

Asymptotic Solutions of Transient Scattered Fields Excited by One of the Edges of a Curved Conducting Surface

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1. Introduction

By recent technological advances in the area of radar cross section, high resolution radar, and target identification, it is becoming important to study the asymptotic analysis methods for the frequency-domain (FD) and the time-domain (TD) electromagnetic scattered fields [1], [2]. When the high-frequency electromagnetic wave is incident on one of the edges or wedges of a curved conducting object, the whispering-gallery (WG) mode, the edge-surface diffracted ray (SDR), and the edge diffracted and reflected ray (EDR) (the edge diffracted ray (ED) is no reflection on the curved surface) are excited in the respective regions by the equivalent current at the edge [1]-[3].

In the present paper, by applying only the Fourier transform method proposed in [4], we derive in the unified manner the TD asymptotic solutions for the WG mode radiation field, the SDR, and the EDR comprising the transient scattered fields when the plane pulse wave is incident on one of the edges of a curved conducting surface. The validity of the TD asymptotic solution derived here is confirmed by comparing with the reference solution calculated numerically.

2. Formulation and Frequency-domain asymptotic solutions

We have shown in Fig. 1 the curved conducting surface (a, θ) defined by radius a and central angle θ and coordinates systems (x, y, z) and (r, ψ) . We have also shown in Fig. 1 the propagation paths along the m th order WG mode radiation field (WG m) (① and ②), the n th order edge-surface diffracted ray (SDR n) (③), and the edge diffracted and k times reflected ray (EDR k) (④ and ⑤). We assume that a plane wave $u_{in}(\omega) = \exp(i\omega x/c)$ propagates in the positive x -axis direction. Here, ω and c denote the angular frequency and the speed of light, respectively. The time factor $\exp(-i\omega t)$ is suppressed throughout this paper.

Each transient scattered field excited by the edge A may be represented by the following inverse Fourier transform of the product $u_i(\omega)S(\omega)$ of the FD scattered field element $u_i(\omega)$ and the frequency spectrum $S(\omega)$ of a pulse source function $s(t)$ [4]

$$y_i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u_i(\omega)S(\omega)\exp(-i\omega t)d\omega, \quad i = \text{WG}m, \text{SDR}n, \text{EDR}k. \quad (1)$$

The FD scattered field element $u_i(\omega)$, $i = \text{WG}m, \text{SDR}n, \text{EDR}k$, can be expressed as follows [5]

$$u_{\text{WG}m}(\omega) = U(P_1)u_{\text{GO}m}(\omega) + u_{\text{ED}m}(\omega), \quad U(P_1) = 1: \text{lit region}, \quad U(P_1) = 0: \text{shadow region}, \quad (2)$$

$$u_{\text{GO}m}(\omega) = A_{\text{GO}m}(\omega) \exp \left[i\omega \left(\frac{L_{Q_0A}}{c} + \frac{L_{\text{GO}m}(\omega)}{c} \right) \right], \quad (3)$$

$$u_{\text{ED}m}(\omega) = A_{\text{ED}m}(\omega) \exp \left[i\omega \left(\frac{L_{Q_0A} + L_{\text{BP}_1}}{c} + \frac{L_{\text{ED}m}(\omega)}{c} \right) \right], \quad (4)$$

$$u_{\text{SDR}n}(\omega) = A_{\text{SDR}n}(\omega) \exp \left[i\omega \left(\frac{L_{Q_0A} + L_{A_1P_2}}{c} + \frac{L_{\text{CW}n}}{v_{p, \text{CW}n}(\omega)} \right) \right], \quad v_{p, \text{CW}n}(\omega) = \frac{c}{c_n(\omega)}, \quad (5)$$

$$u_{\text{EDR}k}(\omega) = A_{\text{EDR}k}(\omega) \exp \left[i\omega \left(\frac{L_{\text{Q}_0\text{A}} + L_{\text{RGO}k}}{c} \right) \right]. \quad (6)$$

Here, $A_i(\omega)$, $i = \text{GOM}, \text{EDM}, \text{SDR}n, \text{EDR}k$, denotes the amplitude term of each FD scattered field which varies slowly as the function of the angular frequency ω . Notations $L_{\text{Q}_0\text{A}}$, L_{BP_1} , $L_{\text{A}_1\text{P}_2}$, and $L_{\text{RGO}k}$ in the phase terms express the propagation distances along the propagation paths $\text{Q}_0 \rightarrow \text{A}$, $\text{B} \rightarrow \text{P}_1$, $\text{A}_1 \rightarrow \text{P}_2$, and $\text{A} \rightarrow \text{R}_1 \rightarrow \text{P}_3$ ($k=1$). While, $L_{\text{GOM}}(\omega)$ ($L_{\text{EDM}}(\omega)$) is a propagation distance along the path $\text{A} \rightarrow \text{Q}_1 \curvearrowright \text{Q}_2 \rightarrow \text{R} \rightarrow \text{P}_1$ ($\text{A} \rightarrow \text{Q}_1 \curvearrowright \text{Q}_3 \rightarrow \text{B}$) including the m th order modal caustic $\text{Q}_1 \curvearrowright \text{Q}_2$ (Q_3) which changes as the function of ω and $L_{\text{CW}n}$ is a propagation distance of the n th order creeping wave (CW n) along the path $\text{A} \curvearrowright \text{A}_1$ (see Fig. 1).

Applying the following transformations in (7), (8), and (9) to (3), these results are readily convertible into (4), (5), and (6), respectively.

$$A_{\text{GOM}}(\omega) \rightarrow A_{\text{EDM}}(\omega), \quad L_{\text{Q}_0\text{A}} \rightarrow L_{\text{Q}_0\text{A}} + L_{\text{BP}_1}, \quad L_{\text{GOM}}(\omega)/c \rightarrow L_{\text{EDM}}(\omega)/c, \quad (7)$$

$$A_{\text{GOM}}(\omega) \rightarrow A_{\text{SDR}n}(\omega), \quad L_{\text{Q}_0\text{A}} \rightarrow L_{\text{Q}_0\text{A}} + L_{\text{A}_1\text{P}_2}, \quad L_{\text{GOM}}(\omega)/c \rightarrow L_{\text{CW}n} c_n(\omega)/c, \quad (8)$$

$$A_{\text{GOM}}(\omega) \rightarrow A_{\text{RGO}k}(\omega), \quad L_{\text{Q}_0\text{A}} \rightarrow L_{\text{Q}_0\text{A}} + L_{\text{RGO}k}, \quad L_{\text{GOM}}(\omega)/c \rightarrow 0, \quad (9)$$

We assume the truncated pulse source modulated by $p(t)$ as follows

$$s(t) = p(t) \exp\{-i\omega_0(t - t_0)\} \text{ for } 0 \leq t \leq 2t_0, \quad s(t) = 0 : \text{ elsewhere}, \quad (10)$$

where ω_0 denotes the central angle frequency and t_0 the constant parameter. Denoting the Fourier transform of $s(t)$ and $p(t)$ by $S(\omega)$ and $P(\omega)$, respectively, the Fourier transform of $s(t)$ can be represented by

$$S(\omega) = P(\omega - \omega_0) \exp(i\omega_0 t_0), \quad P(\omega - \omega_0) = \int_{-\infty}^{\infty} p(t) \exp[i(\omega - \omega_0)t] dt. \quad (11)$$

3. Asymptotic Analyses for transient Scattered Fields

In this section, we will derive in the unified manner the TD asymptotic solutions for the transient scattered fields by applying the Fourier transform method to the integral $y_i(t)$ in (1).

First, we derive the TD asymptotic solution for the m th order geometrical ray $y_{\text{GO},m}(t)$ comprising the transient WG m [5]. Substituting $u_{\text{GOM}}(\omega)$ in (3) and $S(\omega)$ in (11) into (1), $y_{\text{GO},m}(t)$ can be represented as follows

$$y_{\text{GOM}}(t) = \frac{\exp(-i\omega_0 t)}{2\pi} \int_{-\infty}^{\infty} A_{\text{GOM}}(\omega) P(\omega - \omega_0) \exp[-i\{\omega t - h_{\text{GOM}}(\omega)\}] d\omega, \quad (12)$$

$$h_{\text{GOM}}(\omega) = \omega \left(\frac{L_{\text{Q}_0\text{A}}}{c} + \frac{L_{\text{GOM}}(\omega)}{c} \right). \quad (13)$$

Here, we evaluate the inverse Fourier transform in (12) by utilizing the approximations and the definition of the Fourier transform.

When the amplitude function $A_{\text{GOM},m}(\omega)$ and the phase function $h_{\text{GOM},m}(\omega)$ are approximated near the central angular frequency $\omega = \omega_0$ by

$$A_{\text{GOM}}(\omega) \sim A_{\text{GOM}}(\omega_0), \quad h_{\text{GOM}}(\omega) \sim h_{\text{GOM}}(\omega_0) + (\omega - \omega_0) h'_{\text{GOM}}(\omega_0), \quad (14)$$

the integral $y_{\text{GOM}}(t)$ in (12) is reduced and then utilizing the definition that the inverse Fourier transform of $P(\omega)$ is $p(t)$, one can obtain the following TD asymptotic solution.

$$y_{\text{GOM}}(t) = A_{\text{GOM}}(\omega_0) p \left(t - \frac{L_{\text{Q}_0\text{A}}}{c} - \frac{L_{\text{GOM}}(\omega_0)}{v_{g,\text{GOM}}} \right) \exp \left[-i\omega_0 \left(t - t_0 - \frac{L_{\text{Q}_0\text{A}}}{c} - \frac{L_{\text{GOM}}(\omega_0)}{c} \right) \right], \quad (15)$$

$$v_{g,\text{GOM}} = \frac{c}{1 + \omega_0 L'_{\text{GOM}}(\omega_0)/L_{\text{GOM}}(\omega_0)} < c, \quad L'_{\text{GOM}}(\omega_0) (> 0). \quad (16)$$

The total propagation distance of the $y_{GO,m}(t)$ in (15) is $L_{Q_0A} + L_{GO,m}(\omega_0)$ (see \bigcirc in Fig. 1). It is shown that the incident pulse wave element with the distance L_{Q_0A} ($= Q_0 \rightarrow A$) propagates with the phase and the group velocity equal to the speed of light c . While, the GO ray element along the distance $L_{GO,m}(\omega_0)$ ($= A \rightarrow Q_1 \rightarrow Q_2 \rightarrow R \rightarrow P_1$) decided by the angular frequency ω_0 propagates with the phase velocity c and the group velocity $v_{g,GO,m}$ in (16) lower than c .

Next, we derive the asymptotic solution for the transient m th order edge diffracted ray $y_{ED,m}(t)$ comprising the transient WGm . By substituting the transformation in (7) into (15) associated with (16), one may obtain the following TD asymptotic solution for the $y_{ED,m}(t)$

$$y_{ED,m}(t) = A_{ED,m}(\omega_0) p \left(t - \frac{L_{Q_0A} + L_{BP_1}}{c} - \frac{L_{ED,m}(\omega_0)}{v_{g,ED,m}} \right) \cdot \exp \left[-i\omega_0 \left(t - t_0 - \frac{L_{Q_0A} + L_{BP_1}}{c} - \frac{L_{ED,m}(\omega_0)}{c} \right) \right], \quad v_{g,ED,m} = \frac{c}{1 + \omega_0 \frac{L'_{ED,m}(\omega_0)}{L_{ED,m}(\omega_0)}} < c. \quad (17)$$

The TD asymptotic solution $y_{WGm}(t)$ for the transient WGm can be obtained from the summation of the $y_{GO,m}(t)$ in (15) and the $y_{ED,m}(t)$ in (17).

$$y_{WGm}(t) = U(P_1) y_{GO,m}(t) + y_{ED,m}(t), \quad U(P_1) = 1: \text{lit region}, \quad U(P_1) = 0: \text{shadow region}. \quad (18)$$

In the same manner, the TD asymptotic solutions $y_{SDR,n}(t)$ and $y_{EDR,k}(t)$ for the transient $SDRn$ and the transient $EDRk$ may be obtained respectively by substituting the transformation in (8) and (9) into (15) associated with (16). The explicit asymptotic solutions are described in [5].

4. Numerical Results and Discussions

In order to confirm the validity of the TD asymptotic solutions derived in Section 3, we have calculated the transient scattered field elements excited by the edge A of a curved conducting surface illuminated by the incident magnetic-type plane pulse wave (see Fig. 1).

Figure 2 shows the response waveform of the 1st ($m=1$) order transient WG mode radiation field $\text{Re}[y_{WG1}(t)]$ vs. time curves observed at P_1 in the lit region (see Fig. 1). The solid curve (—) is obtained from (18) associated with (15) and (17) and the closed circles (\bullet) are calculated numerically by applying the fast Fourier transform (FFT) numerical code into the inverse Fourier transform $y_{WG1}(t)$ in (1). The numerical solution (\bullet) serves as the reference solution. It is observed that the asymptotic solution agrees excellently with the numerical reference solution. It is confirmed numerically that the group velocity $v_{g,GO,1}$ ($v_{g,ED,1}$) along the propagation distance $L_{GO,1}(\omega_0)$ ($L_{ED,1}(\omega_0)$) constituting transient $y_{GO,1}(t)$ ($y_{ED,1}(t)$) is $v_{g,GO,1} = 0.997c$ ($v_{g,ED,1} = 0.979c$).

Figure 3 and figure 4 show respectively the response waveform $\text{Re}[y_{SDR1}(t)]$ of the transient 1st ($n=1$) order edge-surface diffracted ray (SDR_1) and the response waveform $\text{Re}[y_{EDRk}(t)]$ of the transient edge diffracted and reflected rays (EDR_k) with $k=0,1$. It is observed that the asymptotic solutions (—) agree excellently with the numerical reference solutions (\bullet and \circ).

5. Conclusion

We have derived, by applying only the Fourier transform method, the time-domain (TD) asymptotic solutions for the transient scattered field elements excited by one of the edges of a curved conducting surface. The accuracy of the TD asymptotic solutions derived in this paper has been confirmed by comparing with the numerical reference solution.

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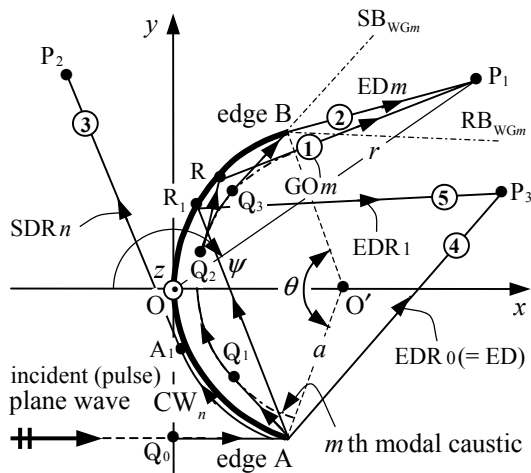


Figure 1 Curved conducting surface (a, θ) with two edges A and B, and coordinate systems (x, y, z) and (r, ψ). Also shown are propagation paths of scattered fields (①~⑤) excited by the edge A.

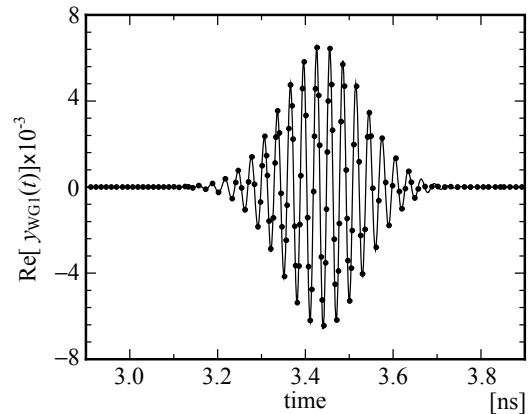


Figure 2 Response waveform of transient 1st order ($m=1$) WG mode radiation field $y_{WG1}(t)$. —: asymptotic solution in (27), $\bullet\bullet\bullet$: reference solution calculated numerically from (9). Pulse source $s(t)$ with $p(t) = \exp[-(t-t_0)^2/(4d^2)]$ used in the calculation. Numerical parameters: (a, θ) = (10.1×10^{-2} m, 99.6°), $P(r, \psi) = (0.8$ m, $130.0^\circ)$, $\omega_0 = 2.105 \times 10^{11}$ rad/s, $t_0 = 3.50 \times 10^{-10}$ s, $d = 6.60 \times 10^{-11}$ rad/s, fractional bandwidth: 21.8%.

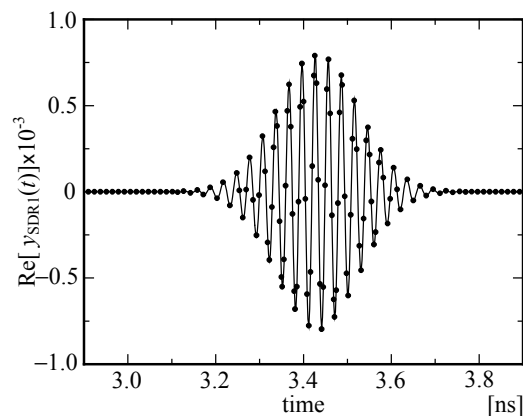


Figure 3 Response waveform of transient 1st order ($n=1$) edge-surface diffracted ray $y_{SDR1}(t)$. —: asymptotic solution, $\bullet\bullet\bullet$: reference solution. Numerical parameters: same as those in Fig. 2 except for $P(r, \psi) = (0.8$ m, $100.0^\circ)$.

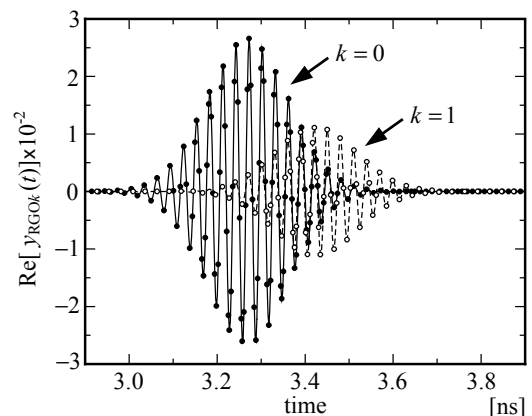


Figure 4 Response waveforms of transient edge diffracted and reflected ray $y_{EDRk}(t)$ ($k=0, 1$). —: asymptotic solution in (37), $\bullet\bullet\bullet$: reference solution calculated numerically from (9). Numerical parameters: same as those in Fig. 3.