ELECTROMAGNETIC SCATTERING FROM A CASCADE CONNECTION OF STRIP GRATINGS

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The cascade connection of periodic strip gratings is one of the typical artificial dielectric structures [1]. We analyze the scattering of obliquely incident plane waves by this geometry. A set of singular integral equations (SIE) is derived and it is solved by the moment method (MM). Some numerical results are presented.

Fig. 1 shows the cross section of the geometry. Strips with negligible thickness and infinite length are allocated with the common period D. The incident angles θ_0 and ϕ_0 are shown in Fig. 2. The angles $\psi = 0$ and $\pi/2$ correspond to ϕ- and θ-polarizations, respectively. The time factor $\exp(i\omega t)$ will be suppressed.

The fields are derived by $\mathbb{E} = \nabla\nabla\cdot \mathbb{\Pi} + k^2 \mathbb{\Pi}$ and $\mathbb{H} = i\omega\varepsilon \nabla\times \mathbb{\Pi}$, where the electric Hertzian potential $\mathbb{\Pi}$ is represented by

$$\mathbb{\Pi} = \mathbb{\Pi}^i + \sum_{q=1}^{P} \mathbb{\Pi}^{(q)} \tag{1}$$

where $\mathbb{\Pi}^i$ and $\mathbb{\Pi}^{(q)}$ are the contributions from the incident field and the scattered field from the q-th plane, respectively;

$$\mathbb{\Pi}^i = k^{-2} \mathbb{A}^i \Phi_0(x,y,z), \qquad \mathbb{\Pi}^{(q)} = k^{-2} \sum_{n=-\infty}^{\infty} \mathbb{A}_n^{(q)} \Phi_n(x,y,-|z+h_q|). \tag{2}$$

The modal function (Floquet harmonics) which satisfies the Helmholtz equation and the periodicity condition is given by

$$\Phi_n(x,y,z) = \exp[i(\alpha_n x + \beta y) + \Gamma_n z], \tag{3}$$

where $\alpha_n = 2n\pi/D + k \sin\theta_0 \cos\phi_0$, $\beta = k \sin\theta_0 \sin\phi_0$, $\Gamma_n = (\alpha_n^2 + \beta^2 - k^2)^{1/2}$ $(Im(\Gamma_n) \geqq 0)$, and $k = \omega\sqrt{\varepsilon\mu} = 2\pi/\lambda$. For convenience we will often use the vector notations for coefficients and functions, and specify their scalar components by subscripts.

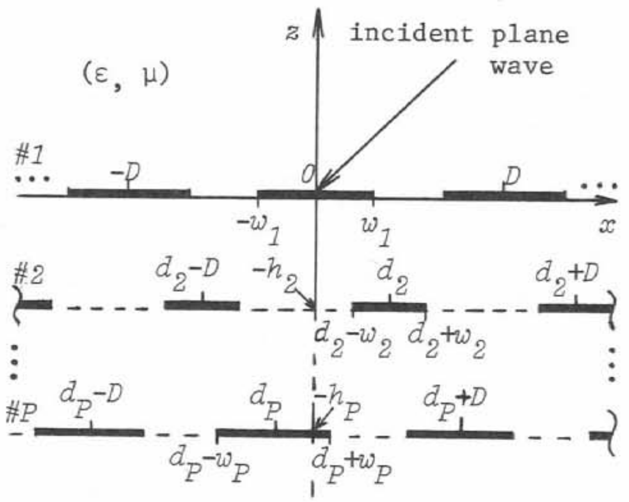

Fig. 1 Geometry

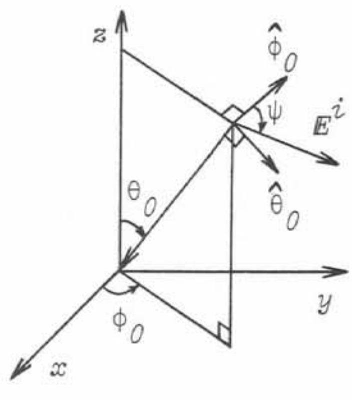

Fig. 2 Incident field

Owing to the orthogonality of the modal functions over one period, we can express the unknown coefficients $\mathbb{A}_n^{(q)}$ as

$$\begin{pmatrix} A_{x,n}^{(q)} \\ A_{y,n}^{(q)} \end{pmatrix} = \frac{\Delta_q}{\pi\Gamma_n D} \int_{-1}^{1} \begin{pmatrix} F_x^{(q)}(t')(i2n\pi)^{-1} \\ F_y^{(q)}(t') \end{pmatrix} e^{-i2n(\pi d_q/D + t'\Delta_q)}\, dt' \quad (4)$$

where $\Delta_q = \pi w_q/D$, and the change of variable $x' = d_q + w_q t'$ is introduced. The unknown functions are related to the surface current density

$$\mathbb{J}^{(q)}(x) = \hat{z} \times \mathbb{H} \Big|_{z=-h_q-0}^{z=-h_q+0} e^{-i\beta y}, \quad (5)$$

$$\begin{pmatrix} F_x^{(q)}(t') \\ F_y^{(q)}(t') \end{pmatrix} = \frac{kDZ}{i2} \begin{pmatrix} D(\mathrm{d}/\mathrm{d}x')[\, J_x^{(q)}(x') \exp(-i\alpha_0 x)] \\ J_y^{(q)}(x') \exp(-i\alpha_0 x) \end{pmatrix}, \quad (6)$$

where Z $(= \sqrt{\mu/\varepsilon})$ is the intrinsic impedance of the space. Substituting (4) into the boundary condition $\hat{z} \times \mathbb{E}\big|_{\mathrm{strip}} = 0$, we obtain the set of conventional integral equations (CIE) for the unknown functions (6). Since the direct application of the moment method often leads us to wrong results [2], we change CIE to the singular integral equations (SIE) before the numerical treatment. The set of SIE is derived by the following steps : A) Acceleration of the convergence of the infinite series included in the kernels ; B) Use of the formulas on infinite series to extract the singularities of the Cauchy type and the logarithmic type. We thus obtain

$$\int_{-1}^{1} \begin{pmatrix} \pi^{-1}[(t'-t)^{-1} + K_x^{(p)}(t,t')] & 0 \\ 0 & -\pi^{-2}\Delta_p(\log|t-t'| + K_y^{(p)}(t,t')) \end{pmatrix}$$

$$\cdot \begin{pmatrix} 1 & i\beta D \\ i\beta D & (k^2-\beta^2)D^2 \end{pmatrix} \begin{pmatrix} F_x^{(p)}(t') \\ F_y^{(p)}(t') \end{pmatrix} dt' = \begin{pmatrix} G_x^{(p)}(t) \\ G_y^{(p)}(t) \end{pmatrix},$$

$$p = 1, 2, \cdots, P; \quad -1 < t < 1, \quad (7)$$

where the bounded kernels are

$$K_x^{(p)}(t,t') = \Delta_p \cot [(t'-t)\Delta_p] - (t'-t)^{-1}, \quad (8)$$

$$K_y^{(p)}(t,t') = \log [2 \sin (|t-t'|\Delta_p) \,/\, |t-t'|]. \quad (9)$$

The right hand side of (7) is composed of the terms related to the incident field and the multiple scattering among the planes.

The set of SIE (7) is solved by the moment method (MM) [3]. First we expand the unknown functions in terms of the Chebyshev polynomials of the first kind $T_n(t') = \cos (n \arccos t')$:

$$\mathbb{F}^{(p)}(t') \cong \sum_{n=0}^{N} \mathbb{f}_n^{(p)} T_n(t') \, (1 - t'^2)^{-1/2}, \quad p = 1, 2, \cdots, P. \quad (10)$$

The above form exactly correspond to the edge condition. The next step is the choice of the testing functions. Taking the characteristics of the singular kernels into account, we find that the weighted Chebyshev polynomials $T_m(t)(1 - t^2)^{-1/2}$ and $U_{m-1}(t)(1 - t^2)^{1/2}$ are the appropriate testing functions, where $U_{m-1}(t) = \sin[\ m \arccos t] / \sin \arccos t$ is the second kind polynomial. We thus obtain the set of linear equations for the unknown coefficients $f^{(p)}$. The elements of the operator matrix include the infinite series related to the Floquet harmonics. In the numerical calculations, the lowest $2L + 1$ (from $-L$ to $+L$) harmonics are retained, where L is chosen to be $3N$.

To demonstrate the accuracy of the SIE-MM solution, we present the convergence of the total transmitted powers for a single grating in Fig. 3. The incident power is normalized to be unity. In the case where $P = 2$, the distance between two planes is set equal to zero and the effect of the interaction is strongest. This is why the curve for $P = 2$ is a little oscillatory. However the both curves approach 67.485 [%] when N is increased, which is the accurate value presented in ref. [4].

Figs. 4 and 5 show the total transmitted powers for doubly and triply stacked gratings, respectively, where the incident field is ϕ-polarized. In Fig. 4 the discrepancies between SIE-MM and ref. [5] are obvious when $h = 0.2D$ and $h = 0.5D$, especially near the cutoff frequency of the higher order modes ($D/\lambda = 1$). The reason for this is in ref. [5] the interactions of evanescent modes are not taken into account in the analysis.

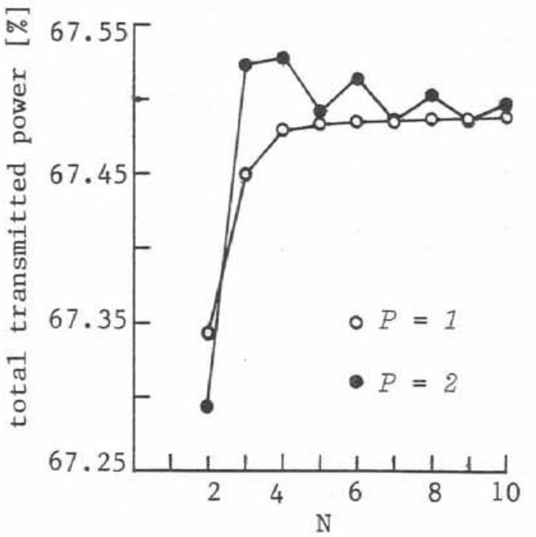

Fig. 3 Convergence of total transmitted powers for two identical structures ($P = 1$ and 2). $\theta_0 = \phi_0 = 45°$, $\psi = 60°$.

For $P = 1$, $D/\lambda = 2$, $2w_1 = D/3$, $d_1 = h_1 = 0$.

For $P = 2$, $D/\lambda = 4$, $2w_1 = 2w_2 = D/6$, $d_1 = 0$, $d_2 = D/2$, and $h_1 = h_2 = 0$.

References

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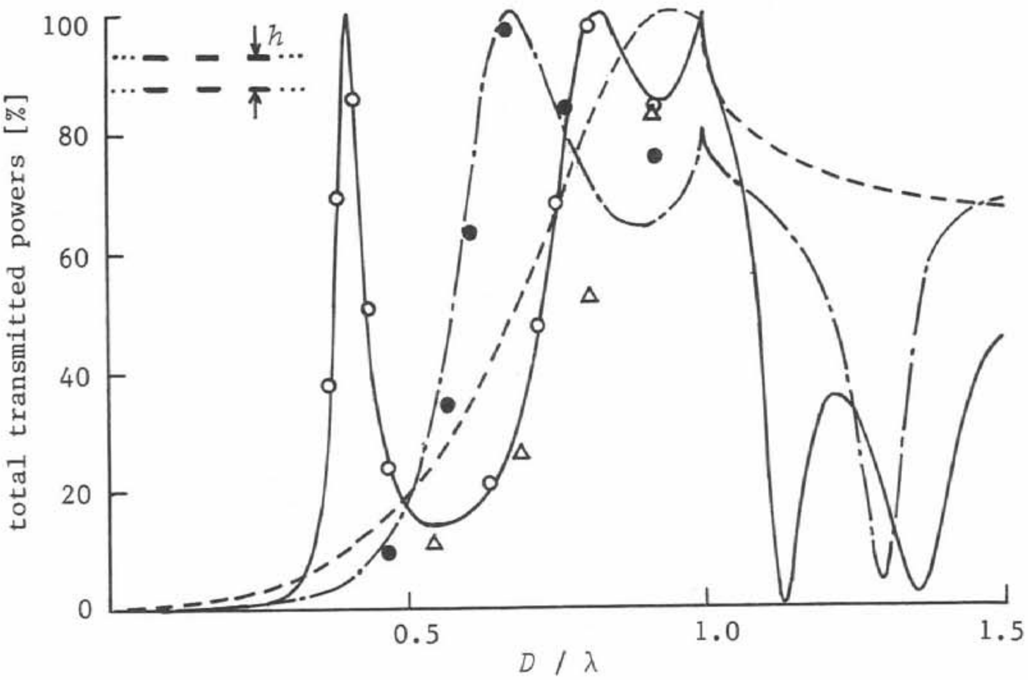

Fig. 4 Total transmitted powers for two symmetrical gratings : $2w_1 = 2w_2 = 0.3D$, $\theta_0 = \phi_0 = 0$, and $\psi = 0$ (E^i // strips). Lines are SIE-MM solutions, and dots are cited from ref. [5].
(– – – △ : $h = 0.2D$, ——- ● : $h = 0.5D$, —— ○ : $h = D$)

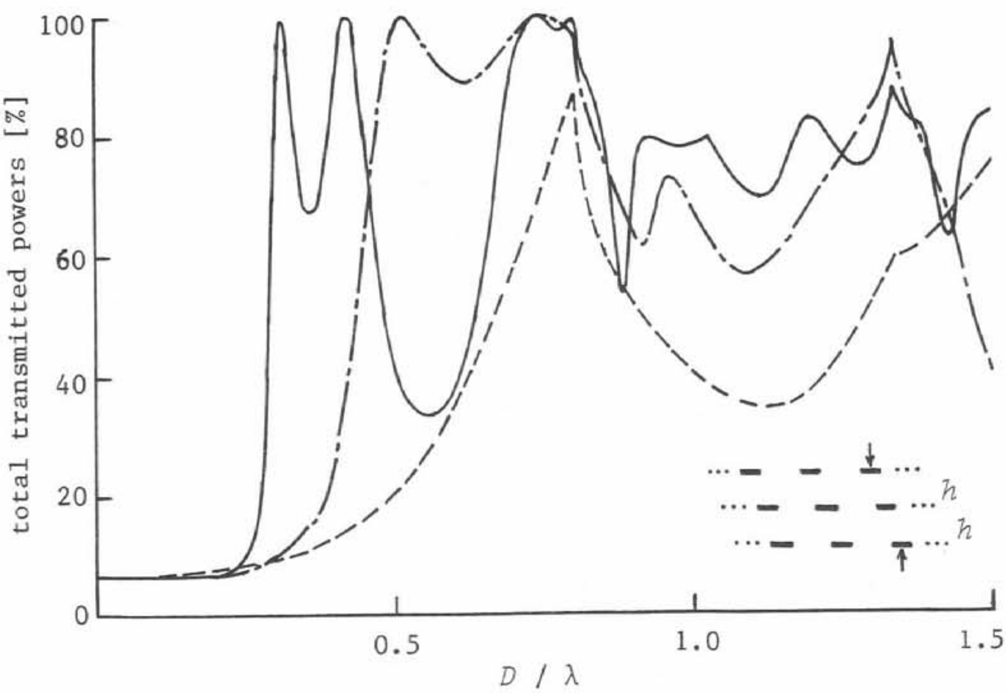

Fig. 5 Total transmitted powers for three staggered gratings at oblique incidence : $2w_1 = 2w_2 = 2w_3 = 0.1D$, $d_1 = 0$, $d_2 = 0.25D$, $d_3 = 0.5D$, $\theta_0 = \phi_0 = 15°$, and $\psi = 0$.
(– – – : $h = 0.2D$, ——- : $h = 0.5D$, —— : $h = D$)