PERTURBATION METHOD FOR CALCULATION OF COUPLING COEFFICIENT BETWEEN TWO RESONATORS

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1.Introduction

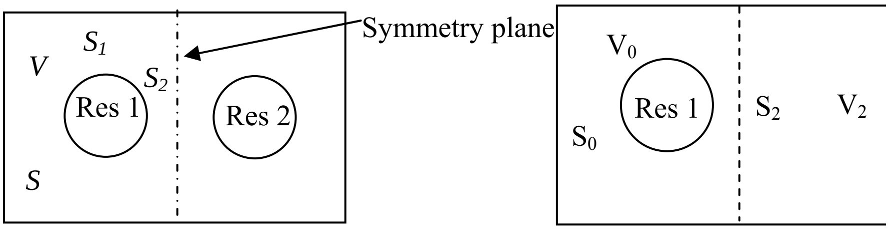
Coupling coefficient of resonators is necessary information for designing an rf bandpass filter. It is usually calculated or measured using two split resonant frequencies of originally the same value before coupling. Considering the versatility of numerical resonant frequency calculation due to the recent advance of EM simulators , the split frequency method looks the best way for determining the coupling coefficient of any resonator structures.

Our proposal, however, is to use the EM field obtained by the simulators instead of the resonant frequencies. Its advantage is the decomposition of electric and magnetic components of the coupling coefficient in compensation for the simplicity of calculation. The principle is based on the perturbation theory[1], which has never been adopted for the calculation of coupling coefficients. The electromagnetic field of a single resonator is calculated and the coupling is given by the ratio of the difference of electric and magnetic energy stored in the limited evanescent region normalized by the total stored energy.

The theory will be derived at first and an example is shown which can be calculated analytically to demonstrate the feasibility of the theory. After that, a numerical calculation will be shown to compare the proposed method with the conventional counterpart.

2.Theory

Two identical resonators couple each other in a closed or open space as shown in Fig.1. The symmetry plane is located in the middle of the resonators. Electric and magnetic walls are assumed at the symmetry plane to calculate the odd and even mode resonant frequency in the split frequency method, respectively. Those frequencies are a little different from the original uncoupled state. Therefore, the structures with those walls are considered the perturbed state from the single resonator shown in Fig.1(b).



(a) two coupled resonators (b) single resonator

Fig.1 Analyzed structure (part of S may be infinite, V_0=V+V_2, S=S_1+S_2)

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The electromagnetic field and the resonant (angular) frequency of the single resonator are assumed $\boldsymbol{E}_o$, $\boldsymbol{H}_o$ and ω_o, respectively, while those for the perturbed one are $\boldsymbol{E}$, $\boldsymbol{H}$ and ω. Maxwell equations hold for both structures

$\nabla\times\boldsymbol{E_0} = -\mathrm{j}\omega_0\mu\boldsymbol{H_0}$ (1.*a*) $\nabla\times\boldsymbol{H_0} = \mathrm{j}\omega_0\varepsilon\boldsymbol{E_0}$ (1.*b*) $\nabla\times\boldsymbol{E} = -\mathrm{j}\omega\mu\boldsymbol{H}$, (2.*a*) $\nabla\times\boldsymbol{H} = \mathrm{j}\omega\varepsilon\boldsymbol{E}$. (2.*b*)

The inner product of (2.b) with $\boldsymbol{E}_0$* and that of the complex conjugate of (1.a) with $\boldsymbol{H}$ give

$$\boldsymbol{E_0^*}\cdot\nabla\times\boldsymbol{H} = \mathrm{j}\omega\varepsilon\boldsymbol{E}\cdot\boldsymbol{E}_0^*, \qquad -\boldsymbol{H}\cdot\nabla\times\boldsymbol{E_0^*} = -\mathrm{j}\omega_0\mu\boldsymbol{H_0^*}\cdot\boldsymbol{H}. \tag{3}$$

Adding these and using the vector identity

$$\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot\nabla\times\boldsymbol{A} - \boldsymbol{A}\cdot\nabla\times\boldsymbol{B}, \tag{4}$$

one obtains

$$\nabla\cdot(\boldsymbol{H}\times\boldsymbol{E_0^*}) = \mathrm{j}\omega\varepsilon\boldsymbol{E}\cdot\boldsymbol{E_0^*} - \mathrm{j}\omega_0\mu\boldsymbol{H_0^*}\cdot\boldsymbol{H}. \tag{5}$$

The similar operations to eqs.(1.b) and (2.a) give

$$\nabla\cdot(\boldsymbol{H}_0^*\times\boldsymbol{E}) = \mathrm{j}\omega\mu\boldsymbol{H}\cdot\boldsymbol{H_0^*} - \mathrm{j}\omega_0\varepsilon\boldsymbol{E_0^*}\cdot\boldsymbol{E}, \tag{6}$$

and hence, integration of (5) ,(6) in V and addition of those will afford the relation

$$\int_S(\boldsymbol{H}\times\boldsymbol{E}_0^* + \boldsymbol{H}_0^*\times\boldsymbol{E})\cdot\boldsymbol{n}\mathrm{d}S = \mathrm{j}(\omega-\omega_0)\int_V(\varepsilon\boldsymbol{E}\cdot\boldsymbol{E_0^*} + \mu\boldsymbol{H}\cdot\boldsymbol{H_0^*})\mathrm{d}v \tag{7}$$

When S_1 is a conductor or infinite surface

$$\int_{S_1}(\boldsymbol{H}\times\boldsymbol{E}_0^*)\cdot\boldsymbol{n}\mathrm{d}S = \int_{S_1}(\boldsymbol{H}_0^*\times\boldsymbol{E})\cdot\boldsymbol{n}\mathrm{d}S = 0, \tag{8}$$

and thus, only the surface integral on S_2 will be important

1) When S_2 is the electric wall, $\boldsymbol{n}\times\boldsymbol{E}$=0 and the resultant term of the left hand side of eq.(7) is

$$I = \int_{S_2}(\boldsymbol{H}\times\boldsymbol{E}_0^*)\cdot\boldsymbol{n}\mathrm{d}S \tag{9}$$

2) When S_2 is the magnetic wall, $\boldsymbol{n}\times\boldsymbol{H}$=0 holds and

$$I = \int_{S_2}(\boldsymbol{H}_0^*\times\boldsymbol{E})\cdot\boldsymbol{n}\mathrm{d}S \tag{10}$$

remains finite. Considering the electromagnetic fields on the perturbed surface S_2 are twice as large as the non-perturbed values, we obtain the resonant frequency for the odd and even mode cases

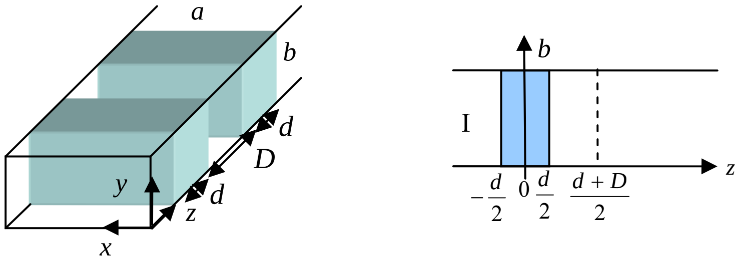
$$\Delta\omega_{od} \cong \frac{-2\mathrm{j}\int_{S_2}(\boldsymbol{H}_0\times\boldsymbol{E}_0^*)\cdot\boldsymbol{n}\mathrm{d}S}{\int(\varepsilon|\boldsymbol{E}_0|^2 + \mu|\boldsymbol{H_0}|^2)\mathrm{d}v} \quad (11) \qquad \Delta\omega_{ev} \cong \frac{-2\mathrm{j}\int_{S_2}(\boldsymbol{H}_0^*\times\boldsymbol{E})\cdot\boldsymbol{n}\mathrm{d}S}{\int(\varepsilon|\boldsymbol{E}_0|^2 + \mu|\boldsymbol{H_0}|^2)\mathrm{d}v} \quad (12)$$

According to the definition of coupling coefficient,

$$k = \frac{2|\omega_{ev}-\omega_{od}|}{\omega_{ev}+\omega_{od}} \cong \frac{|\Delta\omega_{ev}-\Delta\omega_{od}|}{\omega_0} = \frac{2|\mathrm{j}\int_{S_2}(\boldsymbol{H}_0\times\boldsymbol{E}_0^* - \boldsymbol{H}_0^*\times\boldsymbol{E}_0)\cdot\boldsymbol{n}\mathrm{d}S|}{2\omega_0\int_{V_0}\varepsilon|\boldsymbol{E}_0|^2\mathrm{d}v}$$
$$= \frac{2|\mathrm{I_m}\int_{S_2}(\boldsymbol{H}_0\times\boldsymbol{E}_0^*)\cdot\boldsymbol{n}\mathrm{d}S|}{\omega_0\int_{V_0}\varepsilon|\boldsymbol{E}_0|^2\mathrm{d}v} = \frac{2|\int_{V_2}(\varepsilon|\boldsymbol{E}_0|^2 - \mu|\boldsymbol{H_0}|^2)\mathrm{d}v|}{\int_{V_0}\varepsilon|\boldsymbol{E}_0|^2\mathrm{d}v} \tag{13}$$

For fast calculation of k, the surface integral expression is more convenient, while the volume integral expression will give the electric and magnetic contributions separately.

3. Analytical demonstration



(a) Rough sketch (b) Coordinate system for calculation

Fig.2 Dielectric resonators in a rectangular waveguide

The structure shown in Fig.2 is so simple that it can be treated analytically. The EM fields of $TE_{10\delta}$ mode are described in the following for a single resonator [2] when the origin 0 of the *z* direction is taken at the center of the resonator as shown in Fig.2(b)

$$E_y = A\sin\left(\frac{\pi}{a}x\right)e^{\alpha z} \qquad H_x = A\frac{\mathrm{j}\alpha}{\omega\mu_0}\sin\left(\frac{\pi}{a}x\right)e^{\alpha z} \qquad H_z = -A\frac{\mathrm{j}}{\omega\mu_0}\frac{\pi}{a}\cos\left(\frac{\pi}{a}x\right)e^{\alpha z} \qquad \textit{(Region I)} \qquad (14)$$

$$E_y = B\sin\left(\frac{\pi}{a}x\right)\cos(\beta z) \qquad H_x = -B\frac{\mathrm{j}\alpha}{\omega\mu_0}\sin\left(\frac{\pi}{a}x\right)\sin(\beta z) \qquad H_z = -B\frac{\mathrm{j}}{\omega\mu_0}\frac{\pi}{a}\cos\left(\frac{\pi}{a}x\right)\cos(\beta z) \qquad \textit{(Region II)} \qquad (15)$$

$$E_y = C\sin\left(\frac{\pi}{a}x\right)e^{-\alpha z} \qquad H_x = C\frac{\mathrm{j}\alpha}{\omega\mu_0}\sin\left(\frac{\pi}{a}x\right)e^{-\alpha z} \qquad H_z = -C\frac{\mathrm{j}}{\omega\mu_0}\frac{\pi}{a}\cos\left(\frac{\pi}{a}x\right)e^{-\alpha z} \qquad \textit{(Region III)} \qquad (16)$$

where β, α and k satisfy the relations

$$\left(\frac{\pi}{a}\right)^2 + \beta^2 = \varepsilon_r k_0^2, \qquad \left(\frac{\pi}{a}\right)^2 - \alpha^2 = k_0^2, \qquad k_0^2 = \omega^2\varepsilon_0\mu_0, \qquad (17)$$

Due to the boundary conditions at $z=\pm d/2$, the following relations are derived

$$A = C = Be^{\frac{\alpha d}{2}}\cos\frac{\beta d}{2} \qquad \beta\tan\left(\frac{\beta d}{2} - \frac{s\pi}{2}\right) = \alpha \qquad (18)$$

In order to find the coupling coefficient, we substitute eqs.(14)-(16) into each integral in eq.(13).

$$\int_{V_2}\varepsilon|\boldsymbol{E}_0|^2\mathrm{d}v = \varepsilon_0\int_{\frac{d+D}{2}}^{\infty}\int_0^b\int_0^a A^2\sin^2\left(\frac{\pi}{a}x\right)e^{-2\alpha z}\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{\varepsilon_0 abB^2}{4\alpha}\cos^2\left(\frac{\beta d}{2}\right)e^{-\alpha D} \qquad (19)$$

$$\int_{V_2}\mu|\boldsymbol{H}_0|^2\mathrm{d}v = \mu_0\int_{\frac{d+D}{2}}^{\infty}\int_0^b\int_0^a\left(\frac{A}{\omega\mu_0}\right)^2\left(\alpha^2 + \frac{\pi^2}{a^2}\right)\sin^2\left(\frac{\pi}{a}x\right)e^{-2\alpha z}\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{abB^2}{4\alpha\omega^2\mu_0}\left(\alpha^2 + \frac{\pi^2}{a^2}\right)\cos^2\left(\frac{\beta d}{2}\right)e^{-\alpha D} \qquad (20)$$

$$\int_V\varepsilon|\boldsymbol{E}_0|^2\mathrm{d}v = \varepsilon_0\int_0^b\int_0^a\sin^2\left(\frac{\pi}{a}x\right)\mathrm{d}x\mathrm{d}y\left\{A^2\int_{-\infty}^{-\frac{d}{2}}e^{2\alpha z}\mathrm{d}z + B^2\varepsilon_r\int_{-\frac{d}{2}}^{\frac{d}{2}}\cos^2(\beta z)\mathrm{d}z + C^2\int_{\frac{d}{2}}^{\infty}e^{-2\alpha z}\mathrm{d}z\right\}$$

$$= \frac{\varepsilon_0 abB^2}{2}\left\{\frac{1}{\alpha}\cos^2\left(\frac{\beta d}{2}\right) + \varepsilon_r\frac{\sin\beta d + \beta d}{2\beta}\right\} \qquad (21)$$

Hence, the ratio of the electric and magnetic contribution to the coupling coefficient is

$$\frac{k_e}{k_m} = \frac{\int_{V_2}\varepsilon|\boldsymbol{E_0}|^2\,\mathrm{d}v}{\int_{V_2}\mu|\boldsymbol{H_0}|^2\,\mathrm{d}v} = \frac{k^2}{\alpha^2 + (\pi/a)^2} \qquad (22)$$

The total coupling coefficient is given by eq.(13), that is,

$$k = |k_e - k_m| = \frac{\alpha e^{-\alpha D}}{k_0^{\,2}\{1/\alpha + \varepsilon_r(\sin\beta d + \beta d)/(2\beta\cos^2(\beta d/2))\}} \qquad (23)$$

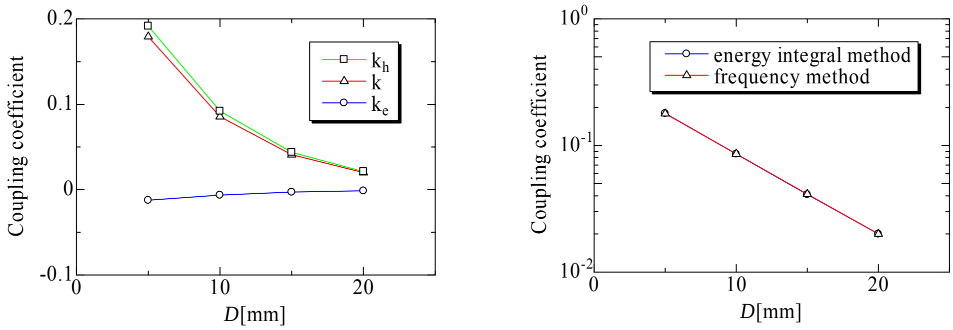
The dispersion relations for the odd and even symmetry in Fig.1(a) are analytically obtained [2], respectively,

$$\left[\beta^2-\alpha^2\coth\left(\frac{\alpha D}{2}\right)\right]\tan(\beta d)=\alpha\beta\left[\coth\left(\frac{\alpha D}{2}\right)+1\right],\quad \left[\beta^2-\alpha^2\tanh\left(\frac{\alpha D}{2}\right)\right]\tan(\beta d)=\alpha\beta\left[\tanh\left(\frac{\alpha D}{2}\right)+1\right] \quad (24)$$

Therefore, solving these equations for the resonant frequency ω_{od} and ω_{ev} by use of eq.(17), the coupling coefficient is given by

$$k=\frac{2|\omega_{od}-\omega_{ev}|}{\omega_{od}+\omega_{ev}} \quad (25)$$

A numerical example is shown in Fig.3 for the structure in Fig.2 with a=20.0mm, b=10.0mm, d=10.0mm, ε_r=16.4. Figure.3(a) shows the contribution of the electric and magnetic coupling, while Fig.3(b) depicts the coincidence of the two results given by eqs.(13) and (25).



(a) Contribution of electric and magnetic coupling (b) Comparison of two calculation method

Fig.3 Coupling coefficient as a function of resonator distance for structure in Fig.2. (a=20.0mm, b=10.0mm, d=10.0mm, ε_r=16.4)

4. Conclusion

We have proposed a new method for calculating the coupling coefficient between two resonators. It needs the EM field distribution for a single resonator instead of the twice calculation for the even and odd mode excitation of coupled resonators. In addition, it discriminates between the electric and magnetic contribution to the coupling. The physical picture of coupling scheme will help to design a variety of bandpass filters.

Reference

[1] R. F. Harrington, "Time-Harmonic Electromagnetic Fields", McGraw-Hill, Inc., pp.317-319(1961).

[2]A, Munir, N. Hamanaga, H. Kubo and I. Awai, "Artificial dielectric rectangular resonator with novel anisotropic permittivity and its $TE_{10\delta}$ mode waveguide filter application", IEICE Trans. Electron., E88-C, No.1, pp.40-46, Jan.2005.