

## TM-MODE RADIATION FROM A FLANGED PARALLEL-PLATE WAVEGUIDE WITH DIELECTRIC PLUG SHAPED LIKE A WEDGE

Vitaliy P. CHUMACHENKO<sup>1</sup>, and Ugur SEVGEN  
 Electronics Engineering Department, Gebze Institute of Technology  
 41400 Gebze, Kocaeli, Turkey  
 E-mail: chumac@penta.gyte.edu.tr

### 1. Introduction

The problem of radiation from flanged waveguides with dielectric plugs has received much attention in the past [1-4] because the plugs are useful as means for protection from the environment and for impedance matching. The previous works have discussed loading the antennas with rectangular inserts. In the present paper we study the radiation structure with a plug shaped from the waveguide end like a wedge. The method of analysis is domain product technique (DPT) [5]. Two distinctive cases, associated with absence or existence of the higher order mode propagating inside the dielectric, are examined. In the latter case, as it is demonstrated with the aid of computational experiment, modifying profile of the insert affects the radiation pattern considerably. In particular, our analysis shows that proper choice of the shape of the plug can provide, along with reduction of the reflected wave amplitude, low level of the side radiation as well.

### 2. Formulation

The geometry of the problem is given in Fig.1. The plug of a lossless dielectric with relative permittivity  $\epsilon$  occupies region 2. The residual part of a finite parallel-plate waveguide forms region 3. Region 1 is half-space  $y > 0$ . The structure is excited by a planar sheet of an invariable in  $x$  magnetic current located at  $y \rightarrow -b$  and polarized in  $z$ -direction. The assumed time dependence is  $\exp(i\omega t)$ . The frequency is sufficiently low so that only a TEM mode can propagate among all modes excited in region 3. Distance  $b$  is sufficiently large to neglect interaction between the back wall of the waveguide and the plug through evanescent modes. The configuration has a plane of symmetry  $x = 0$ .

A local Cartesian coordinate system  $(x_j^{(p)}, y_j^{(p)})$  is introduced for the  $j$ th boundary segment of the  $p$ th region, where  $x_1^{(1)} = x$ ,  $y_1^{(1)} = y$  and  $j = \overline{1,5}$  for  $p = 2$  or  $p = 3$ . Axis  $Oy_j^{(p)}$  is directed into the domain of the field and orientation of the system is chosen in such a way that  $\hat{x}_j^{(p)} \times \hat{y}_j^{(p)} = \hat{z}$ . The length of the  $j$ th segment is  $2f_j^{(p)}$ .

Let  $I$  be the free-space wavelength,  $k_1 = k_3 = k_0 = 2\pi / I$ ,  $k_2 = \sqrt{\epsilon} k_0$  with  $u_0 = \exp(-ik_0(y+b))$  the known contribution of the source. Introduce local elliptic coordinate systems  $(\mathbf{x}_j^{(p)}, \mathbf{h}_j^{(p)})$  as

$$x_j^{(p)} = f_j^{(p)} \cosh \mathbf{x}_j^{(p)} \cos \mathbf{h}_j^{(p)}, \quad y_j^{(p)} = f_j^{(p)} \sinh \mathbf{x}_j^{(p)} \sin \mathbf{h}_j^{(p)} \quad (1)$$

and define the  $z$ -component of the magnetic field sought in regions 1, 2 and 3 as  $u^{(1)}$

$$u^{(2)} = u_s^{(2)}, \quad u^{(3)} = u_s^{(3)} + u_0 \quad (2)$$

Functions  $u^{(1)}$  and  $u_s^{(p)}$  ( $p = 2,3$ ) must satisfy the 2D homogeneous Helmholtz equations with boundary conditions

$$\left. \frac{\partial u^{(1)}}{\partial \mathbf{h}_1^{(1)}} \right|_{\mathbf{h}_1^{(1)}=0} = \left. \frac{\partial u^{(1)}}{\partial \mathbf{h}_1^{(1)}} \right|_{\mathbf{h}_1^{(1)}=p} = 0, \quad \left. \frac{\partial u^{(p)}}{\partial y_j^{(p)}} \right|_{\substack{y_j^{(p)}=0 \\ |x_j^{(p)}| < f_j^{(p)}}} = 0, \quad p = 2,3 \quad (3)$$

<sup>1</sup> On leave from Technical University of Zaporizhzhya, 64 Zhukovsky Str., 330063 Zaporizhzhya, Ukraine

on the conducting parts of the boundaries and conditions

$$u^{(p)} \Big|_{y_j^{(p)}=0+} = u^{(2)} \Big|_{y_j^{(2)}=0+}, \quad \frac{\partial u^{(p)}}{\partial y_j^{(p)}} \Big|_{y_j^{(p)}=0+} = -\mathbf{e} \frac{\partial u^{(2)}}{\partial y_j^{(2)}} \Big|_{y_j^{(2)}=0+}, \quad |x_j^{(2)}| < f_j^{(2)}, \quad p=1,3 \quad (4)$$

on the interfaces. In addition, the edge condition and the radiation condition prescribed in region 1 must be satisfied.

Region 1 has separable geometry in coordinates  $(\mathbf{x}_1^{(1)}, \mathbf{h}_1^{(1)})$ . Solution to the Helmholtz equation  $u^{(1)}$ , satisfying the first boundary condition in (3) and the radiation one, has general form

$$u^{(1)} = \sum_{n=0}^{\infty} {}^1D_n \frac{Me_n^{(2)}(\mathbf{x}_1^{(1)}, q_1^{(1)})}{Me_n^{(2)}(0, q_1^{(1)})} ce(\mathbf{h}_1^{(1)}, q_1^{(1)}) \quad (5)$$

Here,  $ce_n(\mathbf{h}, q)$  is the even Mathieu function of index  $n$ ,  $Me_n^{(2)}(\mathbf{x}, q)$  is the relevant radial Mathieu function,  $q_1^{(1)} = (k_1 f_1^{(1)} / 2)^2$  and  $\{{}^1D_n\}$  is a sequence of expansion coefficients to be sought.

Following the DPT, we write down quantities  $u_s^{(2)}$  and  $u_s^{(3)}$  as

$$u_s^{(p)} = \sum_{j=1}^5 u_j^{(p)}, \quad p=2,3 \quad (6)$$

where every function  $u_j^{(p)}$  can be represented [5] by a similar to (5) fast convergent series

$$u_j^{(p)} = \sum_{n=0}^{\infty} {}^pD_n^j \frac{Me_n^{(2)}(\mathbf{x}_j^{(p)}, q_j^{(p)})}{Me_n^{(2)}(0, q_j^{(p)})} ce(\mathbf{h}_j^{(p)}, q_j^{(p)}), \quad q_j^{(p)} = \left( \frac{k_p f_j^{(p)}}{2} \right)^2 \quad (7)$$

Inserting (2), (5), (6) and (7) into (3) and (4) yields, in the manner usual for the DPT [5], the infinite algebraic system with respect to expansion coefficients  $\{{}^1D_n\}$  and  $\{{}^pD_n^j\}$ . The symmetry of the radiation structure enables reducing twice the order of simultaneous equations in the analysis. After truncation of the system and subsequent inversion, the expansion coefficients are found and, thus, all scattering characteristics of the structure can be evaluated.

Note that  $u_0$  does not describe in full the impinging wave, owing to the re-reflections of the dominant mode from the back wall of the waveguide. Therefore, after solution of the system, both the amplitude  $A$  of the aggregate wave  $Ae^{-ik_0 y_m^{(3)}}$  impinging on the plug and that for the wave  $Be^{ik_0 y_m^{(3)}}$  reflected from the interface are to be found in order to determine reflection coefficient  $R = B / A$ . Under assumption that  $u^{(3)}$  is known quantities  $A$  and  $B$  can be obtained from simple simultaneous equations

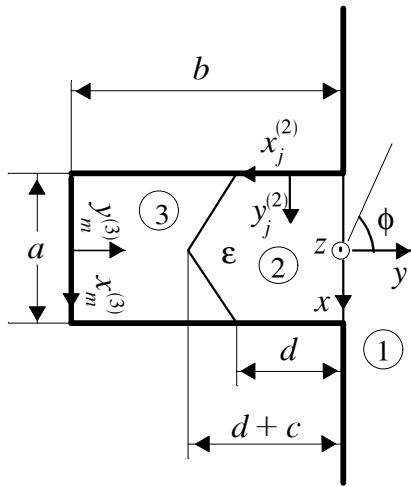
$$\left. \begin{aligned} Ae^{-ik_0 l_1} + Be^{ik_0 l_2} &= \int_{-f_m^{(3)}}^{f_m^{(3)}} u^{(3)} \Big|_{y_m^{(3)}=l_1} dx_m^{(3)} \\ Ae^{-ik_0 l_2} + Be^{ik_0 l_1} &= \int_{-f_m^{(3)}}^{f_m^{(3)}} u^{(3)} \Big|_{y_m^{(3)}=l_2} dx_m^{(3)} \end{aligned} \right\} \quad (8)$$

Equalities (8) follow from the expansion of the  $u^{(3)}$  in terms of eigenmodes after integration over two cross-sections of the waveguide at  $y_m^{(3)} = l_1$  and  $y_m^{(3)} = l_2$ . Here,  $(x_m^{(3)}, y_m^{(3)})$  is the local Cartesian coordinate system related to the back wall of the waveguide,  $0 < l_1, l_2 < b - \max(d, d + c)$ , and  $l_1 \neq l_2$ .

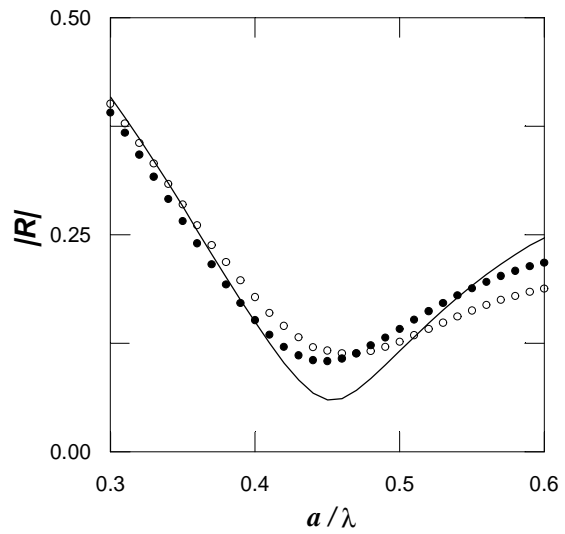
### 3. Numerical results

The validity of the theory and the pertinent computer code was checked for the well-studied case of the unloaded antenna. The comparison of the radiation pattern and the reflection coefficient between [6] and ours when  $\mathbf{e} = 1$  shows excellent agreement. For example, the corresponding values of  $|R|$  are 0.064 (calculated for  $\mathbf{e} = 1, d = 0.3a, c = 0.3a, a / \mathbf{I} = 0.8$ ) and 0.06433 [6].

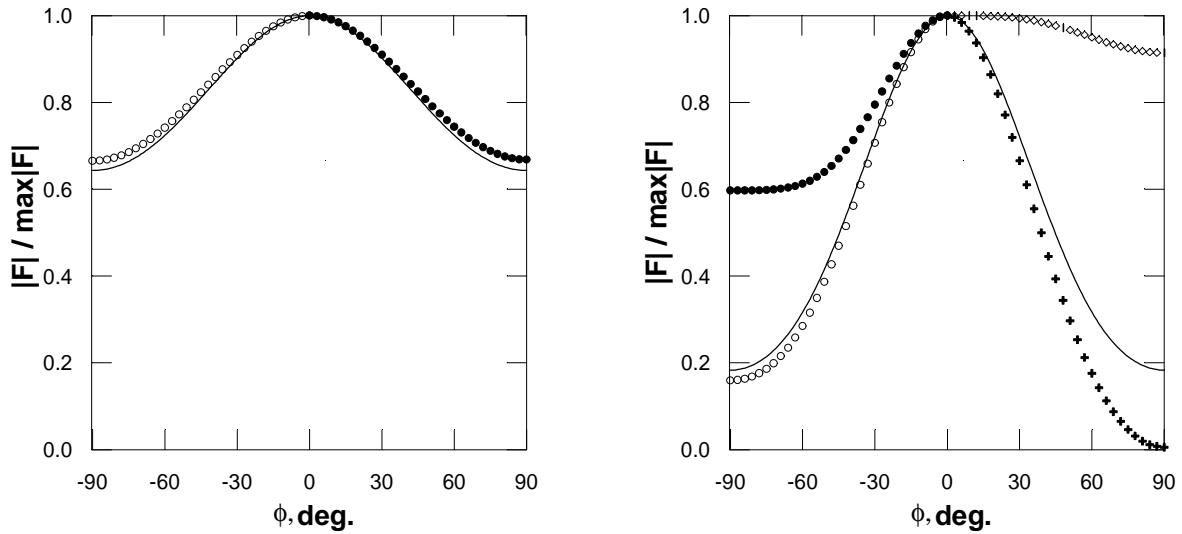
Figs. 2-5 present results obtained for two typical situations when only the TEM mode or two eigenmodes of the waveguide can propagate in the dielectric. In the former case, the reflection coefficient is shown in Fig. 2 as a function of  $a/l$  for fixed values of geometrical parameters.



**Fig. 1** Geometry of the problem.



**Fig. 2**  $|R|$  versus  $a/l$  for  $\epsilon = 2$ . Solid line:  $d = 0.68a, c = 0$ ; circles:  $d = 0.47a, c = 0.45a$ ; dots:  $d = 0.9a, c = -0.37a$ .



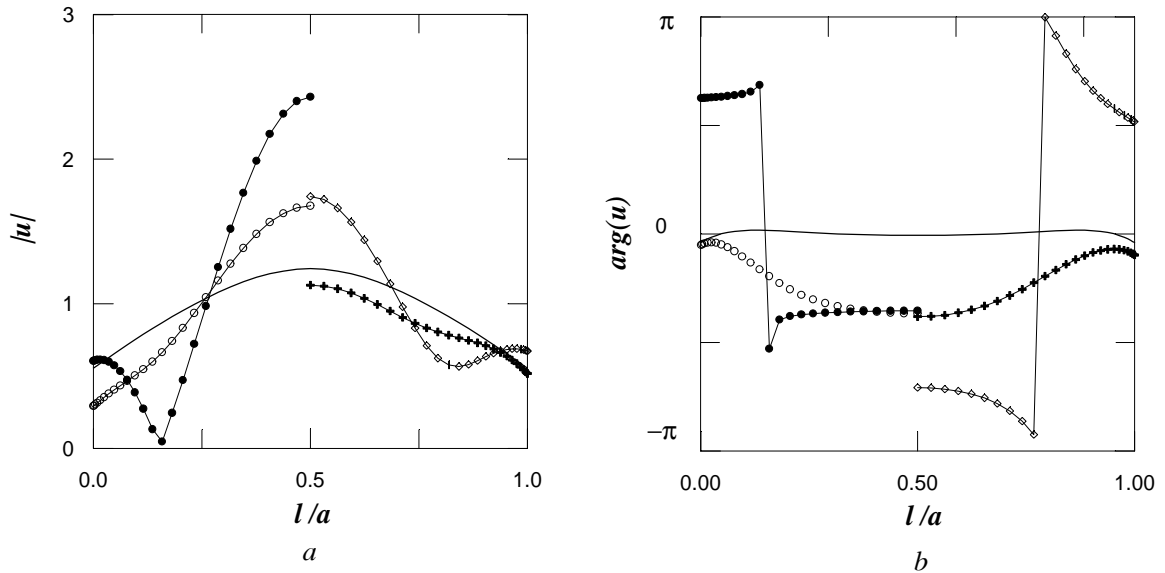
**Fig. 3** Far-field pattern of the unloaded and loaded flanged waveguide for  $a/l = 0.45$ . Solid line:  $\epsilon = 1$ ; circles:  $\epsilon = 2, d = 0.68a, c = 0$ ; dots:  $\epsilon = 2, d = 0.47a, c = 0.45a$ .

**Fig. 4** Far-field pattern of the unloaded and loaded flanged waveguide for  $a/l = 0.85$ . Solid line:  $\epsilon = 1$ ; circles:  $\epsilon = 2, d = 0.42a, c = 0$  ( $|R| = 0.018$ ); dots:  $\epsilon = 2, d = 0.2a, c = 0.62a$  ( $|R| = 0.037$ ); crosses:  $\epsilon = 2, d = 0.57a, c = -0.4a$  ( $|R| = 0.195$ ); squares:  $\epsilon = 4, d = 0.15a, c = 0.7a$  ( $|R| = 0.361$ ).

Values of  $d$  were chosen with the aid of computational experiment to provide reduction in  $|R|$  for  $a/l = 0.45$ . As it is seen, shaping of the plug has no advantages in comparison with the flat

interface. Related radiation patterns are shown in Fig.3. They are close to that for the unloaded antenna.

Field patterns corresponding to the latter situation are given in Fig. 4. As above, the rectangular plug can ensure almost perfect matching of the antenna. The break of the interface between regions 2 and 3 provides excitation of the higher-order mode propagating in the dielectric. In this case, alterations of  $d$  and  $c$  give an opportunity to change the magnitude as well as the phase distribution of the field over the aperture (shown in Fig. 5) and obtain radiation patterns of different type. Specifically, proper choice of geometrical parameters can lower the level of the side radiation considerably.



**Fig. 5** Aperture magnetic field relating to far-field pattern in Fig. 4 versus normalised path length  $l/a = (x + f_1^{(1)})/a$ . (a) Magnitude. (b) Phase ( $b = \max(d, d + c) + a$ ).

### References

- [1] C. P. Wu and Galindo, "Surface wave effects on phased arrays of rectangular waveguides loaded by dielectric plugs," *IEEE Trans. Antennas Propagat.*, vol. AP-16, pp. 358-360, May 1968.
- [2] C. T. Swift, "Admittance of a waveguide-fed aperture loaded with a dielectric plug," *IEEE Trans. Antennas Propagat.*, vol. AP-17, pp. 356-359, May 1969.
- [3] C. P. Wu, "Characteristics of coupling between parallel-plate waveguides with and without dielectric plugs," *IEEE Trans. Antennas Propagat.*, vol. AP-18, pp. 188-194, Mar. 1970.
- [4] S. Gupta, A. Bhattacharya and A. Chakraborty, "Analysis of an open-ended waveguide radiator with dielectric plug," *IEE Proc. Microw. Antennas Propagat.*, vol. 144, pp. 126-130, Apr. 1997.
- [5] V. P. Chumachenko, E. Karacuha and M. Dumanli, "An analysis of TE-scattering from a multiangular groove in a ground plane," *J. Elect. Waves Appl.*, vol. 13, pp. 381-396, Mar. 1999.
- [6] M. S. Leong, P. S. Kooi and Chandra, "Radiation from a flanged parallel-plate waveguide: solution by moment method with inclusion of edge condition," *IEE Proceedings*, vol. 135, Pt. H, pp. 249-255, Aug. 1988.