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REAL RAY TRACING IN AN UNMAGNETIZED ABSORPTIVE IONOSPHERE

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The present paper deals with the numerical investigation of real rays in an unmagnetized collisional ionospheric model, described by means of a single recombination Chapman layer with an exponential profile for the collision frequency. The presence of dissipation, leading to a complex dispersion relation has caused grave difficulties in the past. This is due to the fact that complex index of refraction and group velocity do not vanish in real space, hence the height of reflection could not be determined. This is obviated here by using a recent ray theory which extends Hamilton's equations to absorbing media, still using real space-time.

Results of machine computations for various absorption conditions are graphically displayed. The most interesting result is the fact that absorption does not appreciably distort the ray path, even under severe conditions. This adds credibility to previously used heuristic approximations.

We consider the problem of radio ray propagation in a model ionosphere described by a single recombination Chapman layer with an exponential collision frequency profile. Accordingly the recombination Chapman layer is described by the electron density function

$$N=N_{max}\ \exp\{[1-x-\exp(-x)]/2\}, \tag{1}$$

where N_{max} is the maximal density and, assuming the sun at zenith,

$$x = \frac{h-h_{max}}{H}; \tag{2}$$

here h is the height, h_{max} corresponding to the height where $N = N_{max}$, H is a normalizing scale height. In our example h_{max}=105 Km, H = 8 Km. The collision frequency is described by

$$\nu = \nu_o\ e^{x}. \tag{3}$$

where $\nu = \nu_o$ at x = 0 is a measure of the amount of collisions. This is a typical model for an E layer.

In general a linear medium is described by a dispersion equation

$$F(k_i,\omega,x_i,t) = 0, \quad i = 1,2,3, \tag{4}$$

where k_i are the components of the propagation vector, ω is the (angular) frequency, x_i are the space coordinates and t is the time. The ray equations in an absorbing medium are given by

$$v_i = \frac{dx_i}{dt} = - \frac{\partial F/\partial k_i}{\partial F/\partial \omega} \, , \; Im v_i = 0,$$

$$\frac{dk_i}{dt} = \frac{\partial F/\partial x_i}{\partial F/\partial \omega} + i\beta_i \, ,$$

$$\frac{d\omega}{dt} = - \frac{\partial F/\partial t}{\partial F/\partial \omega} + i\beta_j v_j \, ,$$

$$\beta_j = - \left[Re(\frac{\partial v_i}{\partial k_j} + \frac{\partial v_i}{\partial \omega} v_j) \right]^{-1} Im(\frac{\partial v_i}{\partial k_j} \frac{\partial F/\partial x_i}{\partial F/\partial \omega}$$

$$- \frac{\partial v_i}{\partial \omega} \frac{\partial F/\partial t}{\partial F/\partial \omega} + \frac{\partial v_i}{\partial x_j} v_j + \frac{\partial v_i}{\partial t}) \, , \qquad (5)$$

where Im, Re denote the imaginary, real part, respectively. The boundary condition Im v_i = 0 and the presence of β_i ensure that the ray path is confined to real space-time.

For an unmagnetized cold plasma with collisions we take the physical model

$$F = k_i k_i - \omega^2 \mu_o \varepsilon_o (1 - \frac{\omega_p^2}{\omega(\omega - i\nu)}) = 0 \, , \qquad (6)$$

where ω_p is the plasma frequency, proportional to N, and ν is the collision frequency, as in (1), (3) respectively. The medium is considered to be time independent, hence all partial time derivatives vanish in (5). Time varying ionospheres will be discussed elsewhere. It follows that Re $d\omega/dt$ = 0, i.e., Reω = const. and only the loss term Imω can change along tne ray path.

We have computed the ray paths according to (5), subject to (1) - (3), (6) by using a fourth-order Runge-Kutta subroutine with predictor-corrector, HPCG. The ray is initially transmitted from a height of 65 Km, i.e., practically in free space, with steps of $0.04 \cdot 10^{-6}$ sec. This means space intervals of the order of 10 m. Until the ray is well within the ionospheric layer, theoretically β_i = 0, but in practice the computation entails very small differences, leading to erratic undependable numbers. Therefore the loss term in (6) has been compared to the real part and unless a certain density has been reached, the computer has been instructed to continue as if the ray propagates in free space.

Another problem encountered in the course of solving (5) was the fact that analytic expressions for derivatives, compared to derivatives computed by differences, gave inferior results. The criterion used here is the ray velocity as it reemerges from the ionosphere into free space. Results were considered acceptable only if in free space the ray propagated again with the speed $c = 3.10^8$ m/sec. It is assumed that some of the derivatives, involving differentiation of exponential expressions of the form (1), (3) were conducive to larger errors.

We are more concerned about the following problem, for which we found only a "brute-force" solution. We discovered right away that the computation of (5) can diverge from what one expects analytically, due to an inherent instability existing in the formalism. It is easy to see, by inspection of (5) that errors in computation will give rise to a finite non-vanishing value of $\mathrm{Im}v_i$. This in turn is then included in the computation of β_i, which leads to errors in the later steps. This is then indicated by the fact that $\mathrm{Im}v_i$ is steadily growing. The process can be slowed down by using smaller steps in the computation, however, it appeared more expedient to adjust the phase of k_i every 100 steps, such that $\mathrm{Im}v_i = 0$ again. These changes in k_i were very small and we are satisfied that they did not interfere with the physics of the problem. As another safety measure we wrote $\mathrm{Re}v_i$ instead of v_i in (5) in the equation for $d\omega/dt$.