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Near-Field Target Location Estimation by Using Khatri-Rao Product Array

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1. Introduction

In recent years, high-resolution near-field target location estimation technique for indoor wireless terminal detection, medical imaging and so forth, using an array antenna have been studied actively. In many applications, small number of array elements is desirable. Recently, at simple array signal processing technique called Khatri-Rao (KR) product array [1] [2] has been proposed. By using the technique, degrees-of-freedom (DOF) of the array can be easily increased. Strictly speaking, this holds only for uncorrelated wave incidence. When, the waves are correlated or coherent accuracy of, parameter estimation deteriorates by the signal correlation terms [3]. The estimation error can be decreased when we estimate correlation terms in addition to the signal terms [3]. It costs DOF of the array, and not effective for signal parameter estimation in far-field. On the other hand, if the wave sources are located in near-field, incoming wave becomes spherical wave at the receiving array. Therefore, it can be expected that we can obtain further DOFs by the Khatri-Rao product expansion. In this paper, we consider the effect of increase of DOF by the Khatri-Rao product array for spherical wave numerically, and show that the location estimation accuracy can be improved by using the increased DOFs even when all of the incoming wave are coherent.

2. Data Model

Consider that K waves impinges on the array antenna having M_r-element. The received data vector can be written as follows:

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{a}_k s_k(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

$$\boldsymbol{a}(\theta_k) = \left[e^{-j\frac{2\pi}{\lambda}d_1 \sin(\theta_k)}, e^{-j\frac{2\pi}{\lambda}d_2 \sin(\theta_k)}, \cdots, e^{-j\frac{2\pi}{\lambda}d_{M_r} \sin(\theta_k)} \right]^T, \tag{2}$$

$$\boldsymbol{a}(\boldsymbol{r}_k) = \left[\frac{1}{r_{k1}} e^{-j\frac{2\pi}{\lambda}r_{k1}}, \frac{1}{r_{k2}} e^{-j\frac{2\pi}{\lambda}r_{k2}}, \cdots, \frac{1}{r_{kM_r}} e^{-j\frac{2\pi}{\lambda}r_{kM_r}} \right]^T, \tag{3}$$

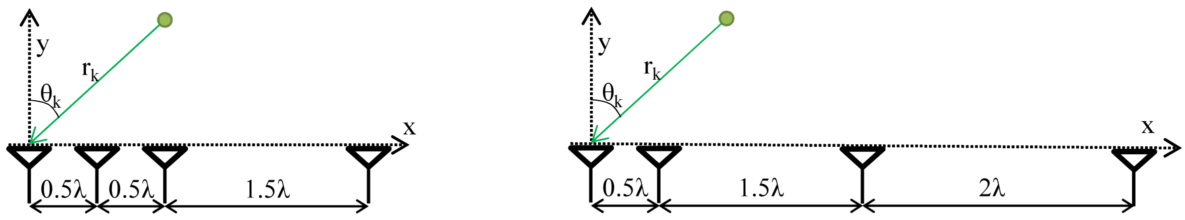
where $s_k(t)$ is the complex amplitude of the k-th incident wave, and $\boldsymbol{n}(t)$ is the noise vector. $\boldsymbol{a}(\theta_k)$ in (2) and $\boldsymbol{a}(\boldsymbol{r}_k)$ in (3) are the mode vector of the k-th wave for far-field source (plane wave) and for near-field source (spherical wave), respcetively. r_{km_r} is the distance between the m_r-th array element and the k-th wave source. The received data correlation matrix is estimated by

$$\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \boldsymbol{R}_N, \tag{4}$$

where $\mathrm{E}[\cdot]$ is the expected value operator, $[\cdot]^H$ is the complex conjugate transpose. In addition, the $M_r \times K$ matrix $\boldsymbol{A}$ is the mode matrix whose column is the mode vector defined by (2) or (3) depending on the source location. $\boldsymbol{S}$ and $\boldsymbol{R}_N$ are the source and the noise correlation matrix, respectively.

3. Khatri-Rao Product Expansion Array

The Khatri-Rao (KR) product is defined by the two matrices having the same number of columns. By using the KR product expansion to the data correlation matrix shown in (4), DOF of the array can be



(a) 4-element 2L-NA

(b) 4-element large aperture array

Figure 1: Element arrangement

increased [1]. When we assume that the incoming waves are uncorrelated with each other, the matrix $\boldsymbol{S}$ becomes a diagonal matrix, hence the following equation holds.

$$
\begin{aligned}
\boldsymbol{z} &= \mathrm{vec}(\boldsymbol{R}) = \mathrm{vec}(\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H) + \mathrm{vec}(\boldsymbol{R}_N) \\
&= (\boldsymbol{A}^* \odot \boldsymbol{A})\bar{\boldsymbol{s}} + \mathrm{vec}(\boldsymbol{R}_N) \\
&= \bar{\boldsymbol{A}}\bar{\boldsymbol{s}} + \boldsymbol{c}, \qquad (5)
\end{aligned}
$$

where $[\cdot]^*$ is the complex conjugate, $\odot$ denotes the KR product operator, and $\mathrm{vec}(\cdot)$ is the operator to transform a matrix to the vector stacking every column of the matrix. Also, $\bar{\boldsymbol{s}}$ denotes the K-dimensional column vector consisting of the diagonal elements of $\boldsymbol{S}$. This equation has the same form as that in (1). Since the $\boldsymbol{z}$ is the M_r^2 dimensional vector, the array has $M_r^2 - 1$ DOFs if the all of the elements are independent. For the uniform linear array (ULA), it is obvious that the DOF becomes $2(M_r - 1)$.

4. Two-Level Nested Array

The 2-Level Nested Array (2L-NA) is the array antenna which consists of two different ULA element spacing [2]. Figure.1(a) shows a 4-element 2L-NA. The first level ULA has M_{r1} with element spacing of Δd_1, and the second level ULA has M_{r2} elements with spacing of Δd_2. Element spacing Δd_2 is arranged so as to satisfy $\Delta d_2 = (M_{r1} + 1)\Delta d_1$. When we apply the KR product array processing to the 2L-NA, the DOF becomes $2M_{r2}(M_{r1} + 1) - 1$ for plane wave incidence.

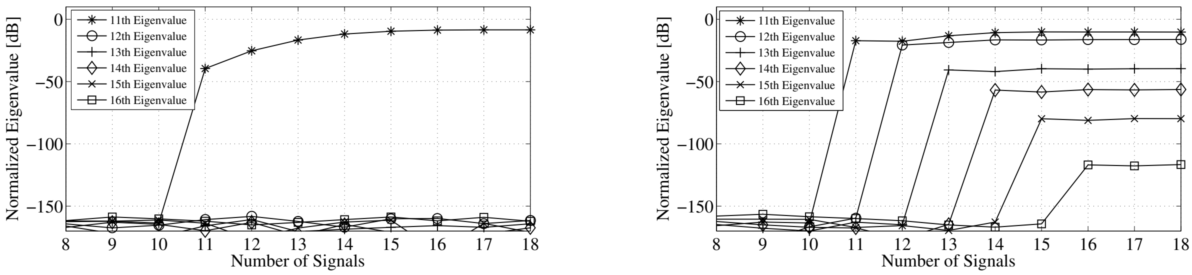
5. Degrees-of-Freedom of the Khatri-Rao Product Array

In this section, we will show characteristic of the DOF of the KR product array for spherical wave and the plane wave incidence, respectively. Order DOF is equivalent to the maximum number of resolvable wavesby the array. This can be identical to the maximum rank of $\bar{\boldsymbol{A}}$. Rank of the matrix is equal to the number of non-zero eigenvalues in the correlation matrix without noise. So, we assume that all the incoming wave is uncorrelated having power of 1, and calculate eigenvalues of $\mathrm{rank}[\boldsymbol{R}] = \mathrm{rank}[\bar{\boldsymbol{A}}\bar{\boldsymbol{A}}^H]$. Figure.2(a) and 2(b) show magnitude of eigenvalues by the maximum value in each K with the 4-element 2L-NA for plane-wave and spherical-wave incidence, respectively. For the plane-wave incidence, the number of non-zero eigenvalues are limited to 11 even when K is greater than 11. On the other hand, the number of non-zero eigenvalues increases for the spherical-wave incidence according to increase of K, therefore, more DOFs are obtained.

6. EM-ML Method for Correlated Waves in Khatri-Rao Product Array

When incident waves are correlated, signal correlation terms remain in the KR product array, hence the DOA/location estimation accuracy often deteriorates. However, it can be improved when we estimate signal parameters including the correlated components [3]. The algorithm listed below is the proposed Expectation-Maximization Maximum Likelihood (EM-ML) algorithm for considering signal correlation terms.

[Step 0:Initialize] Set the number of iterations $l = 0$, and initialize the value $\boldsymbol{\Theta}^{(l)}|_{l=0}$.



(a) Plane-wave incidence (b) Spherical-wave incidence

Figure 2: Magnitude of normalized eigenvalues for each number of incident waves

$$\boldsymbol{\Theta}^{(0)} = [\boldsymbol{r}_1^{(0)}, \cdots, \boldsymbol{r}_K^{(0)}, \phi_1^{(0)}, \cdots, \phi_{K_\phi}^{(0)}]. \tag{6}$$

where $\phi_k = \phi_{ij}(i < j)$ is the phase component of the correlation coefficient ρ_{ij}. The complex amplitude and the mode vectors can be calculated by the following equation.

$$\boldsymbol{s}^{(0)} = (\boldsymbol{A}^{(0)H}\boldsymbol{A}^{(0)})^{-1}\boldsymbol{A}^{(0)H}\boldsymbol{z}, \tag{7}$$

$$\boldsymbol{A}^{(0)} = [\boldsymbol{a}_1^{(l)}, \cdots, \boldsymbol{a}_K^{(l)}, \boldsymbol{a}_{K+1}^{(l)}(\phi_1^{(l)}), \cdots, \boldsymbol{a}_{K+K_\phi}^{(l)}(\phi_{K_\phi}^{(l)})]|_{l=0}, \tag{8}$$

$$\boldsymbol{a}_k^{(l)} = \mathrm{vec}(\boldsymbol{a}(\boldsymbol{r}_k^{(l)})\boldsymbol{a}(\boldsymbol{r}_k^{(l)})^H) \cdots (k \leqq K), \tag{9}$$

$$\boldsymbol{a}_k^{(l)}(\phi_{k_\phi}^{(l)}) = \boldsymbol{a}_{ij}^{(l)} e^{j\phi_{k_\phi}^{(l)}} + \boldsymbol{a}_{ji}^{(l)} e^{-j\phi_{k_\phi}^{(l)}}, \quad \boldsymbol{a}_{ij}^{(l)} = \mathrm{vec}(\boldsymbol{a}(\boldsymbol{r}_i^{(l)})\boldsymbol{a}(\boldsymbol{r}_j^{(l)})^H) \cdots (k > K), \tag{10}$$

where $\boldsymbol{a}_k^{(l)}$ and $\boldsymbol{a}_k^{(l)}(\phi_{k_\phi}^{(l)})$ are the KR-product modevector of each wave source and correlation term, respectively.

[Step 1:E-Step] Estimate each data vector, z_k, as follows

$$\boldsymbol{z}_k^{(l)} = \boldsymbol{a}_k^{(l)}\boldsymbol{s}_k^{(l)} + \beta_k(\boldsymbol{z} - \boldsymbol{A}^{(l)}\boldsymbol{s}^{(l)}) \cdots (k \leqq K), \tag{11}$$

$$\boldsymbol{z}_k^{(l)} = \boldsymbol{a}_k^{(l)}(\phi_{k_\phi}^{(l)})\boldsymbol{s}_k^{(l)} + \beta_k(\boldsymbol{z} - \boldsymbol{A}^{(l)}\boldsymbol{s}^{(l)}) \cdots (k > K). \tag{12}$$

[Step 2:M-Step] Estimate $\boldsymbol{\Theta}^{(l)}$ and $\boldsymbol{s}^{(l)}$ for each $\boldsymbol{z}_k$ and update estimated parameters.

$$\boldsymbol{r}_k^{(l+1)} = \arg\max_{\mathrm{r}} \frac{\boldsymbol{a}(\boldsymbol{r})^H \boldsymbol{C}_k^{(l)} \boldsymbol{a}(\boldsymbol{r})}{\boldsymbol{a}(\boldsymbol{r})^H \boldsymbol{a}(\boldsymbol{r})} \cdots (k \leqq K), \quad \phi_{k_\phi}^{(l+1)} = \arg\max_{\phi} \frac{\boldsymbol{a}(\phi)^H \boldsymbol{C}_K^{(l)} \boldsymbol{a}(\phi)}{\boldsymbol{a}(\phi)^H \boldsymbol{a}(\phi)} \cdots (k > K), \tag{13}$$

where $\boldsymbol{C}_k^{(l)}$ is the correlation matrix of $\boldsymbol{z}_k^{(l)}$. $\boldsymbol{s}^{(l)}$ are updated at once after all the elements of $\boldsymbol{\Theta}^{(l+1)}$ are estimated. These steps are repeated as $l \leftarrow l+1$ until the estimated values are converged.

7. Computer Simulation Results

Table 1 shows the simulation parameters in this study. We assume in this study that distances between the #1 element of the array and each source, r_k, are 2λ in the near-field case (spherical waves) assuming that such as the medical application. We employed two types of array here. One is the 4-element 2L-NA as shown in Fig.1(a) and the other is the 4-el. array with much larger aperture as shown in Fig.1(b). Table 2 shows the estimated RMSE of the estimated parameters. Note that the problem considered here is coherent signal detection, therefore number of resolvable signal becomes half in comparison with that for uncorrelated waves. For the far-field case (plane wave incidence) , both the proposed and conventional methods cannot estimate DOAs correctly. This is because the number of incident waves plus correlated terms exceeds the DOF of the array. On the other hand, the proposed method can reduce estimation error effectively in the near-field case (spherical wave incidence). It is clear that we can decrease RMSE effectively by increasing array length or aperture. Figures 3(a) and 3(b) show the residue at each iteration and RMSE of the distance with the 2L-NA. The residue cannot decrease by the conventional method. However it can decrease rapidly by the proposed method. As shown in Fig.3(b), the proposed technique is effective only for near-field source location estimation problem.

8. Conclusion

In this paper, we examined available of the Khatri-Rao product array for near-field source location estimation having spherical wave incidence. In this problem, we show that the KR product increases the degrees-of-freedom of the array effectively. Furthermore, we propose the EM-ML algorithm which can remove signal correlation error in the KR product array. Simulation results show that the proposed algorithm can improve estimation accuracy effectively.

References

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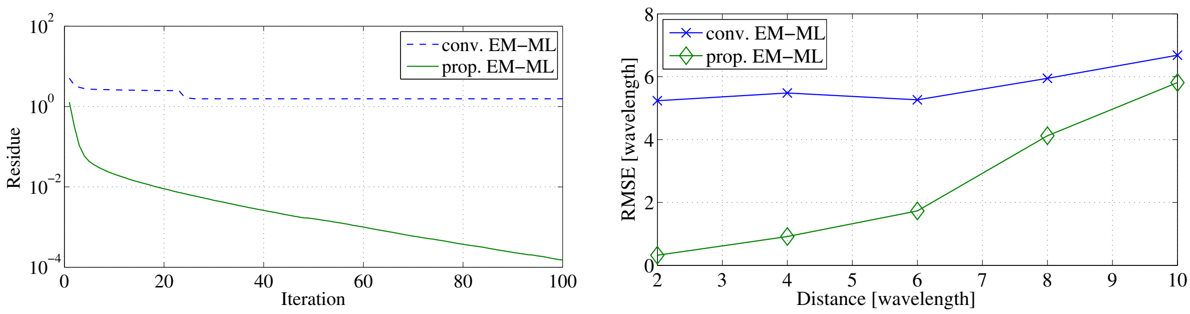
Table 1: Simulation Parameters

Simulation	Array 1	Array 2
Shape of Array	2L-NA	Refer to Figure.1(b)
Number of elements	4	4
Number of waves	3	3
Diretion of arrival θ_k[deg.]	[-35, 25, 45]	[-35, 25, 45]
Correlation coefficient	1	1
SNR [dB]	20	20
Number of snapshots	1000	1000
Number of trial	100	100

Table 2: Simulation Results

method	Plane wave ($r_k \to \infty$)		Spherical wave ($r_k = 2\lambda$)	
	RMSE θ [deg.]		RMSE r [wavelength]	
	Array 1	Array 2	Array 1	Array 2
conv. EM-ML	3.6630	27.9316	5.2363	1.0666
prop. EM-ML	17.8469	28.8052	0.3208	0.1436



(a) Residue in each iteration (2L-NA)

(b) RMSE in each target distance (2L-NA)

Figure 3: Simulation Results