PROCEEDINGS OF ISAP '92, SAPPORO, JAPAN

BEAM POINTING ACCURACY OF THREE DIMENSIONAL ARRAY ANTENNAS

SHAO Jiangda and GUO Yanchang
Nanjing Research Institute of Electronics Technology
P.O.Box 1315, Nanjing, 210013, P.R.China

ABSTRACT The beam pointing accuracy of three dimensional array antennas is studied .The formulas for computation of statistical beam pointing characteristics of phased array antennas in the three dimensional coordinate system are deduced.The theoretical results are in good agreement with simulated statistical ones.

INTRODUCTION There are two kinds of definition for beam pointing direction of sum patterns.One is the direction of maximum point. The other is the direction at the center between two half power points. As for the latter definition, GUO [1] analyzed the beamwidth of spacing density weighted arrays. Here the beam pointing direction means the maximum point of sum pattern. For difference pattern, beam pointing direction means the direction of centeral zero point. But if excitation errors exist, the minimum value between two lobes is not zero. So the minimum point direction of difference pattern is the beam pointing direction.

The beam pointing accuracy of continous line source has been investigated by M.Leichter [2]. L.A.Rondinelli [3] discussed the beam pointing accuracy of planar arrays and 3–dimensional arrays, the formulas for analysis was not given.

The beam pointing accuracy of three dimensional arrays is investigated in this paper.The probability formulas of beam pointing accuracy in (U, V, W) coordinate system is deduced.They are suitable for planar arrays, conformal arrays and 3–dimensional arrays.The theoretical probability of pointing accuracy are in good agreement with simulated statistical ones. The pointing errors in (U, V, W) coordinate system can be translated to those in (θ, φ) coordinate system easily.

STATISTICAL CHARACTERISTICS OF BEAM POINTING ERRORS

In order to evaluate the probability of beam pointing errors, let us compute the mean, variance and correlation function.Assume the beam steering direction is (θ_0, φ_0), the amplitude of excitation current is I_n .The array pattern with excitation errors is as follows :

$$E(U, V, W) = \sum_m J_m \cdot exp\{j[X_m U + Y_m V + Z_m W]\} \tag{1}$$

Where:

$$U = sin\theta cos\varphi - sin\theta_0 cos\varphi_0$$

$$V = sin\theta sin\varphi - sin\theta_0 sin\varphi_0$$

$$W = cos\theta - cos\theta_0$$

$$X_m = K x_m$$

$$Y_m = K\mathrm{Y}_m$$
$$Z_m = K\mathrm{Z}_m$$
$$J_m = I_m (1+\delta_m) exp(j\varphi_m)$$

Here (X_m, Y_m, Z_m) are the position coordinates of the m–th element.(δ_m, φ_m) are the amplitude and phase errors.Their variances are (σ_A^2, σ_φ^2) respectively.If the element arrangement and amplitude distribution are symmetrical to XOZ plane and YOZ plane, and the origin on X_m, Y_m axis is placed on the center of array, the simple results for the mean, variance and correlation function of pointing errors can be obtained :

$$<U_\delta> = <V_\delta> = <W_\delta> = 0 \tag{2}$$

$$K_{uv} = K_{uw} = K_{vw} = 0 \tag{3}$$

$$\sigma_u^2 = \sigma_\varphi^2 \left(1+\sigma_A^2\right) \cdot exp\left(\sigma_\varphi^2\right) \cdot \frac{\sum_m I_m^2 X_m^2}{\left(\sum_m I_m X_m^2\right)^2}$$

$$\sigma_v^2 = \sigma_\varphi^2 \left(1+\sigma_A^2\right) \cdot exp\left(\sigma_\varphi^2\right) \cdot \frac{\sum_m I_m^2 Y_m^2}{\left(\sum_m I_m Y_m^2\right)^2}$$

$$\sigma_w^2 = \sigma_\varphi^2 \left(1+\sigma_A^2\right) \cdot exp\left(\sigma_\varphi^2\right) \cdot \frac{\sum_m I_m^2 Z_m^2}{\left(\sum_m I_m Z_m^2\right)^2} \tag{4}$$

THE PROBABILITY OF BEAM POINTING ERROR

In order to simplify the probability computation, suppose the given beam pointing error area can be bounded with the inequality :

$$\frac{U_\delta^2}{\sigma_u^2} + \frac{V_\delta^2}{\sigma_v^2} + \frac{W_\delta^2}{\sigma_w^2} \leqslant R^2 \tag{5}$$

Certainly, the above hypothesis is not necessary.But with it the complex 3–dimensional integral limits can be simplified.Let:

$$U_\delta = \sigma_u \cdot r\, sin\theta\, cos\varphi$$
$$V_\delta = \sigma_v \cdot r\, sin\theta\, sin\varphi$$
$$W_\delta = \sigma_w \cdot r\, cos\theta$$

Then the probability of pointing error at (U_0, V_0, W_0) over the above given area is

$$P(R)=\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\ \frac{1}{(2\pi)^{3/2}}\cdot exp\left(-\frac{r^2}{2}\right)\cdot r^2\ sin\theta\ dr\ d\theta\ d\varphi$$

$$=\sqrt{\frac{2}{\pi}}\cdot\left\{\frac{R^3}{3}+\sum_{n=1}^{\infty}(-1)^n\frac{R^{(2n+3)}}{2^n\cdot n!\cdot(2n+3)}\right\} \tag{6}$$

SIMULATION RESULTS In order to confirm that analysis of the probability of beam pointing error is correct, the theoretical probabilities and simulated statistical ones in the cases of conformal array and planar array are calculated. Array size: 32 elements along X axis and 16 elements along Y axis. The amplitude distribution along 32 elements is a −30 dB Taylor distribution. Across the 16 elements is an uniform distribution. The surface equation of conformal array is

$$Z=\sqrt{A^2-Y^2}-A \tag{7}$$

where $|X|\leqslant 8\lambda$, $|Y|\leqslant 4\lambda$, $A=8\lambda$. The interelement spacing on XOY plane of planar array is half a wavelength. Assume that conformal array has the same element arrangement on the projection plane XOY as the planar array. When without excitation errors, beam pointing direction is along Z axis. If the r.m.s. of phase error $\sigma_\varphi=0.1$rad, Fig.1 shows that the theoretical (solid line) and simulated statistical probability (symbolic line) of beam pointing error of conformal array are in good agreement to each other. When planar array is under the same conditions, such as size, spacing, aperture distribution, excitation phase errors, etc, good results are also obtained as shown in Fig.2.

CONCLUSIONS In the previously presented paper, the study of the array antenna pointing accuracy is limited to one dimensional linear and two dimensional planar arrays. And only the effect of excitation errors on the mean and variance of pointing error is discussed. Besides, when the element at arrays is with excitation errors, the probability of the beam pointing is not calculated. In this paper, the pointing accuracy problem of three dimensional conformal arrays is investigated. For convenience of calculation, the ellipsoidal integal area is assumed to calculate the probability. It can be seen that for the effect on the pointing accuracy, the phase errors play main roles and the amplitude errors can be ignored.

REFERENCE

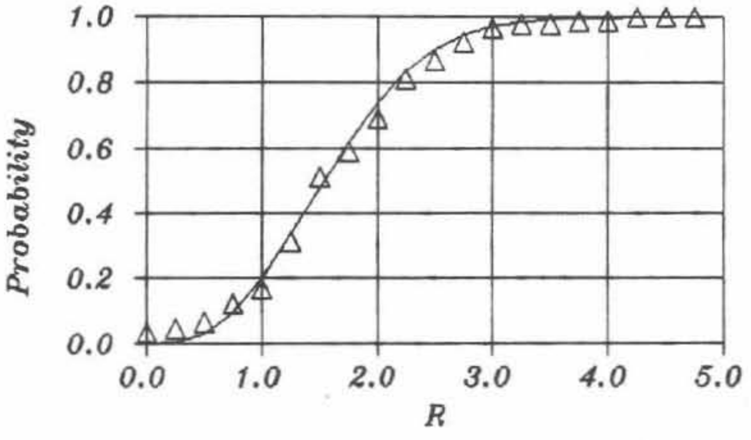
[1] GUO Yanchang, et al, 《Principle of Phased Array and Frequency−Scanning Antennas》 China Defence Industry Press, 1978

[2] M. Leichter, "Beam Pointing Errors of Long Line Sources" IRE Trans. AP, 1960, pp.268∼275

[3] L. A. Rondinelli, "Effects of Random Errors on the Performance of Antenna Arrays of Many Elements" IRE Natl. Conv. Record Pt.I, 1959, pp.174∼189

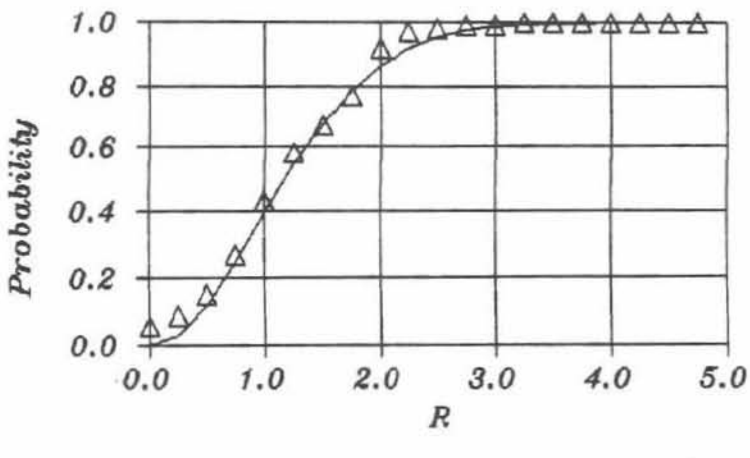
[4] W.H.Nester, "A Study of Tracking Accuracy in Monopulse Phased Arrays" IRE Trans. AP. 1962, pp.237∼246

[5] B. D. Steinberg, 《Principles of Aperture and Array System Design》, JOHN WILEY & SONS, INC, 1976



$\sigma_u = 1.74 \times 10^{-4}$, $\sigma_v = 3.63 \times 10^{-4}$, $\sigma_w = 1.65 \times 10^{-3}$

Fig.1 The comparision of theoretical probability (solid line) and simulated statistical one (△) of beam pointing error of a conformal array



$\sigma_u = 1.74 \times 10^{-4}$, $\sigma_v = 3.63 \times 10^{-4}$

Fig.2 The comparision of theoretical probability (solid line) and simulated statistical one (△) of beam pointing error of a planar array