DECOMPOSITION OF MOMENT METHOD IMPEDANCE MATRIX ELEMENTS FOR PRINTED WIRE ANTENNAS

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1. Introduction

This paper is a sequel to [1], which describes the moment method impedance matrix element $Z_{m,n}$ (composed of $g_{m-1,\,n-1}$, $g_{m-1,\,n}$, $g_{m,\,n-1}$, and $g_{m,\,n}$, all involving triple integrals) used for obtaining the current distribution along an arbitrarily shaped wire antenna printed on a dielectric substrate. To reduce the computational difficulty of $Z_{m,\,n}$, this paper presents a new impedance matrix element $Z^{NEW}{}_{m,n}$, consisting of single-, double-, and triple-integral terms.

2. Basic integral equation for a printed wire

Consider an arbitrarily shaped wire printed on a dielectric substrate of relative permittivity ε_r and thickness B. The wire is assumed to be thin and perfectly conducting. The current I (s') along the wire is obtained by solving the integral equation [1]:

$$\int_{wire} I(s')\,\{-\frac{\partial}{\partial s}\frac{\partial}{\partial s'}(\Pi^{s}(s,s') - \Pi(s,s')) + k_0^2\,(\hat{s}\bullet\hat{s}')\,\Pi^{s}(s,s')\}\,ds' = -E^{i}_{tan}(s) \qquad (1)$$

where s (= s (x, y, z)) and s' (= s'(x', y', z' = B)) are the distances measured from the wire origin to an observation point with coordinates (x, y, z) and a source point with coordinates (x', y', z' = B) along the wire, respectively; $\hat{s}$ and $\hat{s}'$ are the tangential unit vectors at the observation and source points, respectively; $k_0^2 = \omega^2\mu_0\varepsilon_0$; E^{i}_{tan} (s) is the tangential component of an electric field impressed at the observation point; and Π^{s} (s (x, y, z), s'(x', y', B)) and Π (s (x, y, z), s'(x', y', B)) are Green's functions defined as

$$\Pi^{s}(s\,(x,y,z),\, s'(x',y',B)) = 2q \lim_{z\to B}\int_0^{\infty} J_0(\rho\lambda)e^{-u_0\,(z-B)}\frac{\lambda}{D_e(\lambda)}d\lambda \qquad (2)$$

$$\Pi(s\,(x,y,z),\, s'(x',y',B)) = 2q\,(\varepsilon_r - 1)\lim_{z\to B}\int_0^{\infty} J_0(\rho\lambda)e^{-u_0\,(z-B)}\frac{u_0\lambda}{D_m(\lambda)D_e(\lambda)}d\lambda \qquad (3)$$

where $q = -j/(4\pi\omega\varepsilon_0)$; $J_0(\rho\lambda)$ is a Bessel function of the first kind of order zero with $\rho^2 = (x - x')^2 + (y - y')^2$; $D_e(\lambda) = u_0 + u_e \coth u_e B$ and $D_m(\lambda) = u_0\varepsilon_r + u_e \tanh u_e B$ with $u_0 = (\lambda^2 - k_0^2)^{1/2}$ and $u_e = (\lambda^2 - \varepsilon_r k_0^2)^{1/2}$.

Π^s and Π are expressed as

$$\Pi^s = \psi^s + \Delta\psi^s \qquad (4)$$

and

$$\Pi = \psi + \Delta\psi \qquad (5)$$

where $\psi^s = q \lim_{z\to B} e^{-jk_0R} / R$ and $\psi = \tau\psi^s$ with $R^2 = \rho^2 + (z-B)^2$ and $\tau = (\varepsilon_r-1)/(\varepsilon_r+1)$, and

$$\Delta\psi^s = q \lim_{z\to B}\int_0^\infty J_0(\rho\lambda)e^{-u_0(z-B)} \frac{\lambda}{u_0}\left[\frac{2u_0}{D_e(\lambda)}-1\right] d\lambda \qquad (6)$$

$$\Delta\psi = q \lim_{z\to B}\int_0^\infty J_0(\rho\lambda)e^{-u_0(z-B)} \frac{\lambda}{u_0}\left[\frac{2(\varepsilon_r-1)u_0^2}{D_m(\lambda)D_e(\lambda)}-\tau\right] d\lambda \qquad (7)$$

Using Eqs. (4) and (5), the left side of Eq. (1) is decomposed into four terms: $E_{\psi s} + \Delta E_{\psi s} + E_\psi + \Delta E_\psi = -E^i_{tan}(s)$, where

$$E_{\psi s} = \int_{wire} I(s') \left\{-\frac{\partial}{\partial s}\frac{\partial}{\partial s'}\psi^s + k_0^2\,(\hat{s}\bullet\hat{s}')\,\psi^s\right\} ds' \qquad (8)$$

$$\Delta E_{\psi s} = \int_{wire} I(s') \left\{-\frac{\partial}{\partial s}\frac{\partial}{\partial s'}\Delta\psi^s + k_0^2\,(\hat{s}\bullet\hat{s}')\,\Delta\psi^s\right\} ds' \qquad (9)$$

$$E_\psi = \int_{wire} I(s') \frac{\partial}{\partial s}\frac{\partial}{\partial s'}\psi\; ds' \qquad (10)$$

$$\Delta E_\psi = \int_{wire} I(s') \frac{\partial}{\partial s}\frac{\partial}{\partial s'}\Delta\psi\; ds' \qquad (11)$$

Eq. (8) corresponds to the term obtained for an arbitrarily shaped wire in *free space*. The effect of the dielectric substrate appears in Eqs. (10) and (11). In fact, when $\varepsilon_r = 1$, Eqs. (10) and (11) vanish. Note that Eq. (9) for $\varepsilon_r = 1$shows the effect of the ground plane.

Let the current be expanded as $I(s') = \Sigma_n I_n J_n(s')$ with $J_n(s')$ being piecewise sinusoidal functions. Then, Eq. (8) for the n^{th} piecewise sinusoidal expansion function, which is defined in a region from $s' = s_{n-1}$ to s_{n+1}, is given as

$$E_{\psi s} = -I_n \frac{j30}{\sin k_0 d}\left[\left(\frac{\mathbf{P}_{n-1}}{\rho_{n-1}} + \frac{\mathbf{P}_n}{\rho_n}\right) - (\mathbf{S}_{n-1} + \mathbf{S}_n)\right] \bullet \hat{s} \qquad (12)$$

where an equidistant-spacing d ($s_n - s_{n-1} = s_{n+1} - s_n = d$) is adopted; ρ_j ($j = n-1, n$) is the radial distance from the j^{th} wire segment axis to the observation point specified by coordinates (x, y, z); and $\mathbf{P}_{n-1}$, $\mathbf{P}_n$, $\mathbf{S}_{n-1}$, and $\mathbf{S}_n$ are vector quantities. Note that $\mathbf{P}_{n-1}$ is expressed in closed form [1]:

$$.\mathbf{P}_{n-1} = [(\hat{\sigma}_{n-1}\bullet\hat{R}_n)\; e^{-jk_0R_n} \cos k_0 d \;-(\hat{\sigma}_{n-1}\bullet\hat{R}_{n-1})\; e^{-jk_0R_{n-1}} - j\,e^{-jk_0R_n} \sin k_0 d\,]\,\hat{\rho}_{n-1} \qquad (13)$$

where R_j ($j = n-1, n$) is the distance from the wire segment end point specified by distance $s' = s_j$ to the observation point; $\hat{R}_j$ and $\hat{\rho}_j$ are the unit vectors in the R_j and ρ_j directions, respectively; and $\hat{\sigma}_j$ is the unit vector parallel to the j[th] wire segment axis. The remaining vector quantities $\mathbf{P}_n$, $\mathbf{S}_{n-1}$, and $\mathbf{S}_n$, which are not presented due to space limitations, are also expressed in closed form.

3. Impedance matrix element $Z^{NEW}_{m,n}$

After the current I(s') in Eqs. (9)-(11) are expanded using the n[th] piecewise sinusoidal expansion function, the m[th] piecewise sinusoidal weighting function $W_m(s) = J_m(s)$ is applied to Eqs. (9)-(11) and Eq. (12) to obtain an impedance matrix element $Z^{NEW}_{m,n}$:

$$Z^{NEW}_{m,n} = Z^{\psi s}_{m,n} + Z^{\psi}_{m,n} + \Delta Z_{m,n} \tag{14}$$

where

$$Z^{\psi s}_{m,n} = \frac{1}{\sin k_0 d}\int_0^d E_{\psi s} \sin k_0\sigma_{m-1}\, d\sigma_{m-1} + \frac{1}{\sin k_0 d}\int_0^d E_{\psi s} \sin k_0(d-\sigma_m)\, d\sigma_m \tag{15}$$

$$Z^{\psi}_{m,n} = \left(\frac{k_0}{\sin k_0 d}\right)^2 (g^{\psi}_{m-1,\, n-1} + g^{\psi}_{m-1,\, n} + g^{\psi}_{m,\, n-1} + g^{\psi}_{m,\, n}) \tag{16}$$

$$\Delta Z_{m,n} = \left(\frac{k_0}{\sin k_0 d}\right)^2 (\Delta g_{m-1,\, n-1} + \Delta g_{m-1,\, n} + \Delta g_{m,\, n-1} + \Delta g_{m,\, n}) \tag{17}$$

in which

$$g^{\psi}_{m-1,\, n-1} = \int_0^d \cos k_0\sigma_{m-1} \left\{\int_0^d \psi_{m-1,\, n-1} \cos k_0\sigma'_{n-1}\, d\sigma'_{n-1}\right\} d\sigma_{m-1} \tag{18}$$

$$\begin{aligned}\Delta g_{m-1,\, n-1} = &\int_0^d \cos k_0\sigma_{m-1} \left\{\int_0^d (-\Delta\psi^{s}_{m-1,\, n-1} + \Delta\psi_{m-1,\, n-1}) \cos k_0\sigma'_{n-1}\, d\sigma'_{n-1}\right\} d\sigma_{m-1} \\ &+ (\hat{\sigma}_{m-1}\bullet\hat{\sigma}_{n-1}) \int_0^d \sin k_0\sigma_{m-1} \left\{\int_0^d \Delta\psi^{s}_{m-1,\, n-1} \sin k_0\sigma'_{n-1}\, d\sigma'_{n-1}\right\} d\sigma_{m-1}\end{aligned} \tag{19}$$

where new variables σ_{m-1} and σ_m are defined as $\sigma_{m-1} = s - s_{m-1}$ and $\sigma_m = s - s_m$, respectively. Similarly, a new variable σ'_{n-1} is defined as $\sigma'_{n-1} = s' - s_{n-1}$. The subscripts of $\psi_{m-1,\, n-1}$, $\Delta\psi_{m-1,\, n-1}$, and $\Delta\psi^{s}_{m-1,\, n-1}$ show that these are functions of σ_{m-1} and σ'_{n-1}. The remaining terms $g^{\psi}_{i,\, j}$ and $\Delta g_{i,\, j}$ (i = m –1, m: j = n –1, n), which are not presented due to space limitations, take forms similar to $g^{\psi}_{m-1,\, n-1}$ and $\Delta g_{m-1,\, n-1}$, respectively.

$Z^{\psi s}_{m,n}$ in Eq. (15) and $Z^{\psi}_{m,n}$ in Eq. (16) involve single and double integrals, respectively. $\Delta Z_{m,n}$ in Eq. (17) involves triple integrals, because $\Delta g_{i,\, j}$ (i = m–1, m: j = n–1, n) includes Sommerfeld-type integrals in $\Delta\psi^{s}_{i,\, j}$ and $\Delta\psi_{i,\, j}$. Therefore, it can be said that $Z^{NEW}_{m,n}$ is simpler than the conventional $Z_{m,n}$, which is composed of $g_{m-1,\, n-1}$, $g_{m-1,\, n}$, $g_{m,\, n-1}$, and $g_{m,\, n}$ [1], all involving triple integrals.

Using the impedance matrix element $Z^{NEW}_{m,n}$, a grid array antenna printed on a dielectric substrate (see Fig. 1) is analyzed. The computational time using $Z^{NEW}_{m,n}$, where interpolation techniques are applied to $\Delta\psi_{i,j}$ and $\Delta\psi^{s}_{i,j}$, is decreased by a factor of 60, compared with that using the conventional $Z_{m,n}$. The current distribution is shown in Fig. 2.

4. Conclusions

A moment method solution to an integral equation for a printed wire is discussed. The tangential electric field along the printed wire is decomposed into four terms: $E_{\psi s}$, $\Delta E_{\psi s}$, E_{ψ}, and ΔE_{ψ}. When piecewise sinusoidal functions are used as expansion functions (basis functions), the first term $E_{\psi s}$ has a closed form. The weighting (testing) process using the same piecewise sinusoidal functions leads to a new impedance matrix element $Z^{NEW}_{m,n}$, which is composed of single-, double-, and triple- integral terms. The computational time using $Z^{NEW}_{m,n}$ is significantly decreased, compared with that using the conventional $Z_{m,n}$.

Reference

[1] H. Nakano, "Chapter 3, Antenna analysis using integral equations" in *Analysis Methods for Electromagnetic Wave Problems, Volume II*, E. Yamashita (Editor), Artech House, Boston, 1996

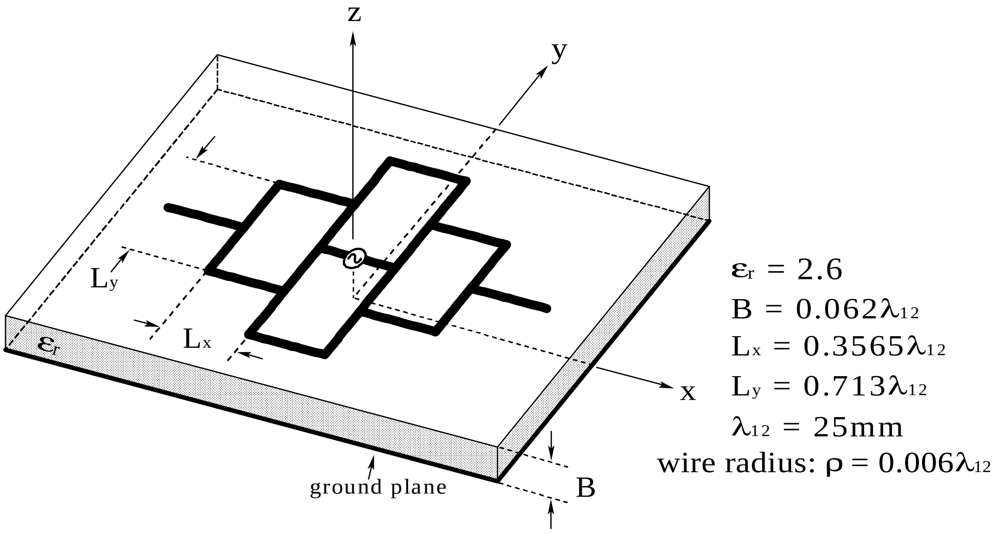

Fig. 1 Configuration of a grid array antenna

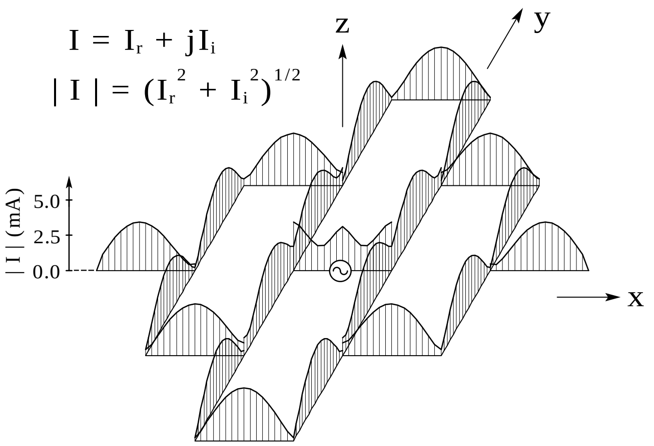

Fig. 2 Current distribution