RAIN DROP SIZE INVERSION BY RANDOM TRIAL-AND-ERROR METHOD

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I. Introduction

When a random medium is discrete we can use the data of wave reflectande, transmittance, or attenuation to obtain parameters representing particle sizes or number densities by means of inversion. In addition to other methods, we have used the modified random trial-and-error (MRT) method based on the inversion by the least squares estimator to determine the size distribution of rain drops with the attenuation data measured in a radio link of 2km with 8mm wave. The result has been compared with that of Phillip and Towmey (PT)[1]. It has been found that the MRT method is better than the PT method.

II. The modified random trial-and-error method

Let $y=f(X,B)$, where $X=(x_1 \ldots x_n)$ and $B=(b_1 \ldots b_m)$ are sets of known independent experimental variables and undetermined parameters respectively. The aim is to find an estimate of B for which the sum of squares

$$Q=\sum_{i=1}^{n}\left[y_i-f(X_i,B)\right]^2 \tag{1}$$

is minimum, where y_i is the ith observation of y. MRT method has the following procedure[2].

1. For an unknown b_k a solution interval is given as follows,

$$E_k-\Delta_k \le b_k \le E_k+\Delta_k \qquad (k=1,\ldots,m)$$

where E_k and Δ_k are the interval and the half interval respectively. For each time, in the above interval of $b_k (k=1,\ldots,m)$, a set of pseudorandom number of m members $B^{(j)}=(b_1^j, b_2^j, \ldots, b_m^j)$ can be generated independently for each $B^{(j)}$ according to a uniform or a triangular distribution. After t times a sequence of sets of B is obtained as

$$B^{(1)}, B^{(2)}, \ldots, B^{(t)}$$

The corresponding sequence $Q^{(j)}$ calculated by (1) with the sequence $B^{(j)}$ can be arranged decreasingly as

$$Q^{(1)} > Q^{(2)} > \ldots > Q^{(t)}$$

2. A weighted sequence defined by $W^{(j)}=Q^{(t)}/Q^{(j)}$ can be obtained accordingly,

$$W^{(1)} < W^{(2)} < \ldots < W^{(t)}=1$$

Now, we can compute the next new searched solution interval E_k and Δ_k with the following formulas

$$E_k=\sum_{j=1}^{t} W^{(j)}\, b_k^{(j)} \Big/ \sum_{j=1}^{t} W^{(j)} \qquad (k=1,\ldots,m) \tag{2}$$

$$\Delta_k=c\sqrt{\sum_{j=1}^{t} W^{(j)}\,(b_k^{(t)}-E_k)^2 \Big/ \sum_{j=1}^{t} W^{(j)}} \qquad (k=1,\ldots,m) \tag{3}$$

where c is a positive number selected by trials.

3. Preceed repeatedly with new E_k and Δ_k. Generally, E_k will approach the expected true value of B and Δ_k will become smaller and smaller, till the condition $Q^{(t)} < \epsilon$ is met. ϵ is the given allowable error bound.

III. Size distribution of rain drops

When multiple scattering and depolarization effect are neglected, the plane electromagnetic wave attenuation induced by rain and the drop size distribution N(D) are related by the following equation[3],

$$G(R_i)=\int_{D_{min}}^{D_{max}} K(D)N(D,R_i)dD \quad (4)$$

where $G(R_i)$ is the attenuation rate in dB/km, R_i is the rain rate in mm/hr, $K(D)$ is the extinction cross section of a single drop, D denotes the diameter of a drop. For the moderate and low rain rate the size distribution density function per unit volume per length N(D) can be assumed as

$$N(D)=N_0\exp(-AD) \quad \text{,and } A=N_1R^{-N_2} \quad (5)$$

where N_0, N_1, and N_2 are parameters to be determined. According to the MRT method described above, sets of random numbers $B=(N_0,N_1,N_2)$ can be taken by the computer so that the following function Q becomes the minimum:

$$Q=\sum_{i=1}^{n}(G-G_{e,i})^2 \quad (6)$$

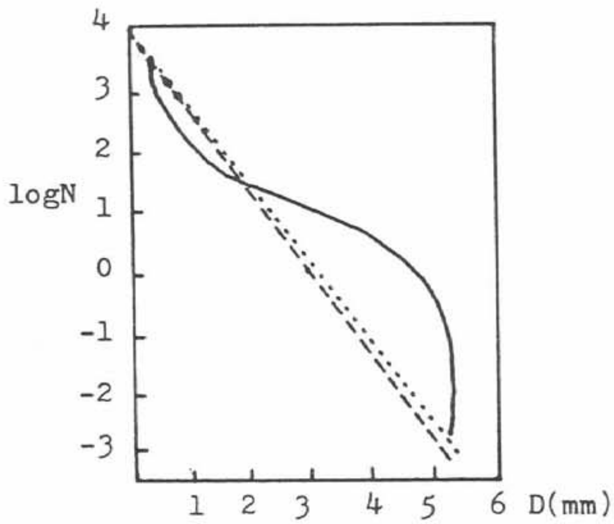

Fig.1 Rain drop size distribution (R 4.2 mm/hr)
..... MRT
——— PT
----- MP

where $G_{e,i}$ is the ith measured attenuation rate when rain rate is R_i. Values of the attenuation rate G can be calculated by the MRT method with (4). Experimental data of $G_{e,i}$ can be obtained from the measured values of the ground radio kink in the local area. The computed results by the MRT method show that in case of moderate and low rain rates (dotted line in Fig.1), the calculated size distribution follows the MP distribution (dashed line in Fig.1)[4]. When PT method is used, equation (4) must be discretized to be a matix [1], and one of the best result is shown in Fig.1 with the solid line.

The result of PT method has more discrepancy between calculated and the MP distribution. Thus, the validity of the MRT method has been confirmed.

IV. Discussions

MRT method is a technique of experience and skill. We give some discussions for its efficiency and effectiveness.

1. In the randomly selected sequence $B^{(j)}(j=1,2,\dots,t)$, the value of t must be appropreiate. If t is too large the effect of contraction interval will be small. When t is too small no end and low efficiency will occur in calculation. In above example we have selected t to be 5.
2. The positive number c is selected by trials. When it is larger than unity, there is a possibility leading the true solution to be located outside the initial trial interval. This is permissible in the MRT method and also is its advantage. By trial, in above example

a suitable choice of c is 2-2.5.

3. For the given trial interval $(E-\Delta, E+\Delta)$, the true value located in the middle of this interval is most probable. Therefore, we may use uniformly and triangularly distributed random numbers alternatively.

4. Values of error depend on the actual problem. Generally they are chosen to have the same order as the accuracy required. If they are too small the computation efficiency is definitely low even no ending. In the above example we have chosen $\epsilon=10^{-4}$.

5. We have not found a theoretical proof of the convergence of the MRT method. But random trial can use the best as a new initial trial.

The algorithm of the MRT method is simple and flexible. In inversion, the MRT method can give the result in a shorter CPU time, while other numerical methods are inefficient. In problems requiring high accuracy, the MRT method can provide a better first step rough solution. Of course, when more than one set of solutions are wanted or the unknown function f(B,X) of (1) varies seriously with respect to B, it may fail.

Refferences

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