Evaluation and Comparison of DOA Estimation Methods with Estimated Number of Signals

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1. Introduction

Recently, the development of mobile communications is making radio wave environments more complicated. To model radio wave environment appropriately, DOA (directions of arrival) estimation of individual incoming waves with array antenna is much effective [1]. For the purpose, many DOA estimation methods have been developed [1]. Among them, the high-resolution DOA estimation methods such as MUSIC and ESPRIT attract considerable attention. One of their features is to require the information on the number of incident waves in advance. In the performance evaluation of MUSIC or ESPRIT, the exact number of incident waves is often assumed to be known a priori. However, it is natural in the practical situation that the number of incident waves should be estimated before the DOA estimation. Thus, the methods of estimating the number of incident waves have been investigated because the incorrect number of waves may cause performance degradation of DOA estimation [2], [3]. In this paper, therefore, we try to evaluate the estimation performance of both the number of waves and DOA [4]. It is just the performance evaluation of overall DOA estimation. In addition, we propose the procedure which enhances the estimation accuracy of DOA by using the estimated number of waves and estimated DOA along with the likelihood function of DOA. Comparing the EVD(eigenvalue-decomposition)-based method, e.g. MUSIC, and non-EVD-based method [5], [6] in computer simulation, we demonstrate the effectiveness of the proposed procedure.

2. Signal Model and Estimation Methods

2.1 Signal Model

Consider that the uniform linear array (ULA) used for the estimation of DOA and the number of waves is composed of M isotropic elements with element spacing d, as shown in Fig. 1. Let the array receive $L(L < M)$ narrow-band waves with DOAs of $\theta_1, \theta_2, \ldots, \theta_L$, respectively, and with complex amplitudes of $s_1(t), s_2(t), \ldots, s_L(t)$, respectively. When the array response vector (mode vector) of lth incoming wave is given by $\boldsymbol{a}(\theta_l)(l = 1, 2, \ldots, L)$, the array input vector $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} \boldsymbol{a}(\theta_l) s_l(\theta) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_L)], \boldsymbol{s}(t) = [s_1(t), \ldots, s_L(t)]^T \tag{2}$$

where $\boldsymbol{A}$ and $\boldsymbol{s}(t)$ are called the array response matrix (mode matrix) and the signal vector, respectively, and $\boldsymbol{n}(t)$ is the internal additive noise vector.

2.2 Estimation Method for the Number of Waves and DOA

In this paper, as the estimation method for the number of waves, we use the improved MENSE [2], [3], denoted by I-MENSE. This method does not employ EVD, resulting in high computational efficiency. As the estimation method for DOA, on the other hand, we use two methods. One is Root-MUSIC with FBSS (forward-backward spatial smoothing) [1], simply called MUSIC here, and the other is the improved Root-SUMWE [5], [6] which is denoted by I-SUMWE. The former is an EVD-based method, and the latter is a non-EVD-based method.

3. Evaluation of DOA Estimation

For evaluation of DOA estimation, RMSE (Root Mean Square Error) of DOA estimates is often used. In this paper, we also utilize RMSE of DOA estimates which is given by

$$\mathrm{RMSE} = \sqrt{\frac{1}{I}\sum_{i=1}^{I}\frac{1}{\hat{L}}\sum_{l=1}^{\hat{L}}\left(\hat{\theta}_{i,l} - \theta_l\right)^2} \tag{3}$$

where I is the number of independent trials of estimation, $\hat{L}$ is the estimated number of waves, and $\hat{\theta}_{i,l}$ is the estimated DOA of the lth wave at the ith trial.

Here, when the estimated number of waves $\hat{L}$ is not equal to the exact number of waves L, we calculate RMSE between the estimated DOA and the exact one closest to it. Using this RMSE, we evaluate the DOA estimation including the case that the estimated number of waves is not correct.

4. Proposed Procedure for Improving DOA Estimation Accuracy

In general, when the SNR (Signal-to-Noise Ratio) is low, it is difficult to estimate accurately the number of waves and DOA. Therefore, we propose the procedure by which we improve the DOA estimation accuracy. The procedure is described below.

1. For the obtained number of waves $\hat{L}$, we set the estimated number of waves as $\hat{L} + \alpha$ where α is an increment for safety in DOA estimation.
2. We compute the DOAs on the condition that there are $(\hat{L} + \alpha)$ waves.
3. Each of $(\hat{L} + \alpha)$ estimated DOAs: $\hat{\theta}_{l'}$ $(l' = 1, \ldots, \hat{L} + \alpha)$ is substituted, in turn, into the following likelihood function of DOA.

$$\begin{aligned} f(\hat{\theta}_{l'}) &= \mathrm{trace}\left[\boldsymbol{a}(\hat{\theta}_{l'})\{\boldsymbol{a}(\hat{\theta}_{l'})^H\boldsymbol{a}(\hat{\theta}_{l'})\}^{-1}\boldsymbol{a}(\hat{\theta}_{l'})^H\boldsymbol{R}_{xx}\right]\ (l' = 1, \ldots, \hat{L} + \alpha) && (4)\\ \boldsymbol{R}_{xx} &= E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] && (5)\end{aligned}$$

4. By choosing $\hat{L}$ DOAs which give the $\hat{L}$ largest values of (4), we can have the DOA estimates for $\hat{L}$ waves.

Using this procedure, we can expect the improved DOA estimation accuracy in low SNR environments.

5. Computer Simulation

Under conditions shown in Tables 1 and 2, the computer simulation is carried out to clarify the effect of the proposed procedure. Average number of waves which are estimated with I-MENSE at each SNR is shown in Fig.2. Also, RMSEs of DOA estimates by MUSIC and I-SUMWE with the estimated number of waves are shown in Figs.3 and 4, respectively. The value represented in parentheses of legends in Figs.3 and 4 means the number of waves used for DOA estimation and “I-MENSE” of legends designates the number of waves estimated by I-MENSE. It is found from Figs.2 and 3 that we can reduce the RMSEs of DOA estimates in both MUSIC and I-SUMWE when the number of waves is larger than the one obtained from I-MENSE. Next, Figs.5 and 6 shows RMSEs of DOA estimates by MUSIC and I-SUMWE with the estimated number of waves and likelihood function and “(ML)” of legends in Fig.5 and 6 means the use oflikelihood function. From Fig.6, it is observed that the method using larger number of waves and extracting the likely DOAs with likelihood function leads to lower RMSE than the method using I-MENSE only. On the other hand, MUSIC using the proposed procedure provides unstable estimation results. It is supposed that it is due to the fact that MUSIC gives several additional DOA estimates close to the exact DOA and so enlarge the value of the likelihood function.

Table 1: Simulation conditions.

Array configuration	ULA of isotropic elements
Element spacing	0.5λ
Number of elements	10
Number of subarray elements (FBSS)	7
Number of subarray elements(I-MENSE)	5
Number of subarray elements(I-SUMWE)	7
Number of snapshots	64
Number of trials of estimation	500

Table 2: Radio environment.

Number of waves	2 (correlated, equal power)
DOA	(0°, 15°)
SNR	-15dB ~ 15dB

6. Conclusion

In this paper, we extended the calculation of RMSE of DOA estimates in order to use the estimated number of waves, and further proposed the procedure to improve the DOA estimation accuracy with the estimated number of waves. This procedure features estimating DOAs with larger number of waves than the one obtained from I-MENSE and extracting the likely DOA estimates with likelihood function of DOA. From computer simulation of DOA estimation using MUSIC and I-SUMWE, it is found that the proposed procedure using I-SUMWE provides a significant improvement over the conventional procedure using just the estimated number of waves by I-MENSE. On the other hand, MUSIC with the proposed procedure shows unstable performance. The future work is to practice in more detail the evaluation of DOA estimation including the estimation of the number of waves.

References

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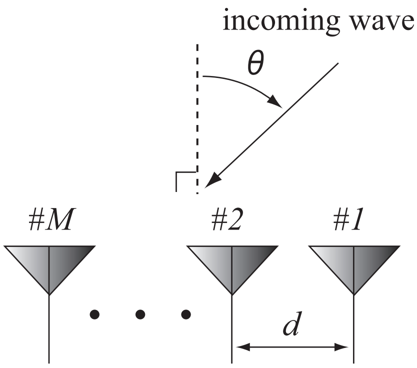

Figure 1: M-element uniform linear array. (element spacing: d)

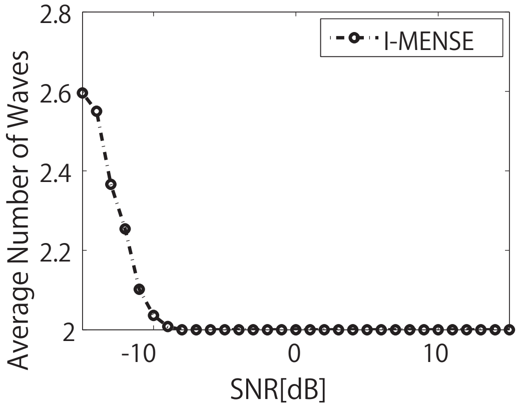

Figure 2: Average of estimated number of wavess by I-MENSE vs. SNR.

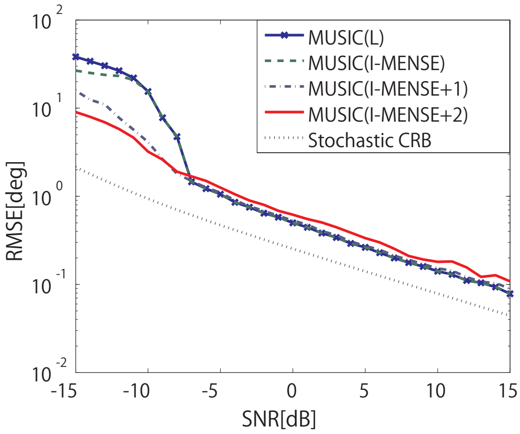

Figure 3: RMSE of DOA estimates vs. SNR in MUSIC algorithm with estimated number of waves.

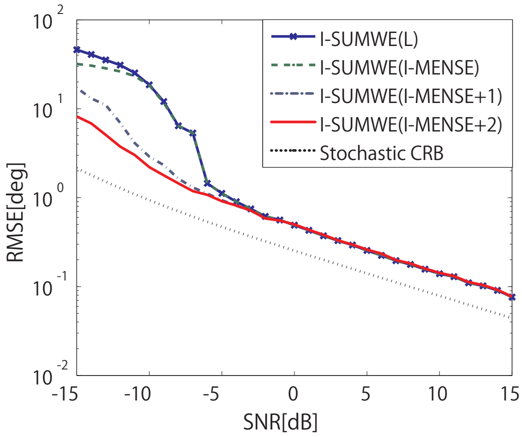

Figure 4: RMSE of DOA estimates vs. SNR in SUMWE algorithm with estimated number of waves.

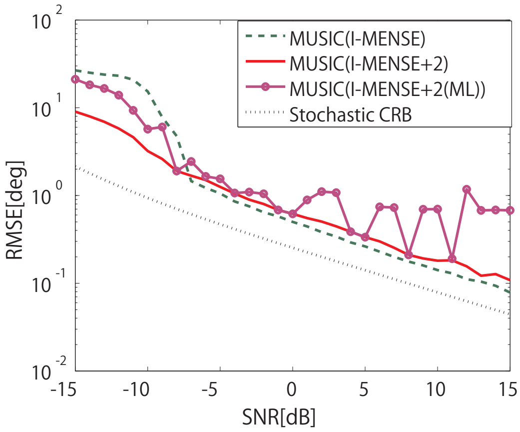

Figure 5: RMSE of DOA estimates vs. SNR in MUSIC algorithm with estimated number of waves and likelihood function.

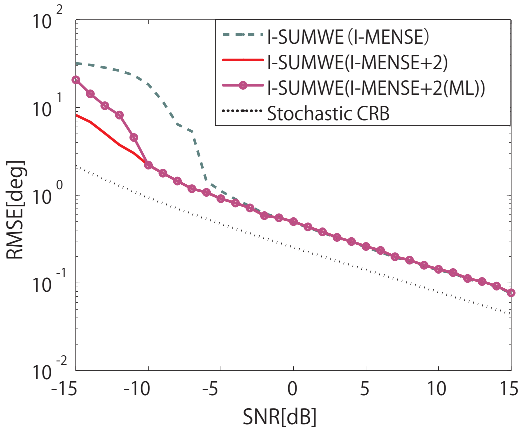

Figure 6: RMSE of DOA estimates vs. SNR in SUMWE algorithm with estimated number of waves and maximum likelihood function.