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ANALYSIS OF A NOVEL METAMATERIAL USING TLM

Wei Xu[1], Hai-Ying Yao[2] and Le-Wei Li[1,2]

[1]Dept of Electrical and Computer Engineering;
[2]HPCES Programme, Singapore-MIT Alliance
National University of Singapore, Kent Ridge, Singapore, 119260.
E-mail: lwli@nus.edu.sg

I. Introduction

Left-handed material represents an artificial dielectric medium, which exhibits some abnormal phenomenons resulted from simultaneous negative permittivity and permeability [1]. So far, there are two main topics to be paid attention to: first, new design and numerical or experimental verification; the other is to investigate the properties of LHM and come up with the corresponding applications. Metamaterials synthesized using the split ring resonators (SRR) or their various counter-parts are found to exhibit high lossy properties [2,3]. In this paper, we first analyze the SRR by using transmission line theory and find a new way to explain the energy loss besides ohm loss. Then we improve the SRR structure by adding the low loss transmission line to the design. A distinct negative refraction phenomena and lower loss in the new metamaterial are observed in the numerical results.

II. Transmission line model for SRR

In this section, a periodic array of SRR are considered, which can behave as an effective material when the wavelength is much longer than the inclusion dimensions and lattice spacings. For a bianisotropic material, the constitutive relations can be written as [4]

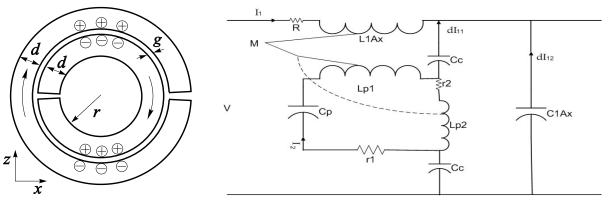
$$\begin{bmatrix} \boldsymbol{D} \\ \boldsymbol{B} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{\epsilon}} & \bar{\boldsymbol{\xi}} \\ \bar{\boldsymbol{\zeta}} & \bar{\boldsymbol{\mu}} \end{bmatrix} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{bmatrix}, \tag{1}$$

where $\bar{\boldsymbol{\epsilon}}$ and $\bar{\boldsymbol{\mu}}$ represent the electric permittivity and magnetic permeability tensors, while $\bar{\boldsymbol{\xi}}$ and $\bar{\boldsymbol{\zeta}}$ denote the magnetoelectric cross coupling tensors, respectively.

Due to its periodicity, our analysis begins with a single SRR in Fig. 1(a). In order to obtain magnetic response, we consider the polarization of the incident magnetic field $\boldsymbol{H}^i$ penetrate through this SRR. The propagation direction $\boldsymbol{k_0}$ and the polarization of the incident electric field $\boldsymbol{E}^i$ are in the xoz-plane. Taking into account the current and charges on the surface of two split rings, we can get its equivalent circuit model by transmission line theory shown in Fig. 1(b). By using transmission line theory, the telegrapher equations for the equivalent circuit model of SRR can be formulated and the effective constitutive parameters can easily be extracted

$$\mu_{eff} = L_1 \left(1 - \frac{jR}{\omega L_1 A_x} - \frac{k^2}{1 - \omega_p^2/\omega^2 - jr_p/\omega L_p} \right) \tag{2a}$$

$$\epsilon_{eff} = C_1 \left(1 + \frac{1}{2C_1 A_x/C_c - \omega^2/\omega_{p2c}^2 + j\omega C_1 A_x r_2} \right) \tag{2b}$$



(a) Geometry model, (b) Equivalent circuit model.

Fig. 1. A single circular split-ring resonator.

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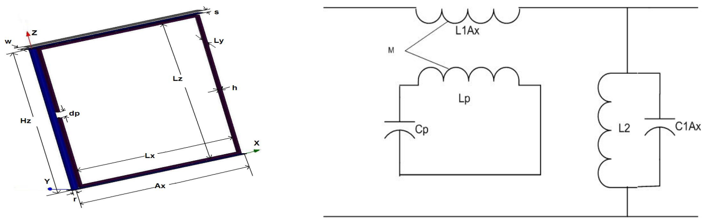


(a) Geometry model (b) Equivalent circuit model

Fig. 2. A single inclusion composited by SRR and transmission line segment.

$$\zeta_{eff} = \frac{jM/(L_p A_x \omega)}{\left(1-\omega_p^2/\omega^2 - jr_2/\omega L_{p_2}\right)\left(1-2\omega_{p_2c}^2/\omega^2 - jr_p/\omega L_p\right)}, \tag{2c}$$

$$\xi_{eff} = 0. \tag{2d}$$

According to the polarization direction of $\boldsymbol{E}^i$ and $\boldsymbol{H}^i$, the dyadic expressions of constitutive parameters for 2-D metamaterial constructed by the SRR are given by

$$\overline{\boldsymbol{\mu}} = \begin{pmatrix} \mu_{eff} & 0 & 0 \\ 0 & \mu_{eff} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{\boldsymbol{\epsilon}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \epsilon_{eff} \end{pmatrix}, \tag{3a}$$

$$\overline{\boldsymbol{\zeta}} = \begin{pmatrix} 0 & 0 & \zeta_{eff} \\ 0 & 0 & \zeta_{eff} \\ 0 & 0 & 0 \end{pmatrix}, \quad \overline{\boldsymbol{\xi}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \xi_{eff} & \xi_{eff} & 0 \end{pmatrix}. \tag{3b}$$

It has been shown that for lossless medium, the time average of the divergence of Poynting's vector $< \nabla \cdot \boldsymbol{S} >$ must vanish for all possible $\boldsymbol{E}$ and $\boldsymbol{H}$ [4] and the lossless conditions are obtained

$$\overline{\boldsymbol{\epsilon}} = \overline{\boldsymbol{\epsilon}}^{\dagger}, \overline{\boldsymbol{\mu}} = \overline{\boldsymbol{\mu}}^{\dagger}, \overline{\boldsymbol{\xi}} = \overline{\boldsymbol{\zeta}}^{\dagger}, \tag{4}$$

where $\dagger$ denotes transpose and complex conjugate. For a periodic SRR array, it is obvious that energy loss include two parts. One part comes from imaginary parts of $\overline{\boldsymbol{\mu}}$ and $\overline{\boldsymbol{\epsilon}}$. The other is contributed by the magnetoelectric cross coupling, that is, $\overline{\boldsymbol{\xi}}$ and $\overline{\boldsymbol{\zeta}}$. It is reasonable to explain a phenomena that metamaterial synthesized by SRR structure exists large loss, even though the split rings are made of very good conductors [2].

III. A novel design of low-loss metamaterial

From our analysis, one way to minimize the energy loss is to make magnetoelectric cross coupling effect weaken or diminish, that is, make the $\overline{\boldsymbol{\xi}}$ and $\overline{\boldsymbol{\zeta}}$ vanish. From Eq. (2c), it is to make $C_c = 0$. In this paper, our approach is to add two coupling transmission lines to the SRR, which is shown in Fig. 2(a). By this way, the upper transmission line can better transmit wave and no charge accumulate easily on the line. Most charges accumulate on C_p, and small amount of charges can stay on C_c. Then C_c can be greatly decreased. In other words, only small amount of current flow through the branch of C_c. As a result, the losses made by bianisotropic effect minimized and the total energy loss is decreased greatly. The corresponding equivalent circuit model of this structure is shown in Fig. 2(b). As above, the effective constitutive parameters for this new structure can be found

$$\mu_{eff} = \mu_0 \left(1 - \frac{k^2}{1-\omega_p^2/\omega^2}\right), \tag{5a}$$

$$\epsilon_{eff} = \epsilon_0 \left(1 - \frac{\omega_1^2}{\omega^2}\right), \tag{5b}$$

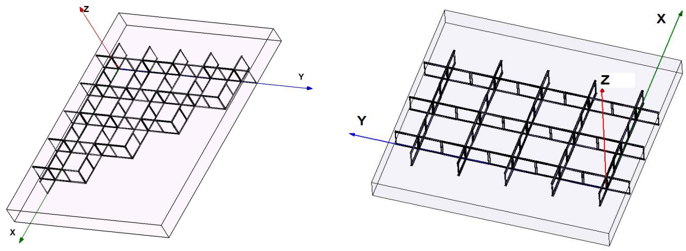
$$\zeta_{eff} = \xi_{eff} = 0, \tag{5c}$$

where $\omega_1 = 1/\sqrt{L_2 C_1 A_x}$.

TABLE I

THE S PARAMETERS OF THE STRUCTURE IN FIG. 3

Frequency (Ghz)	8.2	8.3	8.4
$S_{21}(db)$	-2	-1.5	-2
$S_{11}(db)$	-25	-13	-9



(a) Triangular slab (b) Rectangular slab

Fig. 3. Two different shaped slabs made of the new structure.

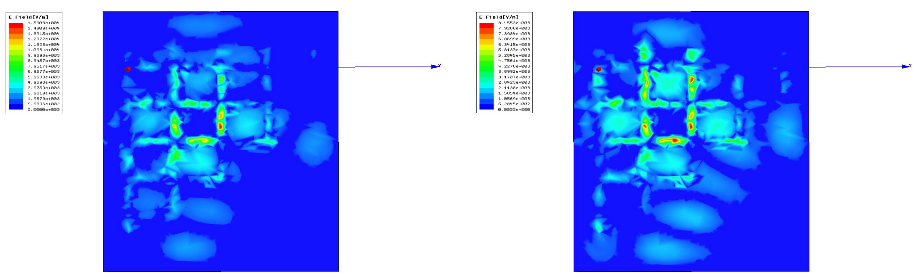
IV. Numerical simulation

To demonstrate the new structure shown in Fig. 2(a) to be left handed material, we first use it constructe a triangular slab and put it in a waveguide shown in Fig. 3(a). Then simulate it by using the commerical software Ansoft HFSS. The sizes of the waveguide is 120 mm × 100 mm × 10.5 mm. The sizes of the new structure is as follows: $A_x = 10$ mm, $H_z = 10$ mm, $w = 0.8$ mm, $s = 0.2$ mm, $d_p = 0.4$ mm, $h = 0.1$ mm, $r = 0.2$ mm, $L_x = 9.2$ mm, $L_y = 0.4$ mm, $L_z = 9.6$ mm and $h = 0.1$ mm. All the elements in the structure is made of copper. At 8.4 Ghz, we saw a very good negative refraction of our structure shown in Fig. 4, and the refraction angel is about $-45°$.

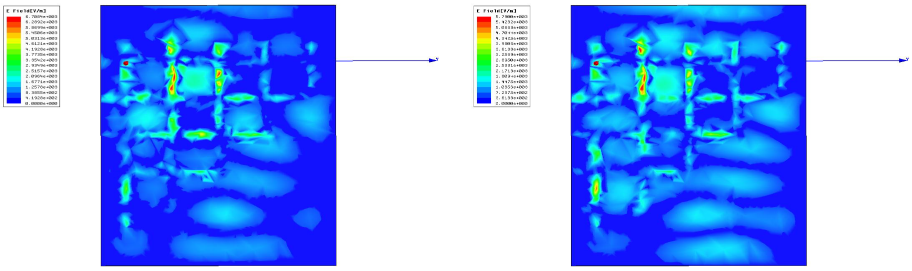
To demonstrate the low loss property of this structure, a rectangular slab shown in Fig. 3(b) is constructed. This time the waveguide is 100 mm × 100.8 mm × 10.5 mm. S paremeters are observed as shown in Table IV. Compared to traditional SRR structure [2] whose S_{21} is normally -7 dB to -10 dB, the energy loss in the present new structure is quite low.

V. Discussion and Remindings

By now, we can obtain the constitutive relations of artificial material constructed by SRR. To demonstrate the validity of these formulas, we consider a periodic SRR array. The geometry and dimensions are taken from [3] and reproduced in Fig. 1(a). Then we build them in a dielectric material whose $\epsilon_r = 4.5$. The dimensions of single SRR are $d = 0.8$ mm, $g = 0.2$ mm, and $r = 1.5$ mm. Here, we consider it as a thin perfect conductor. In this case, the values of the corresponding capacitances and inductances in the model can be estimated in a quasi-static way and by using conformal mapping theory. In our case, we roughly evaluate them by using microstrip line theory [5]. The results of real part of effective permittivity and permeability versus the frequency are

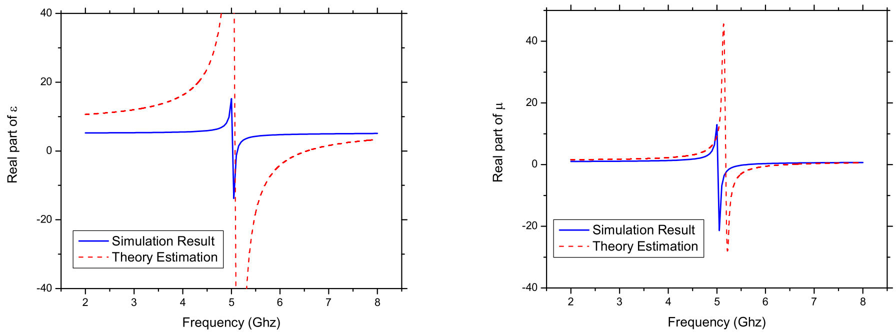


(a)Phase 90° (b)Phase 125°



(c)Phase 145° (d)Phase 160°

Fig. 4. Negative refraction in triangular shape LHM made of the new structure



(a) Real(ϵ) (b) Real(μ)

Fig. 5. Real part of effective permittivity and permeability for SRR-constructed material.

shown in Fig. 5(a) and (b), respectively. In these figures, the solid line is obtained by numerical simulation based on quasi-static Lorentz theory [6], and the dash line is result of the present approach. From them, we see good agreement can be achieved even by quasi-static estimation.

VI. Conclusion

The transmission line theory has been applied to analyze the split-ring resonator in this paper. We find out the origin of the energy losses in the SRR-constructed metamaterial. The effective permittivity and permeability of this material obtained by using equivalent circuit model are reasonable accordant with numerical results obtained by quasi-static Lorentz theory. We are motivated by the analysis and present a new low loss metamaterials, which is composited mixtures of transmission line and split ring. The numerical experiments made by the commercial software (HFSS) are shown. Obvious negative refraction phenomena and low energy loss are observed. The present approach give a possibility for design low loss metamaterials.

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