Asymptotic Solutions for Transmitted Gaussian Beam through a Plane Dielectric Interface

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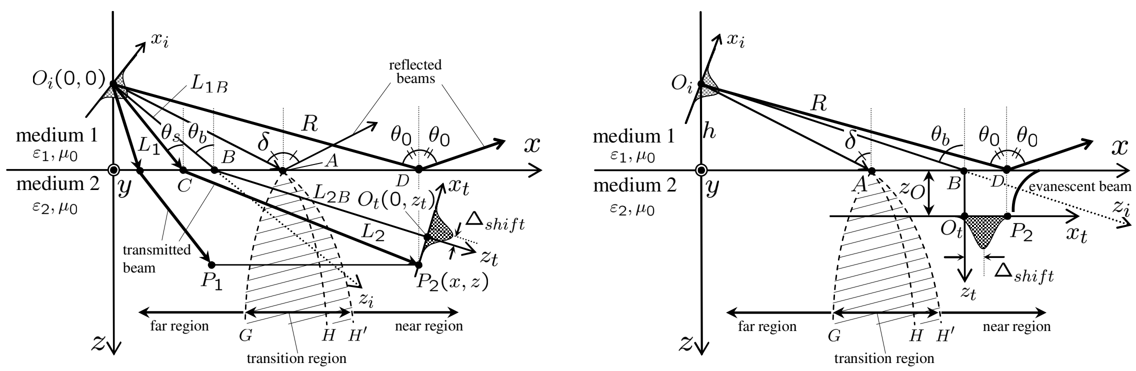
1. Introduction

The reflection and scattering (or the transmission and scattering) of the cylindrical, spherical, and beam waves by (or through) a plane dielectric interface and multiple media interfaces have been the important research subjects for many years and have been studied by many researchers [1]- [8]. Past studies have shown that the Gaussian beam that is incident from a denser medium and is reflected by a single and multiple media interfaces does not exactly follow the path of the geometrical optics [1]- [4], [7] and exhibits the nonspecular effect including lateral shift, longitudinal shift, and angular shift [4].

In this paper, we will derive the asymptotic solutions for the transmitted Gaussian beam through the plane dielectric interface. The validity and the applicable range of the asymptotic solutions are confirmed by comparing with the reference solution calculated numerically from the integral representation of the transmitted Gaussian beam. We will show that, like the reflected Gaussian beam [7], the beam shift is also appeared in the transmitted Gaussian beam.

2. Formulation and Integral Representation for Gaussian Beam Transmission

Fig.1(a) and Fig.1(b) show the transmission and scattering of the Gaussian beam through the plane dielectric interface, the Cartesian coordinate systems (x, y, z), and the beam coordinate systems (x_i, z_i) and (x_t, z_t) in the cases that the incident angle of Gaussian beam θ_b is smaller than the critical angle δ and θ_b is larger than the critical angle δ, respectively. We assume that the medium 1 (ε_1, μ_0) is denser than the medium 2 (ε_2, μ_0). We also assume that the incident Gaussian beam polarized in y-direction along



(a) $\theta_b \ll \delta$ (b) $\theta_b \gg \delta$

Figure 1: Schematic figures for transmission of Gaussian beam through a plane dielectric interface and coordinate systems (x, y, z), (x_i, z_i), and (x_t, z_t), δ: critical angle of total reflection $\theta = \delta$ $(\delta = \sin^{-1} n)$, and θ_b: incident angle of Gaussian beam.

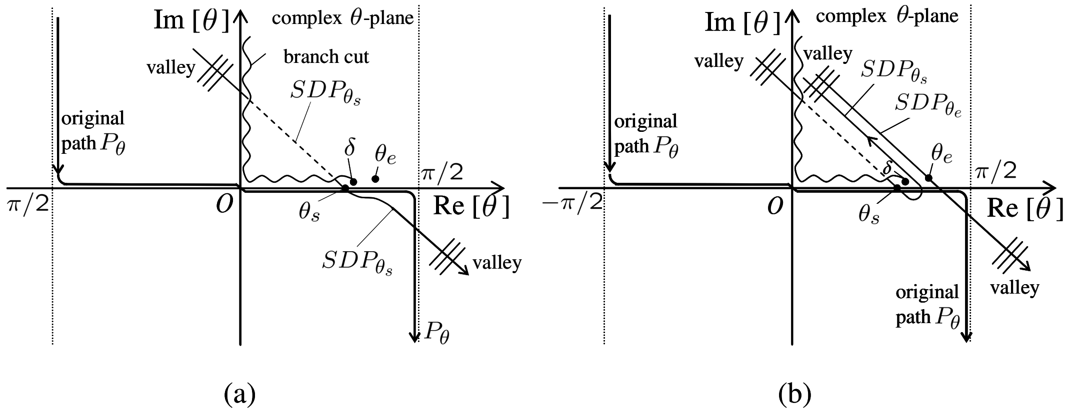


Figure 2: Saddle points θ_s and θ_e and steepest descent paths SDP_{θ_s} and SDP_{θ_e} passing through saddle point θ_s and θ_e in the complex θ-plane. (a): the observation point is located in the near region and (b): the observation point is located in the far region (see Figs.1(a) and 1(b)).

the aperture plane $z_i = 0$ in the (x_i, z_i) coordinate system is given by [2], [4], [7], [9]

$$U^i(x_i, z_i = 0) = \frac{1}{\sqrt{\pi}W} \exp\left\{-(x_i/W)^2\right\} \tag{1}$$

where $2W$ denotes the beam width of the Gaussian beam. Then, the transmitted Gaussian beam observed at the observation point $P_2(x, z)$ in the rarer medium 2 (ε_2, μ_0) can be represented by the following integral

$$U^t = \frac{k_1}{2\pi} \int_{P_\theta} T(\theta) B(\theta, \theta_b) e^{ik_1 q(\theta)} d\theta \tag{2}$$

where the beam function $B(\theta, \theta_b)$, the phase function $q(\theta)$, and the transmission coefficient $T(\theta)$ are defined as follows

$$B(\theta, \theta_b) = \exp\left[-\left\{\frac{k_1 W \sin(\theta - \theta_b)}{2}\right\}^2\right] \cos(\theta - \theta_b), \quad q(\theta) = R\cos(\theta - \theta_0) + z\sqrt{n^2 - \sin^2\theta} \tag{3}$$

$$T(\theta) = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \quad n = \sqrt{\varepsilon_1/\varepsilon_2} < 1 \tag{4}$$

Note that R, θ_0, and θ_b are defined geometrically in Figs.1(a) and 1(b) and that n denotes the refractive index. The original integration path P_θ in (2) is shown in the complex θ-plane in Figs.2(a) and 2(b).

3. Asymptotic Solutions for Transmitted Gaussian Beam

3.1 Incident Angle θ_b Sufficiently Smaller than Critical Angle δ ($\theta_b \ll \delta$)

In this section, we will derive the asymptotic solution for the transmitted Gaussian beam when the incident angle of the Gaussian beam θ_b is sufficiently smaller than the critical angle $\theta = \delta$, i.e., when $\theta_b \ll \delta$.

In this case, as shown in Fig.1(a), the observation point can be located in the near and far regions. Thus the transmitted Gaussian beam may be represented by [10], [11]

$$U^t = U^{go} + u_{AH} U^{eva} \tag{5}$$

where the integral representations of geometrically transmitted beam U^{go} and evanescent beam U^{eva} are given by

$$U^{go} = \frac{k_1}{2\pi} \int_{SDP_{\theta_s}} T(\theta) B(\theta, \theta_b) e^{ik_1 q(\theta)} d\theta, \quad U^{eva} = \frac{k_1}{2\pi} \int_{SDP_{\theta_e}} T(\theta) B(\theta, \theta_b) e^{ik_1 q(\theta)} d\theta \tag{6}$$

In the above integrals U^{go} and U^{eva}, integration paths SDP_{θ_s} and SDP_{θ_e} denote the steepest descent paths shown in Figs.2(a) and 2(b), respectively. In (5), the function u_{AH} denotes the unit step function defined by $u_{AH} = 0$ when the observation point is located in the region on the left of the dotted curve AH and $u_{AH} = 1$ when the observation point is located in the region on the right of the dotted curve AH (see Fig.1(a)). When the incident angle of Gaussian beam θ_b is sufficiently smaller than the critical angler δ, the observation point P_2 is located in the deep region (see Fig.1(a)). As we have shown in the papers [10], [11], the evanescent beam U^{eva} is sufficiently small in the deep region, since the evanescent beam U^{eva} decays exponentially rapidly in the z-direction. By applying the isolated saddle point technique, one may obtain the asymptotic solution for the transmitted Gaussian beam U^t as follows [10], [11]

$$U^t \doteqdot U^{go}, \quad U^{go} \sim T(\theta_s)B(\theta_s, \theta_b)\sqrt{\frac{k_1}{2\pi|q''(\theta_s)|}}e^{ik_1L_1+ik_2L_2-i\pi/4} \tag{7}$$

$$q''(\theta_s) = -\cos\theta_s\left\{R\cos\theta_0\sec^2\theta_s + z\frac{n^2\cos\theta_s}{(n^2-\sin^2\theta_s)^{3/2}}\right\} < 0 \tag{8}$$

The asymptotic solution for the geometrically transmitted beam U^{go} represents clearly the transmitted geometrical ray $O_i \to C \to P_2$ observed at P_2 (see Fig.1(a)). The geometrical parameters L_1, L_2, R, z, and θ_0 are defined in Fig.1(a).

3.2 Incident Angle θ_b Sufficiently Larger than Critical Angle δ ($\theta_b \gg \delta$)

In this section, we will derive the asymptotic solution for the transmitted Gaussian beam U^t when the incident angle of the Gaussian beam θ_b is sufficiently larger than the critical angle δ (see Fig.1(b)). In this case, the observation point is located in the far region and two saddle points θ_s and θ_e contribute to the integral (see Fig.2(b)). Since the saddle point θ_s is located near the branch point δ and far from the beam incident angle θ_b, the beam function $B(\theta_s, \theta_b)$ and therefore the geometrically transmitted beam U^{go} become sufficiently small. Thus, by applying the saddle point technique, one may obtain the transmitted Gaussian beam U^t expressed by using only the evanescent beam U^{eva} as follows [10], [11]

$$U^t \doteqdot U^{eva}, \quad U^{eva} \sim \sqrt{\frac{k_1}{2\pi R}}B(\theta_0, \theta_b)e^{ik_1R-i\pi/4}T(\theta_0)e^{-k_1z\sqrt{\sin^2\theta_0-n^2}}, \quad \theta_0 = \mathrm{Re}[\theta_e] \tag{9}$$

4. Numerical Results and Discussions

Fig.3(a) shows the results of the numerical calculation for the transmitted Gaussian beam when the incident angle of the Gaussian beam $\theta_b = 30°$ is sufficiently smaller than the critical angle of the total

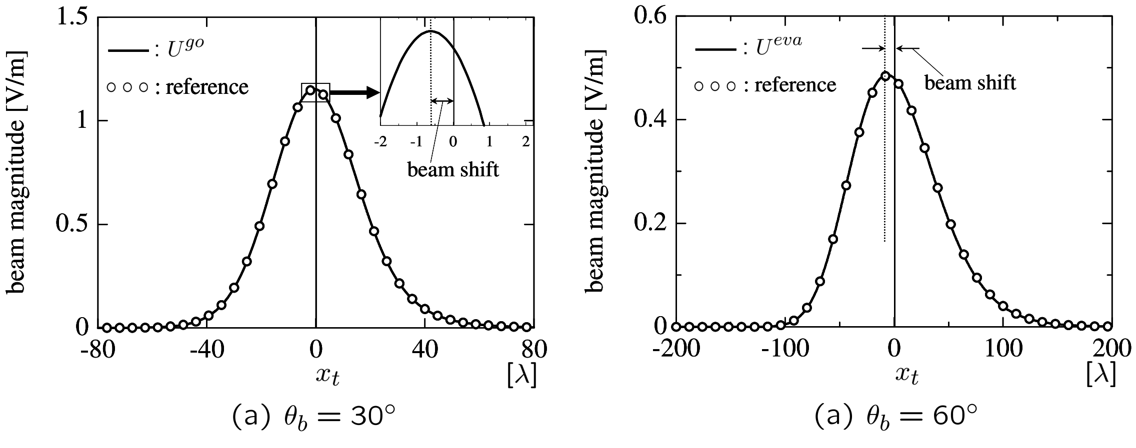


Figure 3: Comparisons of the asymptotic solutions for the transmitted Gaussian beam with the reference solutions.

reflection $\delta = 41.25°$. The numerical parameters are $W = 1.5\lambda$, f=3 GHz, $L_{1B} = 30\lambda$, $L_{2B} = 75\lambda$, $\varepsilon_1 = 2.3\varepsilon_0$, and $\varepsilon_2 = 1.0\varepsilon_0$. The beam magnitudes are calculated as the function of x_t, the beam coordinate $x_t[\lambda]$ (see Fig.1). It is clarified that geometrically transmitted beam U^{go} (——) obtained from (7) agrees excellently with the reference solution (∘ ∘ ∘) calculated numerically from (2).

Fig.3(b) shows the calculation results for the transmitted Gaussian beam when the incident angle of the Gaussian beam $\theta_b = 60°$ is sufficiently larger than the critical angle $\delta = 41.25°$. The numerical parameters are $W = 3\lambda$, $h = 200\lambda$, and $z_O = 0.1\lambda$. The beam magnitude has been calculated along the beam coordinate x_t (seeFig.1(b)). It is shown that the evanescent beam U^{eva} (——) in (9) agrees excellently with the reference solution (∘ ∘ ∘). It is noted that, in this case, only the evanescent beam can provide the excellent solution.

It is very interesting to observe that the maximum value position of the transmitted Gaussian beam is shifted from the beam axis $x_t = 0$. This beam shift Δ_{shift} is produced both in the near region (see Fig.3(a)) and in the far region (see Fig.3(b)).

5. Conclusion

We have derived the asymptotic solutions for the transmitted Gaussian beam observed in the rarer medium when the Gaussian beam is incident on a plane dielectric interface from the denser medium. We have confirmed the validity and the applicable range of the proposed asymptotic solutions by comparing with the reference solution calculated numerically. We have shown that, as observed in the reflection of the beam, the beam shift is also appeared in the transmitted beam.

Acknowledgments

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References

[1] H. K. V. Lotsch, Optik, vol. 32, no. 2, pp. 116-137, April 1970.

[2] B. R. Horowitz and T. Tamir, Appl. Phys., vol. 1, pp. 31-38, 1973.

[3] H. L. Bertoni, L. B. Felsen, and J. W. Ra, IEEE Trans. on Antennas and Propag., pp. 730-732, Sept. 1973.

[4] S. Kozaki and H. Sakurai, J. Opt Soc. Am., vol. 68, no. 4, pp. 508-514, April 1978.

[5] T. Tamir, J. Opt Soc. Am., vol. 3, no. 4, pp. 558-565, Apr. 1986.

[6] H. Shirai, IEICE, C-I, vol. J78-C-I, no. 3, pp. 125-133, March, 1995.

[7] H. Yamada, T. Kawano, K. Goto, and T. Ishihara, The Paper of Technical Meeting on Electromagnetic Theory, IEE Japan, EMT-07-40, pp. 25-30,May 2007.

[8] J. R. Thomas and A. Ishimaru, IEEE Trans. on Antennas and Propag., vol. 53, no. 5, pp. 1591-1599, May 2005.

[9] D. T. Quang, K. Goto, T. Kawano, and T. Ishihara, IEICE Trans. Electron., vol. E95-C, no. 1, pp. 16-26, Jan. 2012.

[10] D. T. Quang, K. Goto, T. Kawano, and T. Ishihara, Proc. of International Symposium on Antennas and Propagation (ISAP) 2011, ISBN: 978-89-5708-211-9 Jeju, Korea, Oct. 2011.

[11] D. T. Quang, K. Goto, T. Kawano, and T. Ishihara, Journal of the Korean Institute of Electromagnetic Engineering and Science, vol. 12, no. 1, pp. 45-54, Jan. 2012.