Modified Surface-normal Vector for improving Physical Optics

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1 Introduction

This paper aims to improve Physical Optics (PO) of the high frequency approximation. PO defines currents induced on the scatterer surface illuminated by the source as $2\hat{\boldsymbol{n}} \times \boldsymbol{H_i}$ ($\hat{\boldsymbol{n}}$: surface-normal vector, $\boldsymbol{H_i}$: incident magnetic field) by assuming the that surface to be an infinite plane. The scattered field of PO is calculated by the radiation surface integral of the PO currents. PO is free of singularities in contrast other high frequency approximations such as Geometrical Theory of Diffraction (GTD). However, PO is generally inaccurate in shadow regions where direct incident waves from sources are screened by a scatterer. The asymptotic and analytical evaluation by the method of stationary phase is applied to the radiation surface integrals of PO for comparison of PO and GTD in terms of the expression for the Equivalent Edge Currents (EEC). The difference between both methods is only the factor of sine function, which causes the diffraction error of PO. Physical Theory of Diffraction (PTD) is well-known as improvement of the diffraction error of PO by using Fringe Wave (FW). The purpose in this paper is to improve the diffraction error of PO by redefinition of the novel surface equivalent currents using modified surface-normal vector $\hat{\boldsymbol{n}}$ in the PO currents, without additional edge equivalent currents such as FW. This unique idea was first suggested in [1]. The similar idea was used to recover the exact solution for a half sheet and the complementary aperture in [2]. This approach of modifying the surface-normal vector is analogous to the idea of Modified Edge Representation (MER) where not the currents definition, but the shape of the scatterer is modified so that the Fermat's principle or reflection law are satisfied [3].

First, based on the equivalence principle, the PO currents $\{2\hat{\boldsymbol{n}} \times \boldsymbol{H_i}\}$ are divided into two pairs of equivalent currents; one is the component radiating into the reflection region of the scatterer $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}\text{: AFIM}^-\}$, the other is the component radiating into the shadow region of the scatterer $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i},\ \boldsymbol{E_i} \times \hat{\boldsymbol{n}}\text{: AFIM}^+\}$ [4, 5]. Here, the pair of the equivalent currents in AFIM$^+$ and AFIM$^-$ match the definition for the equivalent currents in the Aperture Field Integration Method (AFIM) used when approximating, for instance, radiation toward the shadow and the main beam regions in a reflector antenna. Then, the surface-normal vector $\hat{\boldsymbol{n}}$ in these AFIM currents replaces the novel surface-normal vector ($\hat{\boldsymbol{n}}_{\boldsymbol{b}}$ and $\hat{\boldsymbol{n}}_{\boldsymbol{f}}$) which is modified to satisfy the reflection low for the source and the mirror image, respectively. These AFIM currents are redefined as $\{\hat{\boldsymbol{n}}_{\boldsymbol{b}} \times \boldsymbol{H_i},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}_{\boldsymbol{b}}\}$ and $\{\hat{\boldsymbol{n}}_{\boldsymbol{f}} \times \bar{\boldsymbol{H}}_{\boldsymbol{i}},\ -\bar{\boldsymbol{E}}_{\boldsymbol{i}} \times \hat{\boldsymbol{n}}_{\boldsymbol{f}}\}$, respectively, where $\bar{\boldsymbol{H}}_{\boldsymbol{i}}$ and $\bar{\boldsymbol{E}}_{\boldsymbol{i}}$ are assumed the incident field from the mirror image $\{\bar{\boldsymbol{H}}_{\boldsymbol{i}} = -\boldsymbol{H_i} + 2\hat{\boldsymbol{n}}(\boldsymbol{H_i} \cdot \hat{\boldsymbol{n}}),\ \bar{\boldsymbol{E}}_{\boldsymbol{i}} = \boldsymbol{E_i} - 2\hat{\boldsymbol{n}}(\boldsymbol{E_i} \cdot \hat{\boldsymbol{n}})\}$. The scattered field from the set of currents $\{\hat{\boldsymbol{n}}_{\boldsymbol{b}} \times \boldsymbol{H_i} + \hat{\boldsymbol{n}}_{\boldsymbol{f}} \times \bar{\boldsymbol{H}}_{\boldsymbol{i}},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}_{\boldsymbol{b}} - \bar{\boldsymbol{E}}_{\boldsymbol{i}} \times \hat{\boldsymbol{n}}_{\boldsymbol{f}}\}$ are predicted to improved the diffraction error of PO. From viewpoint of the reciprocity theorem, it seems natural the equivalent currents depends both on the source and the observer position via the newly defined surface normal vector, in contrast PO.

2 Modified surface-normal vector

Physical Optics (PO) is the wave optics and the scattered field is calculated by the radiation surface integral of the PO currents $\{2\hat{\boldsymbol{n}} \times \boldsymbol{H_i}\}$ over the lit surface. For applying the stationary phase method, an edge diffraction is expressed by the cylindrical wave from the

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equivalent edge currents with the diffraction coefficients which are only function of incident direction ϕ_i an observer angle ϕ as well as GTD.

$$\boldsymbol{E^d} = \boldsymbol{E^i} D_{//} \sqrt{\frac{1}{2\pi k\rho}} e^{-jk\rho - j\frac{\pi}{4}} \tag{1}$$

where $\boldsymbol{E_i}$ is the incident electric field, $D_{//}$ is the diffraction coefficients, and ρ is the distance from an edge to a observer point.

Table 1 shows the diffraction coefficients of PO, GTD, and so on. The difference of PO and GTD is existence of the sine function in the numerator of the diffraction coefficients. The purpose in this paper is to make up the difference of this sine function and to improve the diffraction error of PO by modified the surface-normal vector $\hat{\boldsymbol{n}}$ of the PO currents.

Table 1: Diffraction coefficients

	$2D^{//}$		$2D_F^{//}$
PO	$\frac{\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}} - \frac{\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$	PO_F	$F_- \frac{\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}} - F_+ \frac{\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$
AFIM^-	$-\frac{\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$	AFIM_F^-	$-F_+ \frac{\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$
AFIM^+	$\frac{\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}}$	AFIM_F^+	$F_- \frac{\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-F\phi_i}{2}}$
GTD	$\frac{1}{\cos\frac{\phi-\phi_i}{2}} - \frac{1}{\cos\frac{\phi+\phi_i}{2}}$	GTD_F (UTD)	$F_- \frac{1}{\cos\frac{\phi-\phi_i}{2}} - F_+ \frac{1}{\cos\frac{\phi+\phi_i}{2}}$
FW	$\frac{1-\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}} - \frac{1-\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$	FM_F	$F_- \frac{1-\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}} - F_+ \frac{1-\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$

$F\pm = F\left(2kd\cos^2\frac{\phi\pm\phi_i}{2}\right),\quad F(x) = 2j\sqrt{x}\exp(jx)\int_{\sqrt{x}}^{\infty}\exp\left(-j\tau^2\right)d\tau$

Based on the equivalence principle, the PO current $\{2\hat{\boldsymbol{n}} \times \boldsymbol{H_i}\}$ are divided into two pairs of equivalent currents; the component radiating into the reflection region of the scatterer $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}$: AFIM$^-\}$ and the component radiating into the shadow region of the scatterer $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i},\ \boldsymbol{E_i} \times \hat{\boldsymbol{n}}$: AFIM$^+\}$. The method of stationary phase is applied to these equivalent currents, the diffraction coefficients are divided into the terms of $-\frac{\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$ and $\frac{\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}}$. The surface-normal vector $\hat{\boldsymbol{n}}$ is modified so that the diffraction coefficients of PO became identical to those of GTD $-\frac{1}{\cos\frac{\phi+\phi_i}{2}}$ and $\frac{1}{\cos\frac{\phi-\phi_i}{2}}$.

An actual surface-normal vector $\boldsymbol{n}$ in the pair of equivalent currents $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}\}$ is modified a novel modified surface-normal vector $\boldsymbol{n_b}$ to satisfy the refraction low to a source as shown in Fig. 1. The scattered field from $\{\hat{\boldsymbol{n}}_{\boldsymbol{b}} \times \boldsymbol{H_i},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}_{\boldsymbol{b}}\}$ becames $\frac{1}{\sin\frac{\phi+\phi_i}{2}}$ than one from the former currents, as a result, the diffraction coefficients become $-\frac{1}{\cos\frac{\phi+\phi_i}{2}}$. A surface-normal vector $\boldsymbol{n}$ in the pair of equivalent currents as well as $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i},\ \boldsymbol{E_i} \times \hat{\boldsymbol{n}}\}$ is modified a novel modified surface-normal vector $\boldsymbol{n_b}$ to satisfy the refraction low to a mirror image as shown in Fig. 2. The scattered field from $\{\hat{\boldsymbol{n}}_{\boldsymbol{f}} \times \boldsymbol{H_i},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}_{\boldsymbol{f}}\}$ became $\frac{1}{\sin\frac{\phi-\phi_i}{2}}$ than one from former currents, as a result, the diffraction coefficient become $\frac{1}{\cos\frac{\phi-\phi_i}{2}}$. The scattered field from the set of currents $\{\hat{\boldsymbol{n}}_{\boldsymbol{b}} \times \boldsymbol{H_i} + \hat{\boldsymbol{n}}_{\boldsymbol{f}} \times \bar{\boldsymbol{H}}_{\boldsymbol{i}},\ -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}_{\boldsymbol{b}} - \bar{\boldsymbol{E}}_{\boldsymbol{i}} \times \hat{\boldsymbol{n}}_{\boldsymbol{f}}\}$ is predicted to be equivalent with GTD diffraction coefficients.

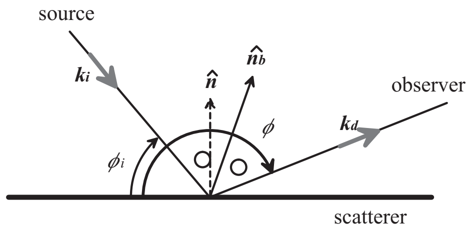


Figure 1: Definition of modified unit normal vector outer $\hat{\boldsymbol{n}}_{\boldsymbol{b}}$ for satisfying the law of reflection to the source

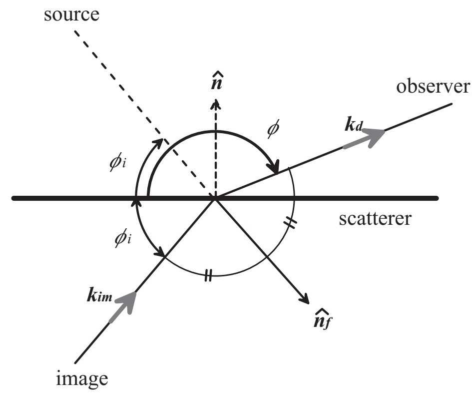


Figure 2: Definition of modified unit normal vector outer $\hat{\boldsymbol{n}}_{\boldsymbol{f}}$ for satisfying the law of reflection to the mirror image

3 Numerical results

For the scattering problem for a 2-D strip in Fig. 3, the scattered field of the component created the reflection wave are examined numerically and is shows in Fig. 4. UTD$^-$ is calculated the diffraction wave with only the diffraction coefficient of $-\frac{1}{\cos\frac{\phi+\phi_i}{2}}$ and the geometrical optics component of only the reflection wave. AFIM$^-$ is calculated the surface integral of the equivalent current of $\{\hat{\boldsymbol{n}} \times \boldsymbol{H_i}, -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}\}$ over the strip. Modified AFIM$^-$ is numerical result by the surface integral of the proposed equivalent current $\{\hat{\boldsymbol{n}}_{\boldsymbol{b}} \times \boldsymbol{H_i}, -\boldsymbol{E_i} \times \hat{\boldsymbol{n}}_{\boldsymbol{b}}\}$. Although modified AFIM$^-$is in good agreement with UTD$^-$in the reflection region (0 deg $< \phi <$ 180 deg), erros are notable in the shadow region (-180 deg $< \phi <$ 0 deg).

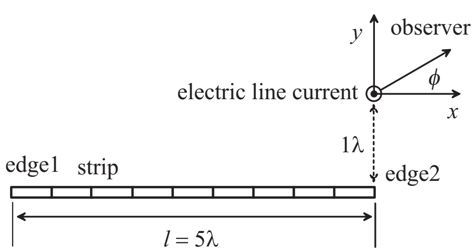


Figure 3: The electromagnetic scattering problem for a 2-D strip illuminated by a line electric current

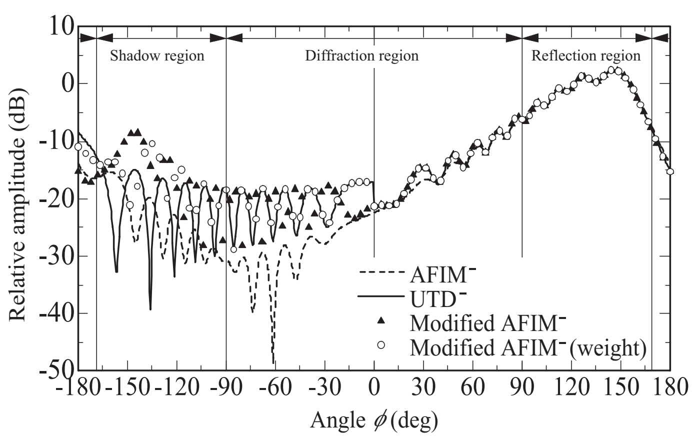
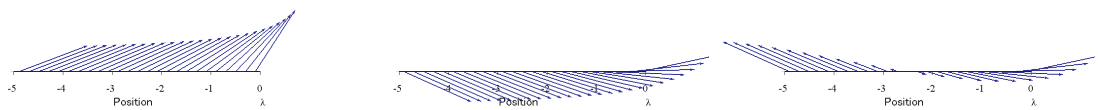


Figure 4: Scattered field of the reflection component for the 2-D strip

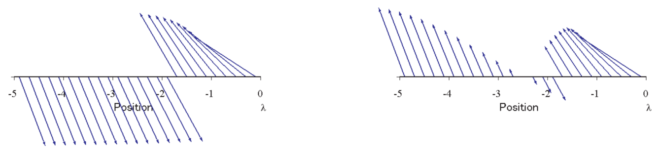
Now, an empirical modification is introduced and is tested. Modified AFIM$^-$ (weight) shows the results when the currents are weighted so that the sign of diffraction coefficients of only the edge1 side is reversed. This achieves the improvement the diffraction in the shadow region (-90 deg $< \phi <$ 0 deg). This weighting function is only empirical and should be discussed in more theoretically.

$$(\hat{\boldsymbol{n}}_{\boldsymbol{b}} \times \boldsymbol{H}_{\boldsymbol{i}}) \cdot \cos\left(\frac{l+2x}{l}\pi\right), (-\boldsymbol{E}_{\boldsymbol{i}} \times \hat{\boldsymbol{n}}_{\boldsymbol{b}}) \cdot \cos\left(\frac{l+2x}{l}\pi\right) \tag{2}$$

Figure 5 shows the modified surface-normal vector $\hat{\boldsymbol{n}}_{\boldsymbol{b}}$ at various observation angles ϕ.



(a) $\phi = 30$ deg (b) $\phi = -60$ deg (c) $\phi = -60$ deg (weight)



(d) $\phi = -150$ deg (e) $\phi = -150$ deg (weight)

Figure 5: Modified surface-normal vector multiplied by the weighting function for the 2-D strip in various observer angles.

4 Conclusion

A novel method which modifies the surface-normal vector in the PO currents is proposed to improve PO. The effectiveness is numerically demonstrated for two-dimensional strip. Although the definition is still immature and an additional weighting function is introduced empirically, which is important especially in the shadow regions.

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