ANALYSIS OF RESONANT FREQUENCY AND COUPLING COEFFICIENT FOR ARTIFICIAL DIELECTRIC CIRCULAR RESONATORS WITH AN ANISOTROPIC PERMITTIVITY

Achmad MUNIR and Hiroshi KUBO
Yamaguchi University, Department of Electrical and Electronic Engineering
2-16-1 Tokiwadai, Ube, Yamaguchi, 755-8611 Japan
E-mail: munir@emlab.eee.yamaguchi-u.ac.jp

I. Introduction

In the microwave field, the application of metamaterials has extensively been studied these 5 years, especially after Pendry proposed the idea to substantiate both negative permeability and permittivity [1]. The notion of metamaterials basically includes all kind of artificial materials such as chiral media, artificial dielectrics, bianisotropic media and double-negative materials. Artificial dielectrics, paraphrased as double-positive materials, are as promising as double-negative counterpart, contrary to the least number of involved researchers.

A long-studied artificial dielectric was also applied for the microwave devices. Some applications for this material were proposed and investigated by many researchers with the intention for a plane wave or TEM wave incidence, such as antenna, wave absorber, polarizer and phase shifter [2]-[3]. In fact, one of the concepts for application of artificial dielectric was proposed over a half century ago when Kock suggested to make a dielectric lens lighter in weight by replacing the refractive material by a mixture of metal disks in a matrix [4]. From the concept, a microwave rectangular resonator using artificial dielectrics has been proposed for the first time [5]-[6], considering that the artificial dielectric should make a resonator if it is worthy of the name.

In this paper, the resonant frequency of artificial dielectric circular resonators for the TE mode and its coupling coefficient including their parametric properties, e.g. novel anisotropic permittivity and separation of higher modes (mode control), are analyzed theoretically. Then a numerical method of simulation by HFSS® version 8.0 is carried out for a comparison. Actually, the resonant frequency and the coupling coefficient of dielectric resonators encapsulated in a circular waveguide were investigated theoretically and experimentally more than 20 years ago [7]. But the investigated results are only for the dielectric resonator with isotropic permittivity. Since one of our research focuses is to control a wanted resonance mode avoiding the unwanted mode as far as possible, the artificial dielectric circular resonator with a novel anisotropic permittivity is expected to have that property.

II. Analytical method for TE mode resonance and its coupling coefficient in a circular waveguide

Figure 1(a) illustrates a cylindrical waveguide with a circular cross section a radius a. In view of cylindrical geometry involved, cylindrical coordinates are most appropriate for the analysis to be carried out. The electromagnetic field distribution of several resonant modes of a circular waveguide is shown in Fig. 1(b). From the figure, it is seem that the $TE_{01\delta}$ and $TM_{01\delta}$ have a simple field distribution since they have no variation of electric field and magnetic field in angular direction, respectively.

The resonant frequency of anisotropic artificial dielectric resonator for the TE mode in a circular waveguide shown in Fig. 2(a) is formulated. The formulation derived in this paper begins with the assumption that the resonator is homogeneous and follows the standard analysis for an isotropic resonator. The permittivity and the permeability of resonator are given by

$$[\varepsilon] = \varepsilon_0 \begin{bmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_\phi & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad \mu = \mu_0 . \qquad (1)$$

where ε_r, ε_ϕ, ε_z are the relative permittivity to the r, ϕ and z directions, respectively, ε_0 and μ_0 are permittivity and permeability in free space, respectively.

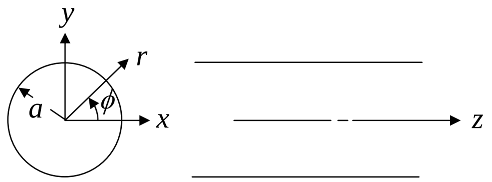
The field in a resonator shown in Fig. 2(a) is assumed to propagate as $\exp(j\omega t - j\beta z)$. By substituting equation (1) into the Maxwell equations, and then solving it with respect to H_z for the TE

ISBN: 89-86522-78-0 94460©KEES

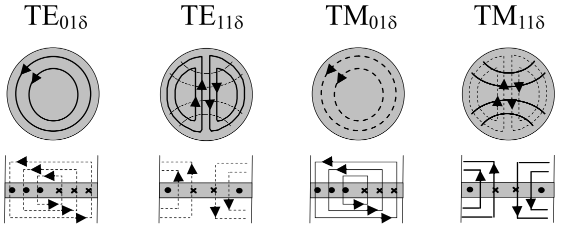
mode, the following equation can be derived,

$$\frac{\partial^2 H_z}{\partial r^2}+\frac{1}{r}\frac{\partial H_z}{\partial r}+\frac{\zeta_\phi^2}{\zeta_r^2}\frac{1}{r^2}\frac{\partial^2 H_z}{\partial \phi^2}+\zeta_\phi^2 H_z=0\,, \tag{2}$$

where $\zeta_r^2=\omega^2\varepsilon_0\varepsilon_r\mu_0-\beta^2$ and $\zeta_\phi^2=\omega^2\varepsilon_0\varepsilon_\phi\mu_0-\beta^2$.

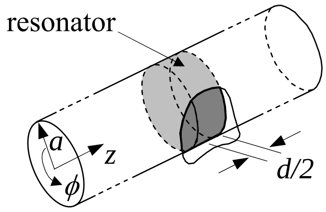


(a) The circular cylindrical waveguide

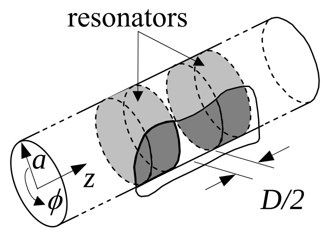


(b) Electromagnetic field distribution

Fig. 1 The circular cylindrical waveguide and its electromagnetic field distribution



(a) Resonant frequency



(b) Coupling coefficient

Fig. 2 Illustration of analytical method for artificial dielectric resonator in a circular waveguide

The solution of equation (2) is given by

$$H_z=AR(r)\Phi(\phi)\,. \tag{3}$$

Substituting the solution into equation (2), the following relation can be obtained,

$$\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2}+\frac{r}{R}\frac{\partial R}{\partial r}+\left(\zeta_\phi r\right)^2=-\frac{\zeta_\phi^2}{\zeta_r^2}\frac{\partial^2\Phi}{\partial\phi^2}\frac{1}{\Phi} \tag{4}$$

The left-hand side is a function of *r* only, whereas the right-hand side depends on ϕ only. As a result, equation (4) is seen to separate into the following two equations,

$$\frac{\partial^2 R}{\partial\left(\zeta_\phi r\right)^2}+\frac{1}{\zeta_\phi r}\frac{\partial R}{\partial\left(\zeta_\phi r\right)}+\left[1-\left(\frac{\zeta_\phi}{\zeta_r}m\right)^2\Big/\left(\zeta_\phi r\right)^2\right]R=0\,, \tag{5}$$

$$\frac{\partial^2\Phi}{\partial\phi^2}+\left(\frac{\zeta_\phi}{\zeta_r}m\right)^2\Phi=0\,. \tag{6}$$

Equation (5) is Bessel's differential equation and has two independent solutions, but since the second solution becomes infinite at r = 0, only the first solution is acceptable for the problem under investigation. As a consequence, an appropriate solution for H_z as may thus be expressed as,

$$H_z=\left(A_1\cos\left[\left(\frac{\zeta_\phi}{\zeta_r}m\right)\phi\right]+A_2\sin\left[\left(\frac{\zeta_\phi}{\zeta_r}m\right)\phi\right]\right)\left(J_{\pm\left(\frac{\zeta_\phi}{\zeta_r}m\right)}\left(\zeta_\phi r\right)\right), \tag{7}$$

with the requirement that E_ϕ must vanish when $r = a$, so

$$dJ_{\pm\left(\frac{\zeta_\phi}{\zeta_r}m\right)}\left(\zeta_\phi r\right)\Big/dr=J'_{\pm\left(\frac{\zeta_\phi}{\zeta_r}m\right)}\left(\zeta_\phi r\right)=0\,,\qquad \text{at } r=a, \tag{8}$$

where *m* is integer (0, 1, 2, …).

The relation for the field outside resonator can be found by replacing ε_r, ε_ϕ, and ε_z in equation (1) with 1 and β in equation (2) with $(-j\alpha)$. Then, calculating the input impedance corresponding to the short or open termination at the center of the resonator (d/2), the following relation can be obtained,

$$\beta \tan\left(\beta \frac{d}{2} - \frac{s\pi}{2}\right) = \alpha , \tag{9}$$

where s is integer (0, 1, 2, …).

Moreover, to analyze the coupling coefficient of TE mode resonance between two resonators with a distance D, the formulation is derived by calculating the resonance frequencies corresponding to the short or open termination between two resonators as shown in Fig. 2(b). The resonant frequency corresponding to the short and open termination can be obtained by solving the following equations, respectively.

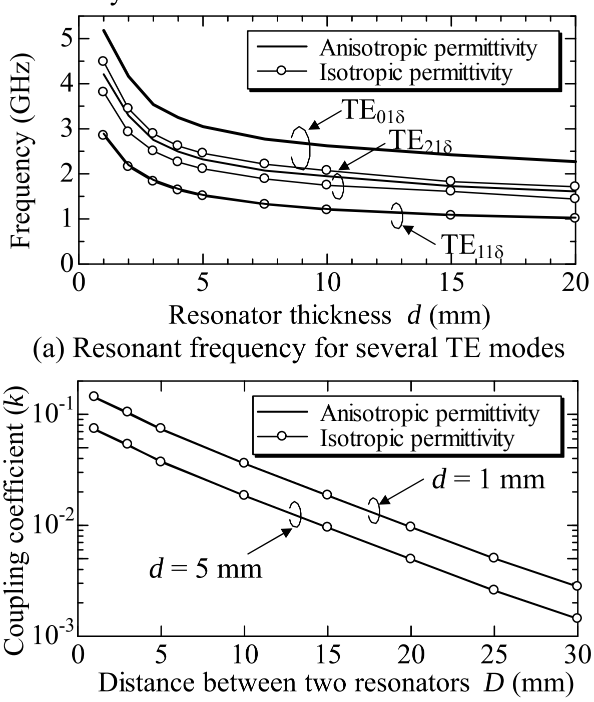
$$\left[\beta^2 - \alpha^2 \coth\left(\alpha \frac{D}{2}\right)\right] \tan(\beta d) = \alpha\beta \left[\coth\left(\alpha \frac{D}{2}\right) + 1\right], \tag{10}$$

$$\left[\beta^2 - \alpha^2 \tanh\left(\alpha \frac{D}{2}\right)\right] \tan(\beta d) = \alpha\beta \left[\tanh\left(\alpha \frac{D}{2}\right) + 1\right]. \tag{11}$$

Substituting equations (8)-(9) into equations (10)-(11), the resonant frequency for the TE mode resonance with shorted termination (f_o) and open termination (f_e) can be obtained. Thence the coupling coefficient for a distance D can be calculated by using the following relation.

$$k = 2\frac{|f_o - f_e|}{f_o + f_e} . \tag{12}$$

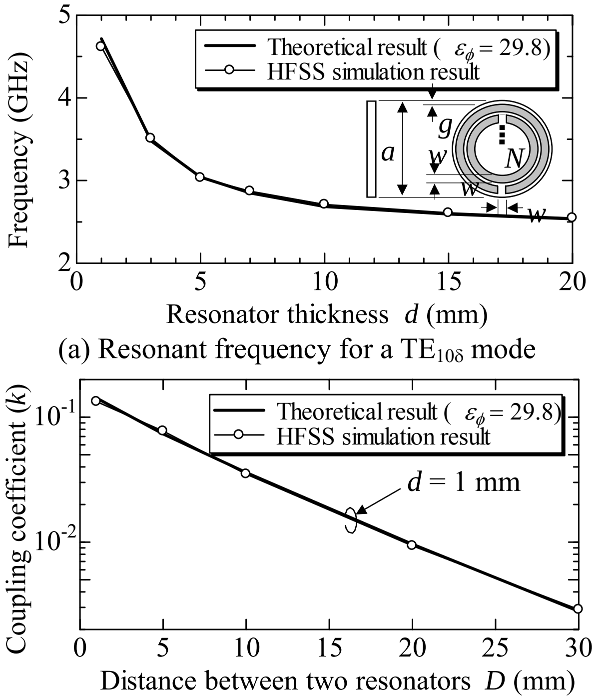
III. Analysis result



(a) Resonant frequency for several TE modes

(b) Coupling coefficient for a $TE_{10\delta}$ mode

Fig. 3 Theoretical result of resonant frequency and coupling coefficient



(a) Resonant frequency for a $TE_{10\delta}$ mode

(b) Coupling coefficient for a $TE_{10\delta}$ mode

Fig. 4 Theoretical and HFSS simulation results of resonant frequency and coupling coefficient

To show a potential of the artificial dielectric resonator in a circular waveguide with a novel anisotropic permittivity, we analyzed theoretically the resonant frequency of the higher order modes for both artificial dielectric resonator and natural dielectric resonator. The relative permittivity of the artificial dielectric resonator is taken as 10, 20 and 2.17 for ε_r, ε_ϕ, and ε_z respectively; while for the natural dielectric resonator it is 20 being isotropic. The resonator thickness (d) is set from 1 mm to 20 mm and the diameter of resonator is 32.54 mm, where the cut-off frequency is around 5.39 GHz. Then, the resonant frequency for $TE_{01\delta}$, $TE_{11\delta}$ and $TE_{21\delta}$ modes is calculated. The result of resonant frequency as a function of resonator thickness (d) is shown in Fig. 3(a) that is obtained by use of equations (8)-(9) for the artificial dielectric resonator and for the natural dielectric resonator replacing ε_r, ε_ϕ and

ε_z by 1. From the results, it can be seen that the resonant frequencies of artificial dielectric resonator with anisotropic permittivity for $TE_{21\delta}$ and $TE_{01\delta}$ modes are higher than the dielectric resonator with isotropic permittivity, while for the $TE_{11\delta}$ modes are not different much.

Then, to investigate the characteristic of artificial dielectric resonator in a circular waveguide comparing with the natural dielectric resonator, the analysis of coupling coefficient between two resonators is performed by using equations (10)-(12). In the analysis, for calculating the dependence of coupling coefficient on the distance (*D*), the resonator thicknesses (*d*) are taken as 1 mm and 5 mm, while the relative permittivity is assumed to be same with the resonant frequency analysis. The calculated result of coupling coefficient as a function of the distance (*D*) for the artificial dielectric resonator and the natural one are plotted in Fig. 3(b). From the figure, it can be understood that for the shorter *D* the coupling is stronger as well as for the thinner *d*, because the overlapping integral of evanescent waves from both resonators is bigger than for the longer *D* or thicker *d*.

Moreover, to confirm the theoretical analysis, by use of commercial software HFSS® we have calculated the resonant frequency and the coupling coefficient for a structure shown in the insert of Fig. 4(a). The structure is assigned for exciting $TE_{01\delta}$ mode in a circular waveguide and it has 32.54 mm of diameter (*a*), 0.5 mm of substrate thickness (*t*) and 2.17 of relative permittivity (ε_r), with 1 mm of strip width (*w*), 100μm of gap between strips (*g*), and 6 of strip number (*N*). From the simulation result of resonant frequency shown in Fig. 4(a), we calculated the relative permittivity ε_ϕ of resonator by employing equations (8)-(9), then averaging all the results. We plotted the theoretical result for 29.8 of ε_ϕ as a function of thickness *d* as depicted in the same figure comparing with the simulation result. While the theoretical and simulation results of coupling coefficient are shown in Fig. 4(b).

IV. Conclusion

The properties of artificial dielectric circular resonators with a novel anisotropic permittivity have been analyzed and compared to the natural one. It was shown that the anisotropy improves the spurious property of resonator, so it can be used for a resonance mode selection. Hence, from the analysis result of coupling coefficient, it should be noted that the coupling coefficient is the same for both isotropic and anisotropic cases. Furthermore, to confirm the theoretical analysis, a numerical analysis for a $TE_{01\delta}$ mode by use of commercial software was also demonstrated and the result agrees with the theoretical result. More realistic study to fabricate a prototype of resonator is underway by using a PCB where the more reliable property and the possibility of material usage for a microwave application will be shown very soon.

V. Acknowledgment

The authors would like to acknowledge a sincere debt Prof. I. Awai, Ryukoku University, Japan, for encouragement, inspiration, and invaluable discussion.

VI. References

[1] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," IEEE Trans. Microwave Theory Tech., vol. 47, no. 11, pp. 2075-2084, Nov. 1999.

[2] M. M. I. Saadoun and N. Engheta, "A reciprocal phase shifter using novel pseudo chiral or Ω medium", Microwave and Optical Tech. Lett., vol.5, no. 4, pp.184-188, Apr. 1992.

[3] I. V. Lindell, A. H. Sihvolla, S. A. Tretyakov and A. J. Vitanen, "Electromagnetic waves in chiral and bi-isotropic media," Artech Hose, Inc. 1994.

[4] W. E. Kock, "Metallic delay lenses," Bell Syst. Tech. J. vol. 27, pp. 58-82, 1948.

[5] H. Kubo, I. Awai, T. Iribe, A. Sanada, and A. Munir, "Artificial dielectric composed of metal strips and calculation method of the permittivity and permeability," IEEJ Trans. FM., vol. 123, no. 3, pp. 265-272, Mar. 2003. (in Japanese).

[6] A. Munir, N. Hamanaga, H. Kubo, and I. Awai, "Artificial dielectric rectangular resonator with novel anisotropic permittivity and its $TE_{10\delta}$ mode waveguide filter application," IEICE Trans. Electron., vol. E88-C, no. 1, pp. 40-46, Jan. 2005.

[7] S. J. Fiedziuszko, "Dual-Mode Dielectric Resonator Loaded Cavity Filters," IEEE Trans. Microwave Theory Tech., vol. MTT-30, No. 9, pp. 1131-1316, Sep. 1982.