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# ON THE DIFFRACTION OF BEAM WAVE BY A CIRCULAR APERTURE

Kazumasa Tanaka
Faculty of Engineering, Nagasaki University, Nagasaki, Japan
and
Otozo Fukumitsu
Faculty of Engineering, Kyushu University, Fukuoka, Japan

## 1. Introduction

The output beam wave from a laser resonator has the amplitude which decreases exponentially as the distance from the propagation axis increases and it has also the surface of constant phase which is approximately spherical.

Apertures such as irises are often used as circuit elements of this beam wave but the diffraction field from them has not been discussed yet.

Besides, it is one of the fundamental problems to investigate the diffraction field of the electromagnetic field from an aperture.

From these points of view, the diffraction field of beam wave from a circular aperture is obtained and the effects of the aperture on the incident beam wave are discussed.

## 2. Diffraction Field

The diffraction field through a plane aperture is often obtained with the Kirchhoff approximation.

This holds when the wavelength of the field is much smaller than the aperture dimension.

Assume that the incident beam wave is linearly polarized and has its smallest spot-size $w_s$ at $z=-z_s$, then this beam wave $\Psi_{mn}(r,\theta,z)$ is given by [1]

$$\Psi_{mn}(r,\theta,z)=A_{mn}(\eta r)^m(\sqrt{1+\xi^2})^{-1}L_{nm}(\eta^2 r^2)\exp[-jkz-\eta^2r^2/2+j\{(2n+m+1)\tan^{-1}\xi-\eta^2\xi r^2/2\}]\cos(m\theta) \quad (1)$$

where $A_{mn}$ : normalization const.
$L_{nm}(x)$ : Laguerre polynomials
$k$ : wave number

and $\xi=2(z+z_s)/kw_s^2$

$\eta=\sqrt{2}/(\sqrt{1+\xi^2}\,w_s)$

The diffraction field $U_{mn}(r,\theta,z)$ of this beam wave through a circular aperture at $z=0$ is, in the Fresnel region, given by

$$U_{mn}(r,\theta,z)=A_{mn}kj^{m+1}\cos(m\theta)/(z\sqrt{1+\xi_0^2})\exp[-jkz-jkr^2/(2z)+j(2n+m+1)\tan^{-1}\xi_0]\sum_{q=0}^{\infty}\sum_{p=0}^{n}\frac{(-1)^{p+q}(p+q+m)!}{p!q!(m+q)!}\binom{n+m}{n-p}\{kr/(2z)\}^{2q+m}\frac{(\eta_0)^{2p+m}(2)^{p+q+m}}{(\eta_0^2\sigma^2)^{p+q+m+1}}\left[1-\exp(-\eta_0^2\sigma^2a^2/2)\sum_{t=0}^{p+q+m}\frac{1}{t!}(\eta_0^2\sigma^2a^2/2)^t\right] \quad (2)$$

where $a$ is the radius of the aperture, $\xi_0$ and $\eta_0$ are the values of $\xi$ and $\eta$ at the aperture and

$$\sigma^2=\sigma_0^2(1+jkw_s\sigma_0^{2*}/2z)$$
$$\sigma_0^2=1+j\xi_0 \quad (3)$$

The result seems to be rather tedious for numerical computations but if we are concerned with only paraxial cases, the infinite series in it converges fast enough.

Fig.1 shows the field intensity distributions on the propagation axis.

## 3. Mode Expansion

The discussions in this section are made for the field in the Fraunhofer region which is easily obtained if $\sigma_0^2$ is used instead of $\sigma^2$ in eq. (2).

The diffraction field of beam wave is not, in general, represented with a single beam mode but may be considered to be the sum of beam modes.

To see this, the diffraction field is expanded into the series of beam modes which have the same beam parameters as the incident beam wave.[2]

$$U_{mn}(r,\theta,z)=\sum_{\bar{m},\bar{n}} C_{mn}^{\bar{m}\bar{n}}\ \Psi_{\bar{m}\bar{n}}(r,\theta,z) \quad (4)$$

The expansion coefficients are easily obtained from the orthogonality of $\{\Psi_{mn}(r,\theta,z)\}$ in the $z$=const. plane.

The physical meanings of $\{C_{mn}^{\bar{m}\bar{n}}\}$ are: $C_{mn}^{mn}$ is the transmission coefficient of the aperture for the incident beam wave and $\{C_{mn}^{\bar{m}\bar{n}} \mid \bar{m}\neq m \text{ or } \bar{n}\neq n\}$ are mode conversion coefficients, which show most clearly the effect of the aperture on the incident beam wave.

Some of $\{C_{mn}^{\bar{m}\bar{n}}\}$ for $m=n=0$ mode incidence are shown in Fig.2.

The expansion (4) is not unique.

In some cases another set of beam mode functions may be appropriate.

For example, we may want to know the position for the aperture to be put and its dimension so that $C_{mn}^{\bar{m}\bar{n}}$ is maximum.

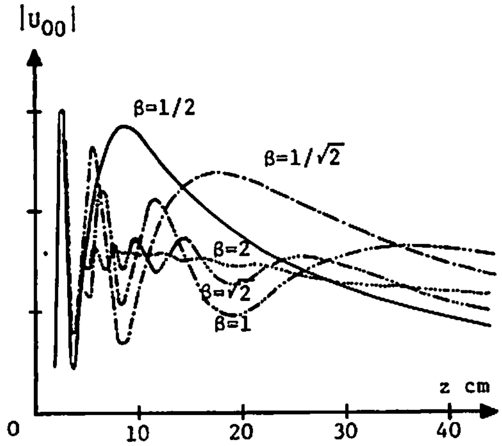


Fig.1 The diffraction field on the propagation axis. $\beta=a/w$ is the ratio of the aperture radius to the spot-size at the aperture.

This mode expansion method will be applied to a mode filter problem.

The effects of the off-axis between the propagation axis of the incident beam wave and the center of the aperture are also discussed. Judging from the results obtained for the field distribution and expansion coefficients, these may be almost negligible if the deviation is less than 1/10 of the aperture radius.

The angular beam width of the diffracted field is also discussed.

The difference between the above width and that of the incident beam wave is the effect of the aperture.

References


1. G.Goubau and F.Schwering: IRE Trans., AP-9, 1961.

2. H.Kogelnik: Proceeding of the symposium on quasi-optics, 1964.


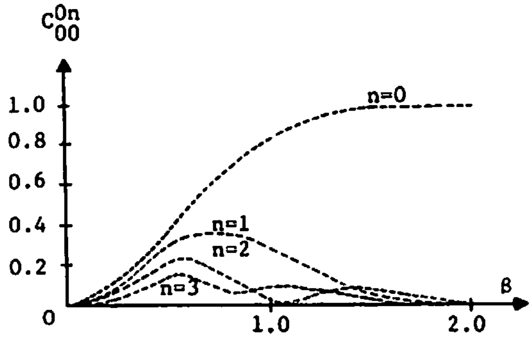


Fig.2 Mode expansion coefficients for $m=n=0$ mode incidence.