int_0^{\infty} \lambda^2 d\lambda J_1(\lambda l)[Q(\lambda)S(a, \mathrm{r}; \mathrm{k}_1, \lambda) + P(\lambda)T(\mathrm{r}, a; \mathrm{k}_1, \lambda)] \quad (20)$$

$$H_\varphi(r, \tfrac{l}{a}) = H_\varphi^{pr}(r, \tfrac{l}{a}) - \frac{ia}{4\pi k_1 r_0}\int_0^\infty \lambda^2 d\lambda J_1(\lambda l)[Q(\lambda)R(r, a; k_1, \lambda) + P(\lambda)S(r, a; k_1, \lambda)] \quad (21)$$

IV. CONCLUDING REMARKS

It is clear from eqs. (9) through (18) that in the flat Earth limit when $a \rightarrow \infty$ with h $= r-a$ fixed at a positive value, eqs. (19) through (21) are reduced to the Sommerfeld integrals [King & Smith, 1981] if E_r is identified with E_Z and E_θ is identified with E_ρ in the cylindrical coordinate system. Furthermore, the point of steepest descent for the asymptotic evaluation of the field integrals, eqs. (19) through (21), corresponds geometrically to the reflection point obtained by Fishback (1951) for calculating fields in the interference region.

The point of steepest descent, $\lambda = \lambda_0$, depends on range l. The radical β contains the combination λ^2/rr_0. Thus the ratio between $\Pi(r, l/a)$ and $\Lambda(r, l/a)$ is range dependent. Even in the asymptotic form, the fields obtained from this formulation are different from those under the Leontovich (surface impedance) approximation (Bremmer, 1958; Wait, 1956).

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