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A TRANSFORMATION METHOD FOR NUMERICAL SOLUTION OF DIFFRACTION PROBLEMS

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Methods based on an integral equation have been used extensively to study the scattering by conducting cylinders of arbitrary cross section. Usually an integral equation for the induced current is obtained, which is then solved numerically by a step or linear approximation for the current. However, if the incident wave is polarized parallel to the axis of the cylinder and the cross section of the scatterer has sharp corners, the induced current tends to be singular reducing the accuracy of the solution using an integral equation method.

For accuracte solution of this type of integral equations Abdelmessih and Sinclair[1] have used Meixner's edge conditions to treat the singularities of the currents on the sharp corners. An alternate method was also used recently by Shafai[2], Where the scattering surface was mapped onto a smooth surface such as a circular cylinder. The transformation introduces a new function into the integral equation, the reciprocal of which gives the singularities of the induced current. Choosing the combination of this function with the current as the unknown function, the integral equation was then solved with a desired degree of accuracy. The method being suitable for the solution of singular integral equations, has a disadvantage that, it requires a conformal transformation to map the scattering surface onto a circular cylinder which in general may not be readily obtainable.

This work is intended to use the transformation of a strip onto a circular cylinder for treatment of the singularities of the current on a cylinder of arbitrary cross section. For a transverse magnetic field normally incident on a conducting cylinder the integral equation has the form

$$E_z = E_z^{inc} - \frac{k\eta}{4}\int_c G(r,r')I_z dc'$$

where $G(r,r')$ is the free space Green's function and c the contour of the cross section. Now, if c is consisted of a smooth section s and a finite number of straight sections c_i bounded by sharp corners, the integral equation may be written in the form

$$E_z = E_z^{inc} - \frac{k\eta}{4}\{\int_s G(r,r')I_{z_s} ds + \int_{c_1} G(r,r')I_{z_1} dc_1 + \ldots + \int_{c_i} G(r,r')I_{z_i} dc_i + \ldots\}$$

the singularity of the current I_{zi} near the limits of the integrals depends on the angle between the sections c_i and c_{i+1}. The strongest singularity is however near the edge of a strip of order of $r^{-\frac{1}{2}}$, r being the distance from the edge. Thus, if a function is introduced into these integrals which behave as the reciprocal of the singularity near a conducting strip, the combination of this function and I_{zi} will be non-singular.

To clarify, let c_i be along the x direction bounded by points $A=x_i$ and $B=x_{i+1}$ with $x_{i+1}-x_i=d$, then the required transformation is given by

$$x = (x_i + d/2) + \frac{d}{2}\sin\theta$$

$$y = 0 \qquad 0 \le \theta \le \pi$$

where the θ coordinate has a scale factor h given by

$$h = \left|\frac{dx}{d\theta}\right| = |d \cos \theta|$$

Using these relations the integral equation takes the form

$$\int_{c_i} G(r,r')I_{z_i} dc_i = \int_0^{\pi} G(r,r')hI_{z_i} d\theta$$

in which the function hI_{zi} is now the new unknown function. Furthermore, since the reciprocal of h gives the singularity of the current near the edge of a conducting strip[2], the function hI_{zi} is always regular, which may be found with any numerical technique.

The method has been applied for scatterers such as fined cylinders (a circular cylinder connected to a strip) and intersecting strips and accurate solutions are found with relatively small computing time. It is hoped that the method be used for similar scattering problems where a direct transformation of the geometry onto a circular cylinder may not be readily feasible.

References

1. Abdelmessih, S.T.M. and Sinclair, G., 1967, Can. J. of Phys., 45, pp. 1305.

2. Shafai, L., 1970, "An improved integral equation for the numerical solution of diffraction problems", Can. J. Phys., Vol. 48, pp. 954 - 963.