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THE ELECTROMAGNETIC WAVE DIFFRACTION FROM A DIELECTRIC COATED FOURIER GRATING

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1 Introduction

In order to prevent the oxidization by air, the metallic Fourier grating is composed of the coated dielectric layer on the metallic surface of grating[1]. There are a few influence in the diffraction characteristics, that is, it is better than thin coating in comparison with wavelength for protection membrane.

The resonance absorption by surface plasmon anomalies exists that the energy of the incident wave in diffraction efficiency by metallic Fourier grating with dielectric coating medium has been absorbed[3]. This resonance absorption depends on oscillation of the surface wave and the guided wave toward the period of the grating that has been studied in details for the past decay[1]-[3].

On the other hand, the absorption of incident light wave is confirmed, because the energy is transmitted toward the period of the grating with the surface wave in a coating dielectric medium. We must study interesting phenomena of the diffraction characteristics of these surface waves in the metallic Fourier grating with the dielectric coating media because the resonance absorption anomalies by coupling diffracted evanescent wave of the -1th mode with surface plasmon wave occur[3].

In this paper, we examine the mechanism of the incident light wave that is absorbed in the grating. The rigorous formulation of the diffraction problem by the metallic Fourier grating with large thickness of the dielectric coating is described by using the T-matrix analysis[6] with the R-matrix expression[2].

2 Analysis

2.1 Geometry of the problem

Let us consider the electromagnetic wave diffraction from a dielectric coated metallic Fourier grating illuminated by a plane wave. The dielectric coated Fourier grating structure and the geometry of the problem are shown in Fig.1. We assume a two-dimensional problem where the surface vary periodically in the x direction and does not vary in the y direction, the plane of incidence is the $x-z$ plane for the incident angle θ_{inc}.

The incident region and substrate of the grating are filled up by material of homogeneous isotropic medium (permittivity ε_0 and ε_2,the permeability μ_0 and μ_2), respectively, and also dielectric coating (uniform medium for the permittivity ε_1 and the permeability μ_1) is considered.

When d means the thickness of the dielectric coating, the surface profile of the dielectric coating and Fourier grating yield $f(x)+d$ and $f(x)$, respectively. While two surfaces have same smooth profile, the Fourier grating is expressed by

$$f(x) = -h\{\cos(Kx) + \gamma\cos(2Kx+\delta)\} \qquad (1)$$

where $K = 2\pi/P$, P is the period of Fourier grating, h is the amplitude of fundamental sinusoidal wave, $h\gamma$ and δ are the amplitude and phase of second hormonic wave, respectively.

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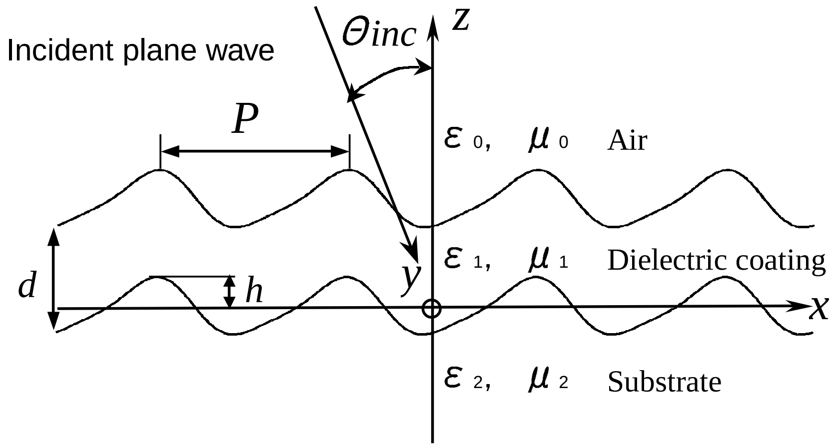


Figure 1: Geometory of the problem

2.2 Formulation by using extinction theorem

Here, we obtain the two-dimensional Helmholtz equation when the Maxwell's equations are derived in the rectangular coordinates for the each region as $\partial/\partial y \equiv 0$, therefore, electric and magnetic field do not vary in the y direction, and time dependence $\exp(j\omega t)$ is suppressed throughout in this paper.

$$\frac{\partial^2\psi_i}{\partial x^2}+\frac{\partial^2\psi_i}{\partial z^2}+k_i^2\psi_i=0 \quad \psi_i=\begin{cases} E_{iy} & : \text{ TE wave} \\ H_{iy} & : \text{ TM wave}\end{cases} \quad (i=0,1,2 : \text{ numbering of the media}) \tag{2}$$

where, $k_i(=\omega\sqrt{\varepsilon_i\mu_i})$ denotes propagation constant in the ith medium, ω is the angular frequency. We yield wave number k_{xm} of the diffraction grating satisfying condition for mth mode number in the x direction as $k_{xm}=k_0\sin\theta_{inc}+2\pi m/P$, where θ_{inc} is the incident angle, above wave number k_{xm} is also replaced by $k_{xm}=k_i\alpha_{im}$. Also, we define wave number k_{izm} satisfying the radiation condition in the z direction, it is repalced by $k_{izm}=k_i\beta_{im}$, that is $\beta_{im}=(1-\alpha_{im}^2)^{1/2} \quad 1\geq\alpha_{im}^2$, $-j(\alpha_{im}^2-1)^{1/2} \quad 1\leq\alpha_{im}^2$ in radiation condition. The incident wave ψ^{inc}, fields ψ_0 in the incident medium (air), fields ψ_2 in the substrate, and fields ψ_1 in the coating medium are defined, respectively. In the each region, the electric and magnetic fields satisfy the integral representations by applying extinction theorem[6].

The two-dimensional Green function with satisfying periodic property is represented as Fourier series. Then, we apply the boundary conditions on the surface.Furthermore, fields expression by using Fourier sires expansion on surface of the grating is assumed.

2.3 T-matrix formulation

In order to find T-matrix formulation by using R-matrix expression, the immedate coefficients are eliminated in the form of the recombined matrix. The reflected coefficient b_m and transmitted coefficient are finally represented for the incident wave coefficients a_m and $B_m\equiv 0$ satisfying radiation condition in the substrate. The total T-matrix formulation $[T_{mn}]$ with R-matrix expression is finally obtained, that is

$$\begin{bmatrix} b_m \\ A_m \end{bmatrix} = [T_{mn}]\begin{bmatrix} a_m \\ B_m \end{bmatrix} \tag{3}$$

where

$$[T_{mn}]=\begin{bmatrix} X_{11}r_{11}+X_{12} & X_{11}r_{12} \\ Y_{21}r_{21} & Y_{21}r_{22}+Y_{22}\end{bmatrix}\begin{bmatrix} X_{21}r_{11}+X_{22} & X_{21}r_{12} \\ Y_{11}r_{21} & Y_{11}r_{22}+Y_{12}\end{bmatrix}^{-1}, \tag{4}$$

$$\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} -\zeta_1^+ Q_N^+(k_1) & \zeta_1^- Q_N^+(k_1) \\ -\zeta_1^- Q_N^-(k_1) & \zeta_1^+ Q_N^-(k_1) \end{bmatrix}^{-1} \begin{bmatrix} \nu_1 \dfrac{k_0}{k_1} \zeta_1^+ Q_D^+(k_1) & -\zeta_1^- Q_D^+(k_1) \\ \nu_1 \dfrac{k_0}{k_1} \zeta_1^- Q_D^-(k_1) & -\zeta_1^+ Q_D^-(k_1) \end{bmatrix}, \tag{5}$$

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = \begin{bmatrix} -Q_D^+(k_0) & -Q_N^+(k_0) \\ Q_D^-(k_0) & Q_N^-(k_0) \end{bmatrix}, \quad \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -\nu_2 \dfrac{k_1}{k_2} Q_D^+(k_2) & -Q_N^+(k_2) \\ \nu_2 \dfrac{k_1}{k_2} Q_D^-(k_2) & Q_N^-(k_2) \end{bmatrix}, \tag{6}$$

and also $\zeta_1^{\pm} = \exp(\pm jk_{1zm}d)$, $\nu_i = \mu_i/\mu_{i-1}$(:TE wave), $\varepsilon_i/\varepsilon_{i-1}$(:TM wave), $(i = 1, 2)$.

In Eqs.(5) and (6), the elements of the Dirichlet matrices $Q_D^{\pm}$ and Neumann matrices $Q_N^{\pm}$ can be analytically expressed by the Bessel functions

$$Q_D^{\pm}(k_i) = \frac{-1}{\sqrt{k_{izm}}} \sum_{l=-\infty}^{\infty} \exp\left\{\mp jl(\frac{\pi}{2} + \delta\right\} (\mp j)^{|n-m\pm 2l|} J_{|n-m\pm 2l|}(k_{izm}h) J_l(k_{izm}h\gamma) \tag{7a}$$

$$Q_N^{\pm}(k_i) = \left[\frac{1 - \alpha_{in}\alpha_{im}}{\pm\beta_{im}}\right] Q_D^{\pm}(k_i) \quad , \quad (i = 0, 1, 2) \tag{7b}$$

This is Fourier grating whose profile is expressed by summation of two sinusoidal functions, for which are rigorously formulated in this paper. Also, the expression of the sinusoidal grating is obtained by setting $\gamma = 0$ in Eq.(1), that is, analysis of the electromagnetic diffraction from a dielectric coated sinusoidal grating has already been done[**?**].

2.4 Diffraction efficiencies

The diffraction power of the reflected, transmitted waves and the incident power are obtained by calculating from time average of the Poynting vector in the each mode. By using the normalized power having the incident wave, we define diffraction efficiencies of the reflected ρ_m^r and the transmitted ρ_m^t waves for the mode number m, $\rho_m^r = |b_m|^2$, $\rho_m^t = \dfrac{k_2}{k_0}\dfrac{1}{\nu_2\nu_1}|A_m|^2$ where b_m is the expansion coefficient of reflected wave, A_m is the expansion coefficient of transmitted wave. Therefore, the total reflected ρ_{total}^r and transmitted ρ_{total}^t diffraction efficiencies are also defined by summing possible modes for the propagating $\rho_{total}^r = \sum_m \rho_m^r \, (m : \quad Re(\beta_{0zm}) > 0)$, $\rho_{total}^t = \sum_m \rho_m^t \, (m : \quad Re(\beta_{2m}) - Im(\beta_{2zm}) > 0)$.

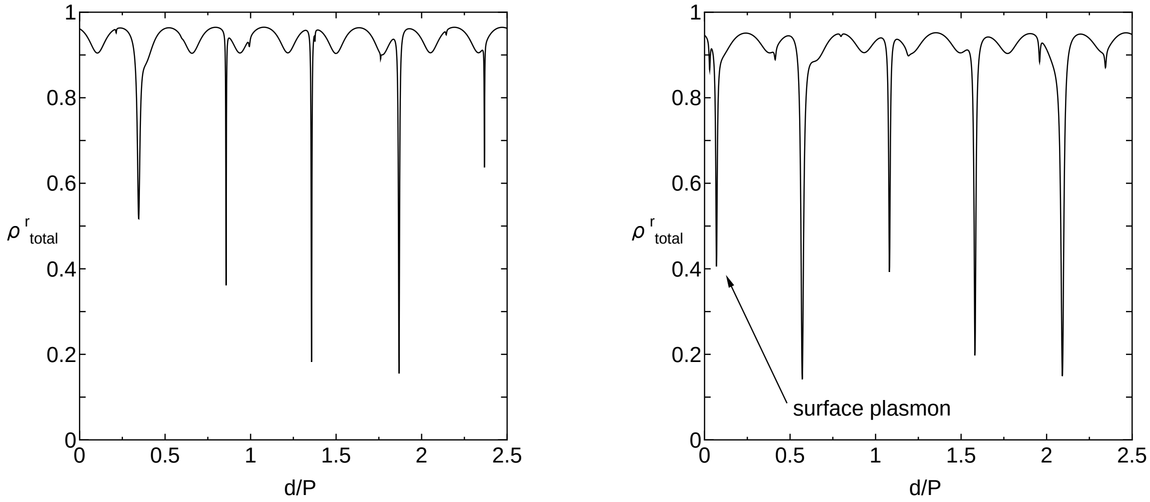
If the medium of all regions is the perfect dielectrics without loss, the energy conversion does supply a good numerical consistency check. The percentage power error ε_{err} is, then, defined by $\varepsilon_{err} = |1 - (\rho_{total}^r + \rho_{total}^t)| \times 100[\%]$

In the perfect dielectrics of all regions, thus, the energy conversion is calculated from time average of the Poynting vector.

3 Numerical examples and discussion

We check the limitation of this formulation compared with a conventional T-matrix method[4]. In this paper, the parameters, $P = 800nm$, $h = 20nm$, $\gamma = 0.2$ and $\delta = \pi/2$, $\theta_{inc} = 30°$ and wavelength $\lambda = 650nm$, substarte(Au , $n_2 = 0.142 - j3.374$), dielectric coating(S_iO, $n_1 = 1.54$) are discussed.

Fig.2(a) shows the metallic Fourier grating with the coating dielectric medium when the TE plane wave with incident angle $\theta_{inc} = 30°$ is illuminated in incidence medium (air, $n_0 = 1.0$). Fig.2(b) also shows the metallic Fourier grating with the coating dielectric medium for the TM plane wave. The total reflected diffraction efficiency $\rho_{total}^r(= \sum |b_m|^2)$ is illustrated versus thickness d of the coating layer with period P, such as, d/P. The numerical results by T-matrix method with R-matrix expression are confirmed in good agreement with the results by S-matrix method[4] in the special case of a sinusoidal profile ($\gamma = 0$) with incident angle ($\theta_{inc} = 30°$), however, the figures are omitted here.



(a) TE wave ($\theta_{inc} = 30°$) (b) TM wave($\theta_{inc} = 30°$)

Figure 2: The total reflected diffraction efficiency ρ^r_{total} versus thickness d/P.

4 Conclusion

The rigorous formulation of the diffraction problem by the metallic Fourier grating with coating dielectric layer using the T-matrix method in R-matrix expression is described. In the numerical examples, the absorption phenomena occur in a certain particular incidence angle depends on existence of guided mode. We found that other absorption arises from the resonance absorption by surface plasmon anomalies for TM wave incidence.

We can possible applying to numerical calculation for the dielectric coating Fourier grating that thickness of dielectric layer is thick and groove depth is a large although the thin coating layer.

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