Analysis on the reflection method for the radiation efficiency measurement using the transmission line model

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Abstract

The reflection method is known as an accurate and simple method for measuring the radiation efficiency of a small antenna. Two big problems arise in the measurement of the radiation efficiency using this method, in which a straight waveguide and two sliding shorts are implemented. To reduce a measurement time, one sliding short is fixed, whereas the other sliding short can be moved. To improve the accuracy, we can set an antenna under test at the middle of two sliding shorts, where the magnitude of the standing wave is locally maximized in the waveguide. When a sliding short is fixed, the efficiency falls into negative territory at some frequencies. We analytically consider the drops using an equivalent transmission line model for the reflection method. Also, we derive overall efficiency for above two cases and examine the way to measure the efficiency with no drops.

1. Introduction

When the antenna is smaller, its gain is also smaller and the magnitude of its reflection coefficient is nearly equal to unity, that is, total reflection occurs at the input port of the antenna. This is because the dimension of the antenna is much less than the wavelength. When the gain is small, the ohmic loss in the antenna's material is larger than the radiation loss of the antenna. In other words, the radiation efficiency, which is defined as the ratio of the radiated power to the input power, is much smaller as the dimension of the antenna is smaller. Therefore, the radiation efficiency is one of the most important parameters in evaluating the performance of the antenna and it is necessary to establish some accurate estimations of the radiation efficiency. The pattern integration method is well known as a typical estimation of the radiation efficiency. However, it requires enormous cost and time to measure the radiated power in the anechoic chamber, and in general it can be said that it is accurate measurement if an appreciate angular separation is selected.

On the contrary, in the reflection method which is studied in this paper, the radiation efficiency can be evaluated by measuring reflection coefficients of the antenna in free space and the waveguide which is shorted by two movable shorts. Therefore, the measurement system is inexpensive and compact and has an advantage of short measurement time. However, it suffers from the disadvantage of some drops in the measured efficiency due to the resonance of the cavity which is formed by the waveguide and two movable shorts. Although some papers refer the resistance of the cavity wall at the resonance, no one has ever proposed any practical models which can analyze the frequency response of the drops in the efficiency.

In this paper, the antenna inserted in the cavity is expressed by use of the transmission line model. Two shorts can be modeled by the resistive load with small resistance, so that the loss at the cavity wall can be included in the transmission line model. Then, the reflection coefficient whose magnitude is unity for the shorts in the previous studies should be replaced by the reflection coefficient including the loss at the cavity wall. After laborious calculation, it is analytically clear that the efficiency in the previous studies can be divided into the true radiation efficiency and the transmission efficiency caused by the waveguide and two shorts.

Moreover, we clarify that the drops in the efficiency which have been observed in the previous studies are caused by the resistance of the cavity wall.

2. Measurement Principle and System of the Reflection Method

A. Measurement Principle in the Previous Studies

An antenna in free space can be considered as a linear, passive, reciprocal two-port network, which is fed at its port 1 and is connected to the free space at its port 2. Because the radiated power from the antenna is supplied with the free space at the port 2, the reflection coefficient at the port 2 is equal to zero. Then, the radiation efficiency η_{ant} is given by

$$\eta_{\mathrm{ant}} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}, \tag{1}$$

where S_{ij}, $i, j = 1, 2$, are S parameters of the network. Now, $|S_{11}|$ can be determined by measuring the magnitude of the reflection coefficient of the antenna in free space. In the

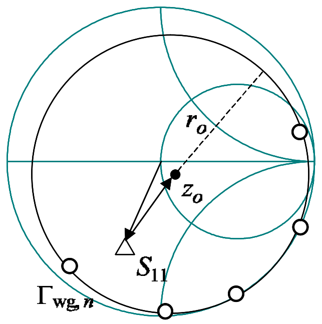

Fig. 1: S_{11} and $\Gamma_{\mathrm{wg},n}$ on the Smith chart

reflection method, $|S_{21}|$ can be determined by measuring the reflection coefficients of the antenna inserted in the shorted waveguide for more than three positional combinations of two sliding shorts. The above coefficients are described as $\Gamma_{\mathrm{wg},n}$, $n = 1, 2, \cdots$. In the following, a variable short-circuited line means the waveguide and two sliding shorts, which are connected to the port 2 of the network. If Γ_n denotes the reflection coefficients of the variable short-circuited line at the port 2, the reflection coefficient at the antenna port is given by

$$\Gamma_{\mathrm{wg},n} = S_{11} + \frac{S_{21}^2 \Gamma_n}{1 - S_{22}\Gamma_n}. \quad (2)$$

If the variable short-circuited line is ideal, then the magnitude of the reflection coefficient is unity. In other words, Γ_n draws a circle, which has a center of 0 and a radius of 1, on the Smith chart. That is, we can describe it as $\Gamma_n = e^{j\theta_n}$, where θ_n is the phase of the reflection coefficient. Due to the natrue of the bilinear transformation on the complex plane, if Γ_n draws a circle, then $\Gamma_{\mathrm{wg},n}$ also draws another circle, as we can see from (2). If $S_{11} + z_o$ and r_o represent the center and radius of the circle, $\Gamma_{\mathrm{wg},n}$, then $|S_{21}|$ is given by

$$|S_{21}|^2 = r_o - \frac{|z_o|^2}{r_o}. \quad (3)$$

Then, from (1), the radiation efficiency can be given by

$$\eta_{\mathrm{ant}} = \frac{1}{1 - |S_{11}|^2}\left(r_o - \frac{|z_o|^2}{r_o}\right). \quad (4)$$

And, $S_{11} + z_o$ and r_o can be determined by plotting the $\Gamma_{\mathrm{wg},n}$ on the Smith chart and fitting them into a circle by the least-square method, as shown in Fig.1.

B. Modified Measurement Principle

In some cases, the variable short-circuited line is not ideal. For example, the resistance of the shorts can not be ignored at or near the resonant frequencies of the cavity resonance. Now, we assume that the magnitude of the reflection coefficient $|\Gamma_n|$ is not unity and Γ_n has a center of z_i and radius of r_i. That is, we can describe Γ_n as $\Gamma_n = z_i + r_i e^{j\theta}$, where θ is real number. Then, $\Gamma_{wg,n}$ draws a circle on the Smith chart, as we can see from (2). A center and radius of the mapped circle are $S_{11} + z_o$ and r_o, respectively, then $|S_{21}|$ is given by

$$|S_{21}|^2 = \frac{r_o - \dfrac{|z_o|^2}{r_o}}{r_i - \dfrac{|z_i|^2}{r_i}}. \quad (5)$$

The derivation of (5) is given in the appendix. Substituting (5) into (1), then the following relation can be obtained:

$$\eta_{\mathrm{net}} = \eta_{\mathrm{ant}} \cdot \eta_{\mathrm{line}}; \quad (6)$$

where,

$$\eta_{\mathrm{net}} = \frac{1}{1 - |S_{11}|^2}\left(r_o - \frac{|z_o|^2}{r_o}\right), \quad (7)$$

$$\eta_{\mathrm{line}} = r_i - \frac{|z_i|^2}{r_i}. \quad (8)$$

η_{ant}, given by (7), is the same as (4), which represents the radiation efficiency in the reflection method if the variable short-circuited line is ideal. Although the efficiency given by (7) includes the efficiency in the variable short-circuited line, given by (8), the previous efficiency, given by (4) does not include the effect of the variable short-circuited line. (6) states that η_{net} which can be determined by measuring $|S_{11}|$ and $\Gamma_{\mathrm{wg},n}$, can be given by the product of the true radiation efficiency η_{ant} by the efficiency in the the variable short-circuited line η_{line}. In other words, (4) can be derived if the loss in the variable short-circuited line is ignored, or $z_i = 0$, $r_i = 1$ or $\eta_{\mathrm{line}} = 1$, so that (4) can be valid if we can ignore the loss at the cavity wall. If not, we must evaluate the radiation efficiency by use of (6) instead of (4).

C. Measurement System

In the following paragraphs, we explain our measurement system for evaluating the radiation efficiency in the reflection method.

1) Waveguide: A rectangular waveguide consists of two U-shaped aluminum castings and two aluminum plates, as shown in Fig. 2. These castings and plates are screwed shut at 200mm intervals. The inside dimension of the cross-section is 150mm$\times$75mm and the length is 1,000mm. An antenna under test can be inserted into the waveguide via a square hole in the broad wall which has an area of 80mm$\times$80mm. The frequency range for the single mode operation of the waveguide is from 1.00GHz to 2.00GHz.

2) Sliding Short: A sliding short consists of an aluminum plate attached to the undersurface of a wooden rectangular box which has a dimension of 147mm $\times$ 72mm $\times$ 500mm, as shown in Fig. 3. To keep the mobility, insulated spacers with a thickness of 1.5mm are attached to the side walls of the sliding short. To avoid the leakage from the clearance, two types of the aluminum types with a length of 39mm or 43mm and a width of 7mm are alternatively attached to the side walls of the sliding short, as shown in Fig. 3. These types are operated as a choke when the sliding short is inserted into the waveguide.

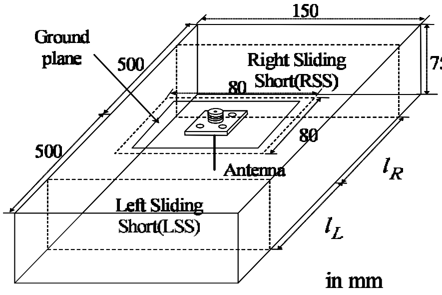

Fig. 2: Rectangular waveguide with a squared aperture

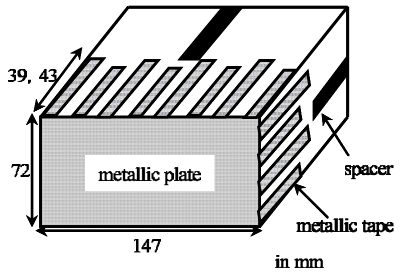

Fig. 3: Sliding short

3) Antenna: In this paper, a monopole antenna with a length of 40mm and a diameter of 1mm is used. The antenna is mounted on a grounded plate with an area of 120mm×120mm.

4) Network Analyzer: S parameters are measured by a network analyzer, Agilent 8720ES. The number of averaging is 32 in the measurement.

3. Analysis on Radiation Efficiency using the Transmission Line Model

The movable shorts can be considered as small resistance, whose normalized resistance is denoted as r_c ($r_c \ll 1$). l_L or l_R represents the distance from the center of the antenna to the left or right movable short. Then, the measurement system can be equivalently represented by a transmission line model as shown in Fig. 4. The normalized admittance looking from the port 2 of the antenna network is given by

$$y_n = \frac{1 + jr_c \tan\beta_g l_L}{r_c + j\tan\beta_g l_L} + \frac{1 + jr_c \tan\beta_g l_R}{r_c + j\tan\beta_g l_R}, \quad (9)$$

where $\beta_g (= 2\pi/\lambda_g)$ is the phase constant and λ_g is the guide wavelength for the TE_{10} mode of the air-filled rectangular waveguide. The corresponding reflection coefficient can be given by

$$\Gamma_n = \frac{1 - y_n}{1 + y_n}. \quad (10)$$

The authors clarify that the drop in the efficiency evaluated by use of (4) leads to small radius of the circle $\Gamma_{wg,n}$ on the Smith chart. As we can see from (3), the small raduis of $\Gamma_{wg,n}$ leads to small radius or shifted center of the circle Γ_n.

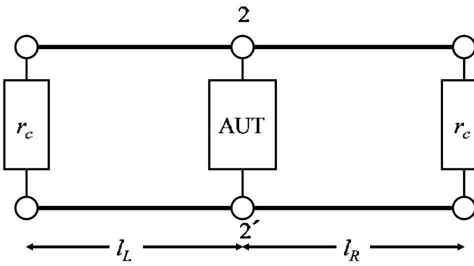

Fig. 4: An equivalent transmission line model

To examine these effects, we derive the center z_i and radius r_i of the circle Γ_n and the efficiency in the section of the sliding shorts, η_{line} in terms of the transmission line model. In the following, we will concentrate two methods for avoiding the drops in the efficiency which we have proposed before.

A. One Sliding Short is Fixed and the Other is Movable

A problem in the evaluation of the radiation efficiency in accordance with the principle of the reflection method is to take a considerable amount of time to measure the reflection coefficients for many positional combinations of two sliding shorts. To overcome this problem, the authors have proposed to evaluate the radiation efficiency with a sliding short fixed and the other sliding short moved.

The above situation can be analytically modeled by the transmission line model. For example, if the left sliding short is fixed, or, l_L is constant and the admittance looking to the left side from the antenna is denoted by y_L as

$$y_L = \frac{1 + jr_c \tan\beta_g l_L}{r_c + j\tan\beta_g l_L}. \quad (11)$$

Since l_L is an arbitrary variable, Γ_n draws a circle on the Smith chart. The center z_i and radius r_i of the circle are given by

$$z_i = -\frac{2r_c|y_L|^2 + (1+r_c)^2 y_L + (1-r_c)^2 y_L^*}{2r_c|y_L|^2 + (1+r_c)^2(y_L + y_L^*) + 2(1+r_c)^2}, \quad (12)$$

$$r_i = \frac{2(1-r_c^2)}{2r_c|y_L|^2 + (1+r_c)^2(y_L + y_L^*) + 2(1+r_c)^2}. \quad (13)$$

Then, from (8), the efficiency for the movable short-circuited line η_{line} is given by

$$\eta_{\text{line}} = -\frac{2r_c|y_L|^2 + (1-r_c)^2(y_L + y_L^*) - 2(1-r_c^2)}{2(1-r_c^2)} = \frac{1 - r_c + r_c^2}{1 - r_c^2} \frac{r_c(r_c - 2) + (1 - 2r_c)\tan^2\beta_g l_L}{r_c^2 + \tan^2\beta_g l_L}. \quad (14)$$

Next, we explain corresponding experimental results. In this measurement, the left sliding short is fixed and the right sliding short is moved from $l_R = 60$mm to $l_R = 130$mm at 10mm intervals and then the reflection coefficients of the antenna inserted into the waveguide are measured. Fig. 5 shows the relation between the frequency and the total efficiency η_{net} for various l_Ls. As we can see from the figure, the drops of the efficiency are observed at 1.5GHz, 1.6GHz, 1.7GHz,

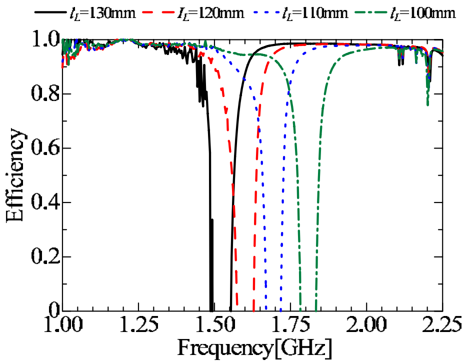

Fig. 5: Measured efficiency η_{net} of 40mm monopole when the left sliding short is fixed

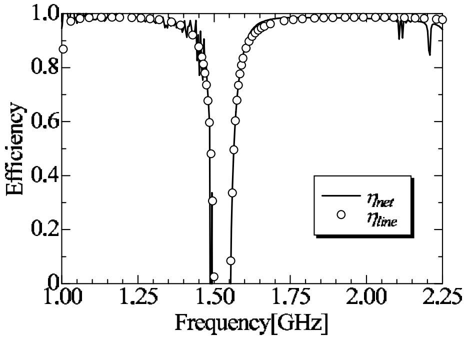

Fig. 6: Comparison between η_{line} and η_{net} for l_L =130mm

and 1.8GHz for l_L = 130mm, 120mm, 110mm and 100mm, respectively. Thus, we can see that the frequency for the drop of the efficiency is higher, as l_L is smaller, or, the left sliding short is closer to the antenna.

Fig. 6 shows η_{net} and η_{line} as a function of the frequency. η_{net} can be evaluated in accordance with the principle of the reflection method, whereas η_{line} can be estimated by (14) with $r_c = 0.003$. The value of the normalized resistance r_c can be determined by the least-square method. As we can see from Fig. 6, η_{net} denotes the same tendency of η_{line}. Then, we can infar from the difference between η_{net} and η_{line} at frequencies without drops that the true radiation efficiency is almost unity, so that the behavior, especially, the sharp falls in the measured efficiency can be determined by the loss at the cavity wall and the cavity resonance. Furthermore, Fig. 6 gives us the validity of the proposed transmission line model.

The range for $\eta_{line} \geq 0$ can be determined by (15), as follows:

$$|\tan\beta_g l_L| \geq \sqrt{\frac{r_c(2-r_c)}{1-2r_c}} \qquad (15)$$

The above inequality means that we should select the proper

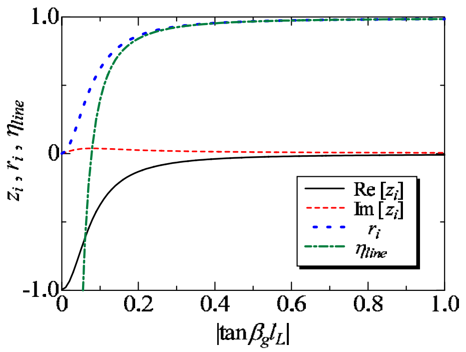

Fig. 7: The center z_i and radius r_i of the circle Γ_n, and η_{line} versus $\tan\beta_g l_L$

distance between the antenna and the left sliding short, l_L, if the efficiency for the section of the sliding shorts is positive. The above inequality also shows that the drop in the efficiency can be observed in a certain frequency band. This fact confirms our experimental results as shown in Fig. 5. And the deepest fall in the efficiency corresponds to the minimum of (14) which is negative and given by

$$\eta_{line}|_{min} = \frac{1-r_c+r_c^2}{1-r_c^2}\,\frac{r_c-2}{r_c} < 0. \qquad (16)$$

The above can be obtained by substituting $\tan\beta_g l_L = 0$ into (14). When $\tan\beta_g l_L = 0$ or $l_L = n\lambda_g$ (n:integer), the antenna is located at the null points of the standing wave.

Fig. 7 shows the center z_i and the radius r_i of the circle Γ_n and the efficiency in the section of the sliding shorts η_{line} as a function of $\tan\beta_g l_L$. When $\tan\beta_g l_L \geq 0.077$, $\eta_{line} \geq 0$. As the value of $\tan\beta_g l_L$ is larger, the efficiency is more stable in the sense that the efficiency is not affected by the loss of the shorts. On the other hand, the efficiency becomes negative or unstable, when $\tan\beta_g l_L = 0$. To examine the effect of selecting $\tan\beta_g l_L$ in detail, Fig. 8 shows the efficiency as a function of the frequency for l_L = 130mm and the locus of the circle Γ_n on the Smith chart. For example, $\tan\beta_g l_L = 1$ corresponds to 1.326GHz or 1.750GHz where η_{net} is nearly equal to unity. And the locus of Γ_n coincides with unit circle on the Smith chart so that the valid efficiency can be obtained. When $\tan\beta_g l_L = 0.077$, the radius of the circle is smaller than unity so that the evaluation of the efficiency is unstable. When $\tan\beta_g l_L = 0$, Γ_n is focused at one point on the Smith chart, or, the circle can not be determined so that the efficiency is indeterminable. As mentioned above, we can find that the efficiency is more stable as the value of $|\tan\beta_g l_L|$ is larger.

B. Both Sliding Shorts are Movable

To avoid the falls in the efficiency due to the loss of the wall at the cavity resonance, we have proposed that two sliding shorts are moved with $l_L = l_R$. This is because the antenna is located at the maxima or minima of the standing wave, then the magnitude of the reflection coefficient is equal to unity.

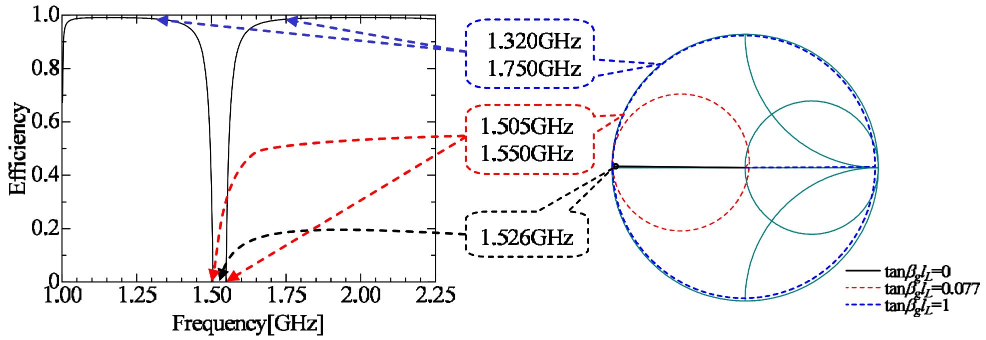

Fig. 8: The relationship between Γ_n and η_{linc}

Since $|\Gamma_n| = 1$, the falls in the efficiency due to the loss of the sliding shorts can not be observed.

The above situation can be analytically modeled by the transmission line model. When $l = l_L = l_R$, the admittance looking from the antenna is given by

$$y_n = 2\frac{1 + jr_c \tan\beta_g l}{r_c + j\tan\beta_g l}. \tag{17}$$

Since l is an arbitrary variable, Γ_n draws a circle on the Smith chart. The center z_i and radius r_i of the circle are given by

$$z_i = -\frac{(3/2)r_c}{1 + (5/2)r_c + r_c^2} \tag{18}$$

$$r_i = \frac{1 - r_c^2}{1 + (5/2)r_c + r_c^2} \tag{19}$$

Then, from (8), the efficiency for the movable short-circuited line, η_{line}, is given by

$$\eta_{\text{line}} = \frac{1 - (5/2)r_c + r_c^2}{1 - r_c^2} \tag{20}$$

The efficiency in the section of the sliding shorts η_{line} is independent of the frequency and the distance between the antenna and the short, and is constant which is nearly equal to unity because of $r_c \ll 1$. That is, although a slight loss due to the resistance of the shorts can be produced, the falls in the efficiency can not be observed.

Next, we explain corresponding experimental results. In this measurement, two sliding shorts are moved from $l = 60$mm to $l = 130$mm at 10mm intervals and then the reflection coefficients of the antenna inserted into the waveguide are measured. Fig. 9 shows the efficiency η_{net} in accordance with the above method as well as the radiation efficiency η_{ant} and the efficiency in the section of the sliding shorts η_{line}, whose value is estimated to be 0.99 when $r_c = 0.003$. As we can see from Fig. 9, we can confirm that the shrap fall in the efficiency can not be observed.

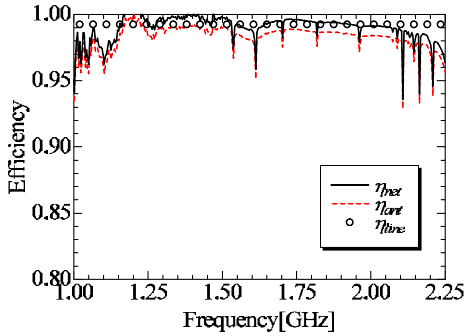

Fig. 9: η_{net}, η_{ant}, η_{linc} versus frequency in the condition of $l_L = l_R$

4. Conclusion

A serious problem in measuring the radiation efficiency of the small antenna in accordance with the reflection method is the drop in the resulting efficiency at and near the resonant frequency of the cavity which is formed by the waveguide and two sliding shorts. In this paper, we introduce the transmission line model including the wall resistance of the cavity, and we analytically clarify that the measured efficiency is equal to the product the true radiation efficiency of the antenna and the efficiency in the section of the sliding shorts. Then, for two ways to avoid the fall in the efficiency, we rigorously derive the expressions for the efficiency in the section of the sliding shorts. We can also see that the drop appears in the efficiency when the radius of the circle which is drawn by the locus of the reflection coefficients is much smaller than unity in the Smith chart. As is well known, this is because the resistance of the wall is larger as the cavity is resonant. This interpretation is consistent with the other researcher's works. By estimating the resistance of the walls properly, we can predict not only the frequency but also the frequency band for the drop in the efficiency. These results can support our transmission line model for the sliding shorts in the reflection method.

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Appendix

When $z = \Gamma_{\mathrm{wg},n} - S_{11}$, $a = S_{22}$, $b = S_{21}^2$, (2) can be reduced to

$$\Gamma_n = \frac{z}{az+b}. \tag{21}$$

The circle Γ_n has a center of z_i and a radius of r_i, then it can be denoted as $\Gamma_n = z_i + r_i e^{j\theta_n}$, where θ_n is real. By substituting Γ_n into (21) and solving for $e^{j\theta_n}$, then

$$e^{j\theta_n} = \frac{(1-az_i)z - bz_i}{ar_i z + br_i} = \frac{Az+B}{Cz+D}, \tag{22}$$

where $A = 1 - az_i$, $B = -bz_i$, $C = ar_i$, $D = br_i$. Note that $|e^{j\theta_i}|^2 = 1$, (22) can be reduced to

$$\left| z - \frac{-A^*B + C^*D}{|A|^2 - |C|^2} \right|^2 = \left| \frac{AD - BC}{|A|^2 - |C|^2} \right|^2. \tag{23}$$

The circle $\Gamma_{\mathrm{wg},n}$ has a center of $S_{11} + z_o$ and a radius r_o, then

$$z_o = \frac{-A^*B + C^*D}{|A|^2 - |C|^2} = b\frac{z_i - a^*(r_i^2 - |z_i|^2)}{|1 - az_i|^2 - |r_i a|^2}, \tag{24}$$

$$r_o = \left| \frac{AD - BC}{|A|^2 - |C|^2} \right| = \frac{|b| r_i}{|1 - az_i|^2 - |r_i a|^2}. \tag{25}$$

Solving (25) for $|b|$,

$$|b| = \frac{r_o}{r_i}\left(|1 - az_i|^2 - |r_i a|^2\right). \tag{26}$$

Therefore, we can find $|z_o|$ from (24) as follows:

$$|z_o| = \frac{r_o}{r_i}\left| z_i - a^*(r_i^2 - |z_i|^2) \right|. \tag{27}$$

Introducing the phase of z_o, ϕ, we can find a as follows:

$$a = \frac{1}{r_i^2 - |z_i|^2}\left(z_i^* - r_i \frac{|z_o|}{r_o} e^{-j\phi} \right). \tag{28}$$

Then,

$$1 - az_i = \frac{r_i}{r_i^2 - |z_i|^2}\left(r_i - z_i \frac{|z_o|}{r_o} e^{-j\phi} \right), \tag{29}$$

(26) can be given by

$$|b| = \frac{r_o}{r_i}\left\{ \left| \frac{r_i}{r_i^2 - |z_i|^2}\left(r_i - z_i \frac{|z_o|}{r_o} e^{-j\phi} \right) \right|^2 - \left| \frac{1}{r_i^2 - |z_i|^2}\left(z_i^* - r_i \frac{|z_o|}{r_o} e^{-j\phi} \right) \right|^2 r_i^2 \right\}. \tag{30}$$

In the above expression, $|b|$ in the left-hand side corresponds to $|S_{21}|^2$ and the right-hand side can be reduced to (5).