2-III D2 FINITE-SIZE CORNER REFLECTOR

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SUMMARY

A corner reflector antenna has been investigated theoretically and experimentally by several authors[1-6]. It seems, however, that the theoretical works almost always assume the reflecting surfaces infinite in extent. This paper presents the analytic method finding the radiation field of the two-dimensional finite-size corner reflector with the aid of the Hertzian function which satisfies the following equation:

[1] $[\Delta^2 + k_o^2]\Pi(\rho,\varphi) = 0,$

and from which the electromagnetic field can be obtained by

[2]
$$E_z = k_o^2\Pi,\quad E_\rho = E_\varphi = H_z = 0,$$
$$H_\rho = i\omega\varepsilon_o \frac{\partial\Pi}{\rho\partial\varphi},\quad H_\varphi = -i\omega\varepsilon_o \frac{\partial\Pi}{\partial\rho},$$

where $k_o = \omega(\varepsilon_o\mu_o)^{1/2}$.

Fig.1 shows the antenna and the cylindrical coordinate system. Let the total field be $(\underline{E}, \underline{H})$ which is divided into two parts as follows:

[3] $(\underline{E}, \underline{H}) = (\underline{E}^\infty, \underline{H}^\infty) + (\underline{E}^s, \underline{H}^s),$

where $(\underline{E}^\infty, \underline{H}^\infty)$ is the radiation field due to the infinite corner reflector which can be calculated by the image theory. For instance, in the case of $2\varphi_1 = \pi/2$ it is derived from

[4]
$$\Pi^\infty = \begin{cases} \sum_{p=0}^{\infty} (-1)^p H_{\eta_p}^{(2)}(k_o\rho) J_{\eta_p}(k_o a) \cdot \sin \eta_p\varphi, & \text{for } 0 \leq \varphi \leq 2\varphi_1 \\ 0, & \text{otherwise} \end{cases}$$

for $\rho > a$, where $\eta_p = 2(2p + 1)$.

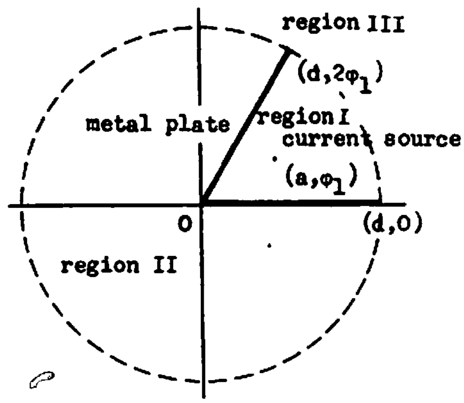

Fig.1 Cross section of the finite -size corner reflector

d: length of reflecting surface
$2\varphi_1$: aperture angle
a: distance of source from apex

$(\underline{E}^s, \underline{H}^s)$ means the corrected field due to the finiteness of length. Then, our problem reduces to find $(\underline{E}^s, \underline{H}^s)$ satisfying the following boundary conditions:

(i) $E_z^s = E_\rho^s = E_\varphi^s = 0$ at $\varphi = 0, 2\varphi_1$; $\rho < d$

(ii) H_ρ is continuous at $\varphi = 0, 2\varphi_1$; $\rho > d$

(iii) $(\underline{E}^s, \underline{H}^s)$ is continuous at $\rho = d$.

The Hertzian function Π^s obeying the condition(i) can be written as

[5]
$$\Pi^s = \sum_{n=1}^{\infty} \begin{cases} A_n J_{\nu_n}(k_o\rho) \sin \nu_n\varphi, \\ B_n J_{\mu_n}(k_o\rho) \sin \mu_n(\varphi - 2\varphi_1), \end{cases}$$

[6] $\nu_n = n\pi/2\varphi_1,\quad \mu_n = n\pi/(2\pi - 2\varphi_1),$

in regions I and II, respectively.

In region III, assuming the

following expansion

$$[7]\quad \Pi^{s} = \sum_{n=0}^{\infty} [R_{cn}(\rho)\cos n\varphi + R_{sn}(\rho)\sin n\varphi],$$

it is found from the condition(ii) that $R_n(\rho)$ satisfies the following equation:

$$[8]\quad \left[\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho} - \frac{n^2}{\rho^2} + k_0^2\right]R_{\substack{c\\s}n}(\rho) = f_{\substack{c\\s}n}(\rho),$$

where

$$[9]\quad f_{cn}(\rho) = \frac{1}{i\omega\epsilon_0\pi\rho(1+\delta_{on})}$$

$$\cdot([H_\rho^{\infty}\cos n\varphi]_{\varphi=2\varphi_1} - [H_\rho^{\infty}\cos n\varphi]_{\varphi=0}),$$

$$[10]\quad f_{sn}(\rho) = \frac{1}{i\omega\epsilon_0\pi\rho}[H_\rho^{\infty}\sin n\varphi]_{\varphi=2\varphi_1}.$$

Then we have

$$[11]\quad R_{\substack{c\\s}n}(\rho) = C_{\substack{c\\s}n}H_n^{(2)}(k_0\rho) + g_{\substack{c\\s}n}(\rho),$$

where

$$[12]\quad g_{\substack{c\\s}n}(\rho) = \frac{i\pi}{4k_0^2}\int_{k_0\rho}^{\infty} dw\, w f_{\substack{c\\s}n}(w/k_0)$$

$$\cdot[H_n^{(1)}(w)H_n^{(2)}(k_0\rho) - H_n^{(2)}(w)H_n^{(1)}(k_0\rho)].$$

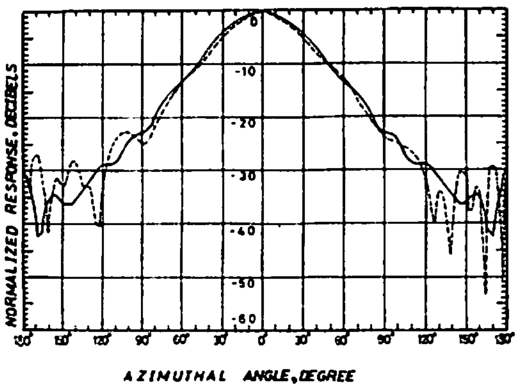

Fig.2 Radiation Pattern

$2\varphi_1 = \pi/2$, $d/\lambda = 1$, $a/\lambda = 0.3$

——— calculated curve ($W/\lambda = \infty$)

------ experimental curve[5] ($W/\lambda = 3$)

(W is the width of reflecting surfaces)

The condition(iii) leads to the infinite simultaneous equations for the unknown coefficients A_n, B_n and C_n. These equations can be solved approximately by the numerical calculation.

It is found from eqs.[11] and [12] that the far-field pattern depends mainly on C_n.

Fig.2 shows one numerical example togather with the experimental curve by H. V. Cottony[5]. In this calculation, the radiation field has been truncated as follows:

$$[13]\quad \Pi^{s} = \sum_{n=0}^{12} [R_{cn}(\rho)\cos n\varphi + R_{sn}(\rho)\sin n\varphi].$$

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