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OPTIMAL DESIGN OF A POLARIZATION-TRANSFORMATION FILTER COMPOSED OF LEFT-HANDED METAMATERIALS AND ISOTROPIC CHIRAL MEDIA BASED ON GENETIC ALGORITHMS

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1. Introduction

Recently, there has been a strong interest in artificial media such as a left-handed metamaterial (LHM) and a chiral medium. An LHM with simultaneously negative permittivity and permeability was theoretically investigated by Veselago [1]. Smith et al. first realized an LHM consisting of periodic arrays of thin wires and split ring resonators [2]. In the LHM, the refractive index is negative and the Poynting vector is antiparallel to the wave vector of a plane electromagnetic wave [1],[3]. A chiral medium, which is fabricated by embedding small wire helices in a host medium, has many applications in the development of a variety of electromagnetic devices [4]-[6]. As is well known [5], a stratified chiral slab acts as a polarization-transformer that passes only a cross-polarized component of the transmitted wave at some frequency band.

The purpose of this paper is to investigate the optimal design of a polarization-transformation transmission filter, which is composed of multiple layers of LHMs and isotropic chiral media. We propose a two-stage procedure based on genetic algorithms (GAs) to design the filter [7],[8]. A plane electromagnetic wave with arbitrary polarization is normally incident from free space upon the stratified slab located on a dielectric substrate. A chain-matrix formulation is used to obtain the cross- and co-polarized powers carried by the transmitted and reflected waves. Two objective functions in the GAs are defined according to specifications for a filter without polarization transformation and a polarization-transformation filter at the first and the second stages. Application of the GAs to the maximization of the objective functions gives the optimal number of layers and the optimal material parameters of the stratified slab. Numerical results show that the stratified slab can be used as an efficient polarization-transformation band-pass filter for the transmitted wave.

2. Formulation of the problem

Consider a plane electromagnetic wave with arbitrary polarization normally incident from free space upon a stratified slab, which is composed of LHMs and isotropic chiral media. The geometry of the problem is illustrated in Fig. 1. The slab consists of M layers with different material parameters and thicknesses. The l-th LHM layer with thickness d_l has simultaneously negative permittivity ε_l and permeability μ_l. The m-th chiral layer has the parameters of permittivity ε_m, permeability μ_m, chiral admittance ξ_{cm}, and thickness d_m, where $l \neq m$. The permittivities and the permeabilities of free space and the dielectric substrate are ε_0, μ_0 and ε_s, μ_s, respectively.

Assuming $\exp(-j\omega t)$ time dependence and noting the negative values of ε_l and μ_l, the constitutive relations for the l-th layer are expressed as

$$\mathbf{D}_l = -|\varepsilon_l|\mathbf{E}_l, \tag{1}$$

$$\mathbf{B}_l = -|\mu_l|\mathbf{H}_l, \tag{2}$$

where $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, and $\mathbf{B}$ are the vectors of the electric field, the magnetic field, the electric flux density, and the magnetic flux density, respectively.

Since the Poynting vector is opposite to the wave vector of a plane electromagnetic wave in the LHM [1], the electric field inside the l-th layer is written as

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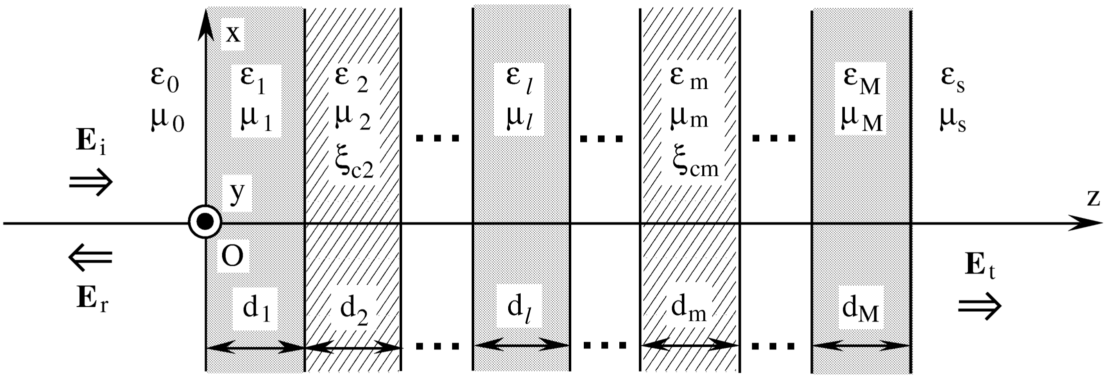

Fig. 1 Geometry of the problem.

$$\mathbf{E}_l = \left(-E_{lx}^{+}\mathbf{u}_x + E_{ly}^{+}\mathbf{u}_y\right)\exp\left(jk_0|n_l|z\right) + \left(E_{lx}^{-}\mathbf{u}_x + E_{ly}^{-}\mathbf{u}_y\right)\exp\left(-jk_0|n_l|z\right), \tag{3}$$

where

$$n_l = -\sqrt{(\varepsilon_l\,\mu_l)\,/\,(\varepsilon_0\,\mu_0)}\,. \tag{4}$$

In Eq. (3), k_0 is the wavenumber in free space and $\mathbf{u}_\alpha(\alpha = x, y)$ are the unit Cartesian vectors. Furthermore, the superscripts $+$ and $-$ refer to the waves propagating toward the positive and negative z-direction in the l-th layer.

The constitutive relations for the m-th chiral layer are represented as [4]

$$\mathbf{D}_m = \varepsilon_m\mathbf{E}_m + j\xi_{cm}\mathbf{B}_m, \tag{5}$$

$$\mathbf{H}_m = j\xi_{cm}\mathbf{E}_m + \mathbf{B}_m/\mu_m, \tag{6}$$

where the chiral admittance ξ_{cm} denotes the degree of chirality of the layer. Note that the electric field inside the m-th layer is expressed as a sum of the left- and right-circularly polarized (LCP and RCP) plane electromagnetic waves of different phase velocities [5]. Then

$$\mathbf{E}_m = \mathbf{E}_{mL}^{+}\exp\left(jk_{mL}z\right) + \mathbf{E}_{mR}^{+}\exp\left(jk_{mR}z\right) + \mathbf{E}_{mL}^{-}\exp\left(-jk_{mL}z\right) + \mathbf{E}_{mR}^{-}\exp\left(-jk_{mR}z\right), \tag{7}$$

where

$$\mathbf{E}_{mL}^{+} = E_{mL}^{+}\left(\mathbf{u}_x - j\mathbf{u}_y\right), \tag{8}$$

$$\mathbf{E}_{mR}^{+} = E_{mR}^{+}\left(\mathbf{u}_x + j\mathbf{u}_y\right), \tag{9}$$

$$\mathbf{E}_{mL}^{-} = E_{mL}^{-}\left(-\mathbf{u}_x - j\mathbf{u}_y\right), \tag{10}$$

$$\mathbf{E}_{mR}^{-} = E_{mR}^{-}\left(-\mathbf{u}_x + j\mathbf{u}_y\right). \tag{11}$$

The parameters k_{mL} and k_{mR} are the wavenumbers of the LCP and RCP plane electromagnetic waves, respectively.

The magnetic fields, $\mathbf{H}_l$ and $\mathbf{H}_m$, may be obtained from Maxwell's equations. Imposing the boundary conditions at the interface of two adjacent layers and using a chain-matrix formulation, we can obtain the x and y components of the transmitted and reflected electric fields. Then the transmitted and reflected powers P_{tx}, P_{ty}, P_{rx}, and P_{ry} are expressed by using these electric field components.

A procedure for designing a polarization-transformation transmission filter is divided into two stages. At the first stage, we design a band-pass filter without polarization transformation, which is composed of LHMs and dielectric media. The design variables are the number M of layers and the refractive indices of the stratified *non*-chiral slab. At the second stage, a polarization-transformation band-pass filter is designed with a stratified slab, which is composed of LHMs and isotropic chiral media. The design variables in this case are only the chiral admittances of the slab. It should be noted that the number of layers and the refractive indices of the slab are the same as those obtained at the first stage.

To find the optimal design variables of the stratified non-chiral slab at the first stage, we define an objective function,

$$g_A(\mathbf{X}_A) = \left[\sum_{i=1}^{I} P_r(f_i; \mathbf{X}_A) + \sum_{i=I}^{I+J} P_t(f_i; \mathbf{X}_A) + \sum_{i=I+J}^{I+J+K} P_r(f_i; \mathbf{X}_A) \right] / \, q_A, \tag{12}$$

where $\mathbf{X}_A$ is a M-dimensional row vector constructed from the refractive indices of the slab. The parameter q_A denotes the number of frequencies used. The parameters f_1, f_I, f_{I+J} and f_I, f_{I+J}, f_{I+J+K} are the lowest and highest frequencies for the stop- and the pass-bands, respectively. Since $\xi_{cm} = 0$ for $m = 1, 2, \ldots, M$, $P_t = P_{ty}$, $P_r = P_{ry}$ and $P_t = P_{tx}$, $P_r = P_{rx}$ for $E_{ix} = 0$ and $E_{iy} = 0$, respectively.

At the second stage, the chiral admittances of the stratified slab are found to design a polarization-transformation band-pass filter for the transmitted wave. Then we define the following objective function:

$$g_B(\mathbf{X}_B) = \sum_{i=I}^{I+J} [\, P_{t,\,cr}(f_i; \mathbf{X}_B) - P_{t,\,co}(f_i; \mathbf{X}_B) \,] \, / \, q_B, \tag{13}$$

where $\mathbf{X}_B$ is a N-dimensional ($N \leq M$) row vector composed of the chiral admittances of the slab. The parameter q_B is the number of frequencies used. Furthermore, $P_{t,\,cr}$ and $P_{t,\,co}$ are the cross- and co-polarized transmitted powers, respectively.

The problem of designing each filter can be formulated as an optimization problem, where the objective function given by Eq. (12) or (13) is maximized. We apply GAs with variable- and fixed-length chromosomes [7],[8] to the maximization of Eqs. (12) and (13), respectively. Then one may obtain the optimal number of layers, refractive indices, and chiral admittances of the stratified slab.

3. Numerical results

Numerical results are presented for the incident wave of perpendicular polarization, i.e., $E_{ix} = 0$. The incident power is now normalized to unity. The refractive index and the permeability of the dielectric substrate are assumed to be $n_s = 1.52$ and $\mu_s = \mu_0$. We employ the parameters of $|n_p| d_p = \lambda_c/4$ and $|\mu_p| = \mu_0$ ($p = 1, 2, \ldots, M$), where $2 \leq M \leq 10$. Here n_p is the refractive index of the p-th layer, and λ_c is the wavelength in free space at the central frequency f_c. The fitness functions in the GAs at the first and the second stages are given by Eqs. (12) and (13).

The frequencies used at the first stage are $f_1 = 0.20 f_c$, $f_I = 0.95 f_c$, $f_{I+J} = 1.05 f_c$, and $f_{I+J+K} = 1.80 f_c$. Applying the GA with variable-length chromosomes to the maximization of Eq. (12), we can obtain $M = 10$, $n_1 = 4.47$, $n_2 = n_{10} = -1.29$, $n_3 = -1.21$, $n_4 = 4.70$, $n_5 = -1.28$, $n_6 = 4.04$, $n_7 = -1.32$, $n_8 = -1.30$, and $n_9 = 3.59$ at the 93-th generation. At the second stage, we use the number of layers and the refractive indices obtained at the first stage. The negative values of n_2, n_3, n_5, n_7, n_8, and n_{10} indicate that the 2nd, 3rd, 5-th, 7-th, 8-th, and 10-th layers are composed of LHMs with $\xi_{c2} = \xi_{c3} = \xi_{c5} = \xi_{c7} = \xi_{c8} = \xi_{c10} = 0$, and $N = 4$. The other material parameters of the slab are the same as those used at the first stage. Application of the GA with fixed-length chromosomes to the maximization of Eq. (13) gives $\xi_{c1} = 3.453 \times 10^{-3}$S, $\xi_{c4} = 2.993 \times 10^{-3}$S, $\xi_{c6} = 1.852 \times 10^{-3}$S, and $\xi_{c9} = 2.743 \times 10^{-3}$ S at the 42-th generation. Figure 2 presents the cross- and co-polarized transmitted powers for the 10-layered slab constructed from LHMs and chiral media.

The second example is a stratified slab composed of only chiral layers, i.e. $M = N$. Specifications for this case are the same as those in the first example. Using the GA with variable-length chromosomes in the maximization of Eq. (12), one can obtain $M = 10$, $n_1 = 4.29$, $n_2 = 5.87$,

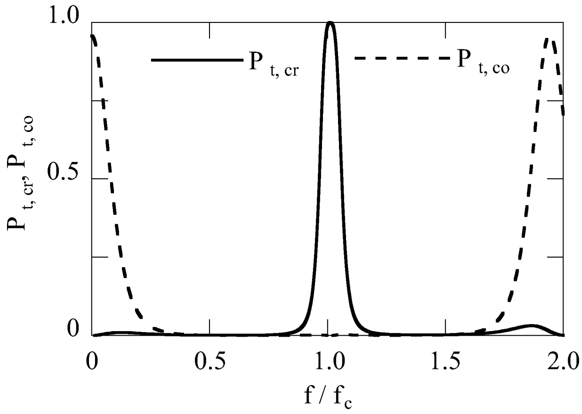

Fig. 2 Cross- and co-polarized transmitted powers for the 10-layered slab composed of LHMs and chiral media.

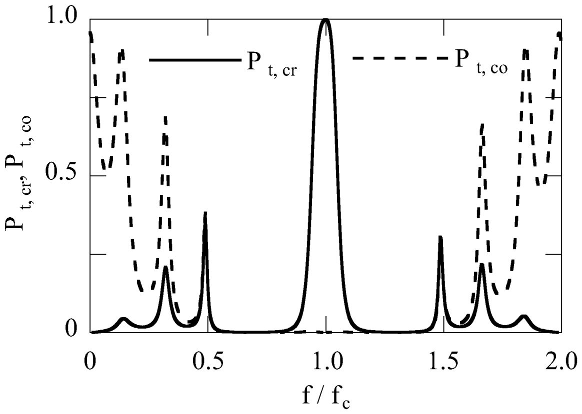

Fig. 3 Cross- and co-polarized transmitted powers for the 10-layered chiral slab.

$n_3 = n_6 = n_8 = 1.47$, $n_4 = 5.47$, $n_5 = 2.51$, $n_7 = 3.59$, $n_9 = 5.99$, and $n_{10} = 5.95$ at the 34-th generation. Applying the GA with fixed-length chromosomes to the maximization of Eq. (13), the optimal values of the chiral admittances are found to be $\xi_{c1} = 2.121 \times 10^{-5}$ S, $\xi_{c2} = 1.990 \times 10^{-3}$S, $\xi_{c3} = 3.425 \times 10^{-4}$S, $\xi_{c4} = 1.173 \times 10^{-3}$S, $\xi_{c5} = 2.198 \times 10^{-4}$S, $\xi_{c6} = 4.063 \times 10^{-4}$S, $\xi_{c7} = 1.103 \times 10^{-3}$S, $\xi_{c8} = 1.163 \times 10^{-3}$S, $\xi_{c9} = 2.443 \times 10^{-3}$S, and $\xi_{c10} = 9.629 \times 10^{-6}$S at the 44-th generation. Figure 3 shows the cross- and co-polarized transmitted powers for the 10-layered chiral slab.

It is seen from Figs. 2 and 3 that both slabs act as efficient polarization-transformation filters with narrow pass-band. Furthermore, the cross-polarized transmitted power at the both sides of the pass-band can be suppressed when LHMs are used in combination with chiral media.

4. Conclusion

A two-stage design procedure based on GAs for a polarization-transformation transmission filter, which is composed of multiple layers of LHMs and chiral media, has been presented. It is confirmed from the numerical results that the proposed procedure is effective in the optimal design of a polarization-transformation band-pass filter. Furthermore, the transmission characteristics of the filter can be improved by the combination of LHMs and chiral media.

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