Proceedings of ISAP'04, Sendai, JAPAN

ACCURACY IMPROVEMENT IN FDTD ANALYSIS OF DIPOLE ANTENNA WITH END CAPS

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1. Introduction

The Finite Difference Time Domain (FDTD) method [1],[2] are widely used for the various electromagnetic problems. This method is efficient for modeling the electromagnetic fields in complex geometries within a practical level of the accuracy. However, when the highly accurate result is required, a great amount of computer resource is often needed even if the geometry is relatively simple. In order to deal with this difficulty, for example, a thin wire structure, the approach using subcellular technique [3] has been introduced. In this technique, the electric and magnetic fields near the wire are assumed to vary as $1/\rho$, where ρ is the radial distance from the wire axis. This technique is effective for the scattering analysis of an infinitely long cylinder, but is not for the finite-length antenna problems.

For this problem, we previously introduced a technique using a quasi-static approximation to modify the FDTD update equations [4]. The effectiveness of the approach was confirmed in the input impedance calculation of both short and long dipole antenna since the quasi-static field is considered dominant near the antenna conductor.

On the other hand, it has been indicated that the modeling of the end caps of dipole antenna affects the improvement of the accuracy as well [5],[6]. The approach including the effect of charge accumulation at the wire-end caps to the FDTD method has been introduced. However, the sufficiently accurate input impedance has not been obtained.

In this paper, the quasi-static approximation technique proposed in [4] is extended to the current distribution on the whole dipole antenna including the end caps. The new modified FDTD update equations are derived and the input impedances obtained by this proposed method and by the $1/\rho$-Subcell method [3] are compared with the result of the Method of Moment (MoM) [7].

2. Quasi-static approximation

In this technique, we consider a dipole antenna consists of a perfectly conducting cylinder whose radius a is very small compared with the length, and fed by a delta-gap voltage, as shown in Fig. 1. The quasi-static approximation of the current distribution along the length of this dipole antenna and on the end caps can be given by

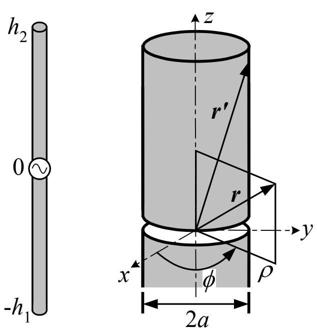

Fig. 1 Dipole antenna models.

$$J_z(z)=\begin{cases}\dfrac{I}{2\pi a}\left(1+\dfrac{z}{h_1+a}\right), & -h_1\le z\le 0\\[2ex] \dfrac{I}{2\pi a}\left(1-\dfrac{z}{h_2+a}\right), & 0\le z\le h_2\end{cases} \tag{1}$$

$$J_\rho(\rho)=\begin{cases}\dfrac{I}{2\pi}\left(\dfrac{1}{h_1+a}\right)\cdot\dfrac{\rho}{a}, & z=-h_1\\[2ex] -\dfrac{I}{2\pi}\left(\dfrac{1}{h_2+a}\right)\cdot\dfrac{\rho}{a}, & z=h_2\end{cases} \tag{2}$$

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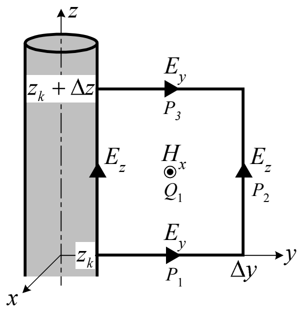


Fig. 2 FDTD cell of electric fields.

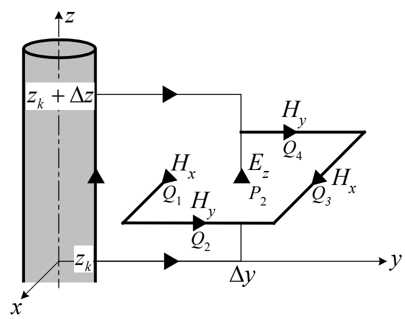


Fig. 3 FDTD cell of magnetic fields.

Therefore, the quasi-static electric and magnetic fields can be derived from a static vector potential as

$$\boldsymbol{E}(\boldsymbol{r})=\frac{1}{j\omega\mu_0\varepsilon_0}\nabla\nabla\cdot\boldsymbol{A}(\boldsymbol{r})-j\omega\boldsymbol{A}(\boldsymbol{r})\approx\frac{1}{j\omega\mu_0\varepsilon_0}\left\{\frac{\partial F(\boldsymbol{r})}{\partial x}\hat{x}+\frac{\partial F(\boldsymbol{r})}{\partial y}\hat{y}+\frac{\partial F(\boldsymbol{r})}{\partial z}\hat{z}\right\} \tag{3}$$

$$\boldsymbol{H}(\boldsymbol{r})=\frac{1}{\mu_0}\nabla\times\boldsymbol{A}(\boldsymbol{r})=\frac{1}{\mu_0}\left[\left\{\frac{\partial A_z(\boldsymbol{r})}{\partial y}-\frac{\partial A_y(\boldsymbol{r})}{\partial z}\right\}\hat{x}+\left\{\frac{\partial A_x(\boldsymbol{r})}{\partial z}-\frac{\partial A_z(\boldsymbol{r})}{\partial x}\right\}\hat{y}\right] \tag{4}$$

where,

$$\boldsymbol{A}(\boldsymbol{r})=\frac{\mu_0}{4\pi}\int_V\frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}dv' \tag{5}$$

$$F(\boldsymbol{r})=\frac{\partial A_x(\boldsymbol{r})}{\partial x}+\frac{\partial A_y(\boldsymbol{r})}{\partial y}+\frac{\partial A_z(\boldsymbol{r})}{\partial z} \tag{6}$$

By substituting the current distribution (1) and (2) into (5), then (3) and (4) yield the quasi-static expressions of both electric and magnetic fields of dipole antenna.

3. Modification of the FDTD update equations

In this section, we show how the quasi-static approximation is incorporated into the FDTD update equation.

3.1 Magnetic field

In this method, an integral form of the Faraday's law

$$\oint_C\boldsymbol{E}(\boldsymbol{s},t)\cdot d\boldsymbol{s}=-\mu_0\frac{\partial}{\partial t}\int_S\boldsymbol{H}(\boldsymbol{r},t)\cdot\boldsymbol{n}dS \tag{7}$$

is applied to the FDTD electric cell near the antenna conductor and end caps as shown in Fig. 2. Therefore, since Δy must be considerably smaller than wavelength, the electric and magnetic fields in the cell should be dominated by the quasi-static field, and the spatial dependence of the fields can be approximated as

$$\begin{cases}E_y(\boldsymbol{r},t)=E_y(P_i,t)\dfrac{\partial F(\boldsymbol{r})/\partial y}{\partial F(\boldsymbol{r})/\partial y\big|_{\boldsymbol{r}=P_i}}, & i=1,3\\[2ex] E_z(\boldsymbol{r},t)=E_z(P_2,t)\dfrac{\partial F(\boldsymbol{r})/\partial z}{\partial F(\boldsymbol{r})/\partial z\big|_{\boldsymbol{r}=P_2}}\end{cases} \tag{8}$$

along the contour, and

$$H_x(\boldsymbol{r},t)=H_x(Q_1,t)\frac{\partial A_z(\boldsymbol{r})/\partial y-\partial A_y(\boldsymbol{r})/\partial z}{\left\{\partial A_z(\boldsymbol{r})/\partial y-\partial A_y(\boldsymbol{r})/\partial z\right\}\big|_{\boldsymbol{r}=Q_1}} \tag{9}$$

within the cell, where the $E_y(P_i,t)$, $E_z(P_2,t)$, and $H_x(Q_1,t)$ are electric and magnetic fields located on the FDTD cell edges.

As shown in [4], by substituting these electric and magnetic field approximations into an integral form of the Faraday's law and applying $t = n\Delta t$, the modified FDTD update equation of the magnetic field can be derived. Furthermore, the update equations for the $H_y(\boldsymbol{r},t)$ can be derived by the same procedure as well.

3.2 Electric field

An integral form of the Ampere's law given by

$$\oint_C \boldsymbol{H}(\boldsymbol{s},t)\cdot d\boldsymbol{s} = \frac{\partial}{\partial t}\int_S \varepsilon \boldsymbol{E}(\boldsymbol{r},t)\cdot \boldsymbol{n}dS \tag{10}$$

is applied to the FDTD magnetic cell near the antenna conductor and end caps as shown in Fig. 3. Again, the quasi-static fields are assumed dominant in this region. Therefore, the spatial dependence of the fields are approximated as

$$\begin{cases} H_x(\boldsymbol{r},t) = H_x(Q_i,t)\dfrac{\partial A_z(\boldsymbol{r})/\partial y - \partial A_y(\boldsymbol{r})/\partial z}{\left\{\partial A_z(\boldsymbol{r})/\partial y - \partial A_y(\boldsymbol{r})/\partial z\right\}\Big|_{\boldsymbol{r}=Q_i}}, & i = 1,3 \\ H_y(\boldsymbol{r},t) = H_y(Q_i,t)\dfrac{\partial A_x(\boldsymbol{r})/\partial z - \partial A_z(\boldsymbol{r})/\partial x}{\left\{\partial A_x(\boldsymbol{r})/\partial z - \partial A_z(\boldsymbol{r})/\partial x\right\}\Big|_{\boldsymbol{r}=Q_i}}, & i = 2,4 \end{cases} \tag{11}$$

along the contour, and the electric field within the contour is approximated as given in (8).

Also, by substituting these field approximations into an integral form of the Ampere's law and applying $t = (n\text{-}1/2)\Delta t$, the modified FDTD update equation of the electric field can be derived. The update equations for $E_x(\boldsymbol{r},t)$ and $E_y(\boldsymbol{r},t)$ can also be derived by the same procedure.

3.3 Feeding current

As described in [4], since the feeding current calculated in conventional FDTD analysis by the Ampere's Circuital law

$$\begin{aligned} I(t) &= \oint_C \boldsymbol{H}\cdot dl \\ &= \left\{H_x(0,\tfrac{\Delta y}{2},0,t) - H_x(0,-\tfrac{\Delta y}{2},0,t)\right\}\Delta x - \left\{H_y(\tfrac{\Delta x}{2},0,0,t) - H_y(-\tfrac{\Delta x}{2},0,0,t)\right\}\Delta y \end{aligned} \tag{12}$$

contains the displacement current flowing through the closed contour, the contour is then shrunk to the surface of the antenna conductor in order to eliminate the error caused from the displacement current. Therefore, the feeding current of a dipole antenna is obtained by

$$I(t) = 2\pi a H_x(0,a_+,0,t) \tag{13}$$

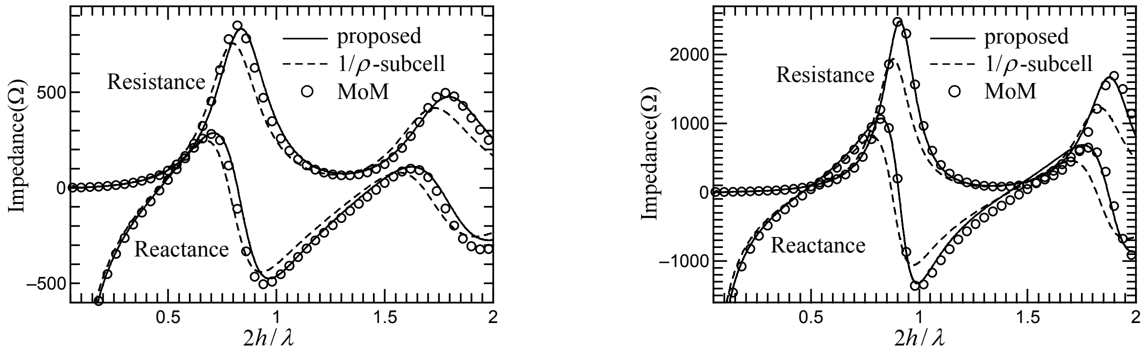
where $H_x(0,a_+,0,t)$ is the magnetic field at the outer surface.

4. Numerical results

In this section, a numerical example is shown to verify the accuracy of the proposed quasi-static approximation technique. A dipole antenna with the length $h_1 = h_2 = h = 75.0mm$ as shown in Fig. 1 is used. The ratio of the length of antenna ($2h$) and cell size is set to 33. The Gauss pulse $e^{-\alpha(t-t_0)^2}$ is applied as an input voltage. The results analyzed by this method are shown comparing with those of the $1/\rho$-Subcell method and the 9-segment Method of Moment.

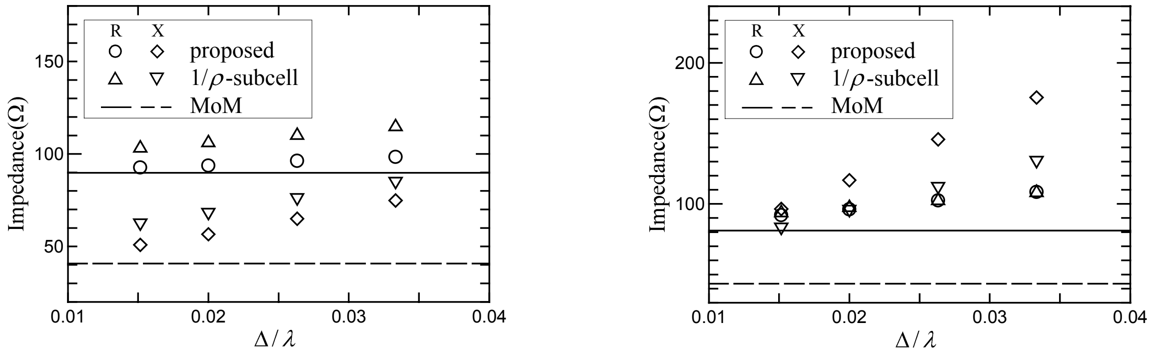
The input impedances of a thick dipole antenna with h/a = 71.3 are shown in Fig. 4(a), and those of a thin dipole antenna with h/a = 904 are shown in Fig. 4(b). The results show that this proposed method of including the quasi-static approximation of the current distribution on the end caps can achieve higher accuracy on both thick and thin dipole antenna, and agree well with the results of the MoM.

Moreover, the input impedance of half-wavelength dipole antenna analyzed with various cell size are shown in Fig. 5. For a thick dipole antenna with h/a = 71.3 shown in Fig. 5(a), the results of the proposed method converge to the MoM with higher accuracy than the $1/\rho$-Subcell method, while



(a) h/a = 71.3 (b) h/a = 904

Fig. 4 Input impedances of a dipole antenna.



(a) h/a = 71.3 (b) h/a = 904

Fig. 5 Input impedances of a half-wavelength dipole antenna as a function of cell size.

for a thin dipole antenna with h/a = 904 shown in Fig. 5(b), the results of the proposed method and the $1/\rho$-Subcell method give the same level of accuracy.

5. Conclusion

This paper has introduced the method for improving the accuracy of a dipole antenna analysis. This proposed method included the quasi-static approximation of the current distribution on the end caps of dipole antenna with the cylindrical surface. The field approximations derived by utilizing these current distribution approximations are incorporated into the Faraday's and Ampere's law contour-path integral formulation of the FDTD method, and the new modified FDTD update equations are derived. The numerical results indicated that our proposed method can achieve higher accuracy for both thick and thin dipole antenna.

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